# (DME 221)

## B. Tech. DEGREE EXAMINATION, MAY - 2015

## (Examination at the end of Second Year)

## MECHANICAL ENGINEERING

## Paper - I : Engineering Mathematics - IV

**Time : 3 Hours** **Maximum Marks : 75**

***Answer question No.1 compulsory*** ***(15)***

***Answer ONE question from each unit*** ***(4 × 15 = 60)***

***1)*** a) Define one dimensional wave equation.

b) What are the possible solutions of the wave equation.

c) Write Laplace equation in Cartesian form.

d) Define Cauchy Riemann equations in both Cartesian and polar coordinates.

e) Define harmonic function and conjugate of harmonic function.

f) Define poisson's integral formula.

g) Find the Laurent's expansion of $z^2 e^{½}$ with center o.

h) Write Laurent's series.

**UNIT - I**

***2)*** A string is stretched and fastened to two points $l$ apart motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$ show that the displacement of any point at a distance x from one end at time $t$ is given by $u(x,t) = a \sin\frac{\pi x}{l} \cos\left(\frac{\pi t}{l}\right)$.

OR

***3)*** Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with the boundary conditions.

$u(0, y) = u(\pi, y) = 0$ for ally

$u(x, \infty) = 0$ in $0 < x < \pi$

$u(x, 0) = 40 \quad 0 < x < \pi$

## UNIT – II

**4)** a) Evaluate $\oint_c \frac{e^{tz}}{z^2+1} dz$ where c is |z| = 3.

b) Show that $v(x, y) = -\sin x \sinh y$ is harmonic find the conjugate harmonic of V.

OR

**5)** a) Using the Cauchy's integral formula $\oint_c \frac{\cos \pi z}{z^2-1} dz$ around a rectangle with vertices 2$\pm$ i, - 2 $\pm$ i

b) Compute $\int_0^{\infty} \frac{dx}{(a^2+x^2)^2}$.

## UNIT - III

**6)** Evaluate $\int_{-\infty}^{\infty} \frac{\cos mx}{(x^2+a^2)(x^2+b^2)} dx$.

OR

**7)** State and prove Laurent's series.

## UNIT – IV

**8)** Show that $W = \frac{i-z}{i+z}$ maps the real axis of z plane into the cirde |w| = 1 and the half plane y > 0 into interior of unit cirde |w| = 1 in the w – plane.

OR

**9)** Prove that cross ratio of four points is invariant under bilinear transformation.

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