



4. From the first principle, find the value of $\int_0^1 (x^3 + 4) dx$.
5. Show that $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$.
6. Verify Euler's theorem for $u(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$.
7. Find the equation of the normal to the curve $y = 2x^3 - 3x + 5$ at the point $(1, 4)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 3 & -2 \end{pmatrix}$.
- b) Find the equation of the hyperbola whose vertices are $(2, 3)$, $(8, 3)$ and the distance between foci is 12.
- c) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$.
9. a) Find $\frac{dy}{dx}$ when $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$.
- b) Evaluate : $\int \frac{xe^x}{(1+x)^2} dx$.
- c) If $x = \sin t$, $y = \cos pt$,
prove that $(1-x^2)y'' - xy_1' + p^2y = 0$.



10. a) Evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$.

- b) Find the equations of the tangent and normal to the curve $y = x^2 - 4x - 2$ at the point $(5, 3)$. 3 + 2
- c) Find the area of the region bounded by $y = x^2$, y -axis and the straight line $y = 4$.

11. a) If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log_e x}{(1 + \log_e x)^2}$.

- b) Determine the matrices A and B when

$$2A + B = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix} \text{ \& \ } d \ 2A - B = \begin{pmatrix} 2 & 1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

- c) If $v = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$, then show that

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = \tan v.$$

12. a) Find the maximum value of $\left(\frac{1}{x}\right)^x$.

- b) Verify that the function $f(x)$ as defined below is continuous or not at $x = 1$:

$$f(x) = \begin{cases} x^2 + 2 & , x > 1 \\ 3 & , x = 1 \\ 2x + 1 & , x < 1 \end{cases}$$

c) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$, then

find AB .



13. a) Find y_n , if $y = (2 - 3x)^n$.

b) Find the value of $\lim_{x \rightarrow 0} \frac{7^x - 5^x}{5^x - 3^x}$.

c) Express the matrix A as a sum of a symmetric and a skew-symmetric matrix, where

$$A = \begin{pmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix}.$$

