

Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics - III

Time: 3 hrs.

Max. Marks: 100

Note : Answer any FIVE full questions choosing at least TWO from each part.

Part A

- 1 a. Find the Fourier series for the function $f(x) = x + x^2$ from $x = -\pi$ to $x = \pi$ and deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (07 Marks)
- b. Obtain the cosine half-range Fourier series for $f(x) = Kx$, in $0 \leq x \leq \frac{l}{2}$
 $= K(l-x)$ in $\frac{l}{2} \leq x \leq l$ (07 Marks)
- c. The following table gives the varying of periodic current over a period:
- | | | | | | | | |
|---------|------|-------|-------|-------|--------|--------|------|
| t (sec) | 0 | $T/6$ | $T/3$ | $T/2$ | $2T/3$ | $5T/6$ | T |
| A (Amp) | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |
- Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (06 Marks)
- 2 a. Obtain the finite Fourier Cosine transform of the function $f(x) = e^{ax}$ in $(0, l)$. (07 Marks)
- b. Find the Fourier sine and cosine transforms of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ (07 Marks)
- c. Solve the integral equation, $\int_0^{\infty} f(x) \cos(\alpha x) dx = \begin{cases} 1-\alpha, & 0 \leq \alpha \leq 1 \\ 0, & \alpha > 1 \end{cases}$.
Hence evaluate $\int_0^{\infty} \frac{1-\cos x}{x^2} dx$. (06 Marks)
- 3 a. Form the P.D.E by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (07 Marks)
- b. Solve $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$ by the method of separation of variables. (07 Marks)
- c. Solve $(y^2 + z^2)p + x(yq - z) = 0$. (06 Marks)
- 4 a. Derive the one dimensional heat equation. (07 Marks)
- b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0$; $u(l, t) = 0$; $\frac{\partial u}{\partial t} = 0$ when $t = 0$
and $u(x, 0) = u_0 \sin \frac{\pi x}{l}$. (07 Marks)
- c. Obtain the various possible solutions of the Laplace's equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. (06 Marks)

Part B

- 5 a. Find the real root of the equation $3x = \cos x + 1$ correct to four decimal places using Newton's method. (07 Marks)
- b. Solve the system of equations,
 $2x + y + z = 10$
 $3x + 2y + 3z = 18$
 $x + 4y + 9z = 16$
 by Gauss-Jordan method. (07 Marks)
- c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method. (07 Marks)
- $$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$
- Taking $[1 \ 0 \ 0]^T$ as the initial eigen vector. Carry out four iterations. (06 Marks)
- 6 a. Given $f(0) = 1$, $f(1) = 3$, $f(2) = 7$, $f(3) = 13$. Find $f(0.1)$ and $f(2.9)$ using Newton Interpolation formula. (07 Marks)
- b. Using Newton's divided difference formula evaluate $f(8)$ and $f(15)$, given that (07 Marks)
- | | | | | | | |
|------|----|-----|-----|-----|------|------|
| x | 4 | 5 | 7 | 10 | 11 | 13 |
| f(x) | 48 | 100 | 294 | 900 | 1210 | 2028 |
- c. Evaluate $\int_4^{5.2} \log_e x dx$ by using Weddle's rule, taking 7 ordinates. (06 Marks)
- 7 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (07 Marks)
- b. Find the extremal of the functional $\int_0^{\pi/2} [y^2 - (y')^2 - 2y \sin x] dx$ under the conditions $y(0) = y(\pi/2) = 0$. (07 Marks)
- c. Find the geodesics on a surface, given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x[1 + (y')^2]} dx$. (06 Marks)
- 8 a. Find the z-transforms of i) $(n+1)^2$ ii) $\sin(3n+5)$. (07 Marks)
- b. Obtain the inverse Z transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (07 Marks)
- c. Solve the difference equation, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z transforms. (06 Marks)