11-11-13-DTP7-RM-24 Sem III / App. maths - III / mech / Prod.

Con. 7845-13. **GX-12025**

(3 Hours) 25-11-13 **[Total Marks : 80**

N.B. : (1) Question No 1 is Compulsory
(2) **Answer** any **Three** from remaining.
(3) **Figures** to the **right** indicate **marks**.

1. (a) Find laplace of $\sin\sqrt{t}$ **5**

(b) Show that the set of functions $\mathrm{Sin}\left(\frac{\pi x}{2L}\right)$, $\mathrm{Sin}\left(\frac{3\pi x}{2L}\right)$, $\mathrm{Sin}\left(\frac{5\pi x}{2L}\right)$ is orthogonal over (O, L). **5**

(c) Show that $u = \sin x \cos hy + 2\cos x \sin hy + x^2 - y^2 + 4xy$ Satisfies laplace equation and find its corresponding analytic function $f(z) = u + iv$. **5**

(d) Determine constants a,b,c,d if $f(z) = x^2+2axy+by^2 + i(cx^2+2dxy+y^2)$ is analytic. **5**

2. (a) Find complex form of fourier series $f(x) = e^{3x}$ in $0<x<3$. **6**

(b) Using Crank Nicholson Method solve $u_t = u_{xx}$ subject to $u(x,0) = 0$ $u(0, t)=0$ and $u(l,t) = t$ for two time steps. **6**

(c) Solve using laplace transforms $\frac{d^2y}{dt^2} + y = t$, $y(0) = 1$, $y^1(0) = 0$. **8**

3. (a) Find bilinear transformation that maps the points $0, 1-\infty$ of the z plane into -5, -1, 3 of w plane. **6**

(b) By using Convolution Theorem find inverse laplace transform of $\frac{1}{(S^2+4S+13)^2}$ **6**

(c) Find fourier series of $f(x) = x^2$ $-\pi \le x \le \pi$ and prove that **8**

(i) $\frac{\pi^2}{6} = \sum_{1}^{\infty} \frac{1}{n^2}$

(ii) $\frac{\pi^2}{12} = \sum_{1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

(iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

4. (a) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t}\, dt$ **6**

(b) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$ by **6**

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Bender schmidt method subject to conditions $u(o, t) = 0$ $u(x, 0) = 0$
$u(l, t) = t$ taking $h = 0.25$ $0 < x < 1$

(c) Obtain two distinct Laurent's Series for $f(z) = \dfrac{2z-3}{Z^2 - 4z - 3}$ in Powers of (z-4) indicating Region of Convergence. 8

5. (a) Evaluate $\int_0^{1+i} Z^2 dZ$ along 6

(i) line $y = x$
(ii) Parabola $x = y^2$
Is line independent of path? Explian.

(b) Find half range Cosine Series for $f(x) = e^x$ $0 < x < 1$. 6

(c) Find analytic function 8
$f(z) = u + iv$ such that

$$u\text{-}v = \frac{\text{Cos}x + \text{Sin}x - e^{-y}}{2\text{Cos}x - e^{y} - e^{-y}}$$

when $f(\pi/2) = 0$

6. (a) A tightly streched sting with fixed end points $x = 0$ and $x = \ell$ in the shape defined by $y = K x (l - x)$ where K is a Constant is released from this position of rest. Find y(x,t) The vertical displacement 6

if $\dfrac{\partial^2 y}{\partial t^2} = C^2 \dfrac{\partial^2 y}{\partial x^2}$

(b) Find image of region bounded by $x = 0$, $x = 2$ $y = 0$ $y = 2$ in the z plane under the transformation $w = (1 + i) Z$ 6

(c) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{25 - 16\cos^2\theta}$ 8