

9/5/13

Comp Sem IV

D:sneha /April 2013 (C) 14

APPLIED Mathematics - IV

Con.6426-13.

GS-6897

(3 Hours)

[Total Marks :100

- N.B.** (1) Question No.1 is compulsory.
 (2) Attempt any **four** questions out of remaining six questions.
 (3) Marks to the **right** indicate **full** marks.

1. (a) Check if the following function is harmonic. $f(\gamma, \theta) = \left(\gamma + \frac{a^2}{\gamma} \right) \cos \theta$ 5
 (b) Integrate function $f(z) = x^2 + iy$ from A(1, 1) to B(2, 4) along the curve $x = t, y = t^2$ 5
 (c) Prove that the eigen values of an orthogonal matrix are +1 or -1. 5
 (d) Construct the dual of the following LPP : 5
- Maximize $z = x_1 + 3x_2 - 2x_3 + 5x_4$
 Subject to $3x_1 - x_2 + x_3 - 4x_4 = 6$
 $5x_1 + 3x_2 - x_3 - 2x_4 = 4$
 $x_1, x_2 \geq 0, \quad x_3, x_4$ unrestricted.

2. (a) Evaluate $\oint_c \frac{e^z}{\cos \pi z} dz$ c is the circle $|z|=1$. 6
 (b) Diagonalise the Hermitian matrix $A = \begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$ 6
 (c) Use Simplex method to solve the LPP : 8
 Maximise $z = 1000x_1 + 4000x_2 + 5000x_3$
 Subject to $x_1 + 2x_2 + 3x_3 \leq 14$
 $3x_1 + 2x_2 \leq 14$
 and $x_1, x_2, x_3 \geq 0$

3. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$ using contour integration. 6
 (b) State Cayley-Hamilton theorem. Use it to find A^{-1} and A^4 6

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

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(c) Use Penalty method

to Minimise $z = x_1 + 2x_2 + x_3$ Subject to $x_1 + \frac{x_2}{2} + \frac{x_3}{2} \leq 1$

$$\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0$$

8

4. (a) If $A = \begin{bmatrix} 4 & 3 \\ 7 & 8 \end{bmatrix}$ find A^{100}

6

(b) If $f(z)$ is analytic function,

6

prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$

(c) Use Dual simplex method to Minimise $z = 3x_1 + 2x_2 + x_3 + 4x_4$

8

Subject to

$$2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$$

$$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$$

$$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$$

$$x_1, x_2, x_3, x_4 \geq 0$$

5. (a) Find the bilinear transformation that maps the points 1, $-i$, 2 in z -plane onto the points 0, 2, $-i$ in w -plane.

6

(b) $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ is A^3 derogatory?

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(c) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} d\theta$ where $0 < b < a$

8

6. (a) If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are positive integers, then prove that

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(i) $a + b + c$ is an eigen value of A and(ii) If A is non-singular, one of the eigen values is negative.(b) Find the image of region bounded by $x = 1$, $y = 1$ and $x + y = 1$ under the transformation $w = z^2$

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(c) Use Lagrangian Multiplier Method to Optimise

8

$$z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

$$\text{s.t. } x_1 + x_2 + x_3 = 20$$

$$x_1, x_2, x_3 \geq 0$$

7. (a) Find Laurent's series for the function

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$$f(z) = \frac{1}{(z-1)(z-2)} \text{ in the regions}$$

$$(i) 1 < |z-1| < 2$$

$$(ii) 1 < |z-3| < 2$$

(b) Find the analytic function $f(z)$ whose imaginary part is

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$$e^{-x}[2xy \cos y + (y^2 - x^2) \sin y]$$

(c) Using Kuhn Tucker method,

8

Optimise the function $2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$

$$\text{s. t. } x_1 + x_2 \leq 1$$

$$2x_1 + 3x_2 \leq 6$$

$$x_1, x_2 \geq 0$$