



Name :

Roll No. :

Invigilator's Signature :

CS/B.Sc(H)/BT/GEN/Micro.Bio./MOLBIO/SEM-1/BMT-104/2012-13

2012

BIO-MATHEMATICS-I

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The modulus of the complex number $\frac{(1+i)^2}{3-i}$ is

a) $\sqrt{\frac{2}{5}}$

b) $\sqrt{\frac{1}{5}}$

c) $\frac{2}{5}$

d) $\frac{1}{5}$.

ii) The amplitude of the complex number $\frac{i}{1-i}$ is

a) $\frac{\pi}{4}$

b) $\frac{3\pi}{4}$

c) $\frac{5\pi}{4}$

d) $\frac{7\pi}{4}$.



iii) If x, y are real and $x + iy = -i(-2 + 3i)$ then the values of x any y are

- a) $x = 2, y = 3$ b) $x = -2, y = 3$
 c) $x = 3, y = 2$ d) $x = -2, y = -3$.

iv) The value of $\log_{2\sqrt{3}} 1728$ is

- a) 2 b) 3
 c) 4 d) 6.

v) If $\log_x 81 = 4$, then the value of x is

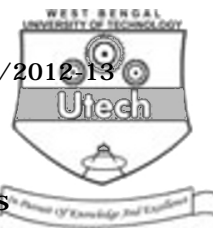
- a) 3 b) 4
 c) 2 d) 1.

vi) The number of terms in the expansion $\left(x^2 - \frac{1}{x^3}\right)^{12}$ is

- a) 13 b) 12
 c) 11 d) 10.

vii) If the coefficient of $(r + 3)^{\text{th}}$ term and $(3r + 2)^{\text{th}}$ terms in the expansion $(1 + x)^{47}$ be equal then the value of r is

- a) 11 b) 10
 c) 12 d) 13.



xi) If $A = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $B = [3 \ 5 \ 7]$ then AB is

a) $[14]$

b) 14

c) $\begin{bmatrix} 6 & 10 & 14 \\ 9 & 15 & 21 \\ -3 & -5 & -7 \end{bmatrix}$

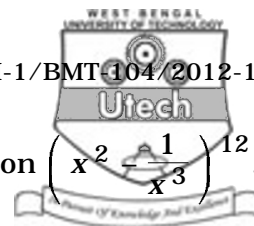
d) $\begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix}$.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If Z_1 and Z_2 be two complex numbers such that $|Z_1| = |Z_2| = 1$ and $\text{amp } Z_1 + \text{amp } Z_2 = 0$, then show that $Z_1 = \frac{1}{Z_2}$.
3. If $Z_r = \cos \frac{\pi}{3^r} + i \sin \frac{\pi}{3^r}$, ($r = 1, 2, \dots$) then prove that $Z_1 Z_2 \dots \text{to } \infty = i$.
4. If $\log \frac{x+y}{5} = \frac{1}{2} (\log x + \log y)$, then show that $\frac{x}{y} + \frac{y}{x} = 23$.



5. Find the coefficient of x^{-11} in the expansion $\left(x^2 - \frac{1}{x^3}\right)^{12}$.

6. Show that

$$\begin{vmatrix} x^2 + y^2 + 1 & x^2 + 2y^2 + 3 & x^2 + 3y^2 + 4 \\ y^2 + 2 & 2y^2 + 6 & 3y^2 + 8 \\ y^2 + 1 & 2y^2 + 3 & 3y^2 + 4 \end{vmatrix} = x^2 y^2$$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the matrix $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal. 5

b) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 2 & 0 & 1 \\ 3 & -4 & -4 & -7 \\ -7 & 5 & 6 & 10 \end{bmatrix}$ by diagonalization method. 5

c) Solve the following equation by Cramer's rule :

$$2x - z = 1$$

$$2x + 4y - z = 1$$

$$x - 8y - 3z = -2 \quad \text{5}$$



8. a) A function $f(x)$ is defined as follows :

$$f(x) = x, \text{ when } 0 < x < 1$$

$$= 2 - x, \text{ when } 1 \leq x \leq 2$$

$$= x - \frac{1}{2}x^2, \text{ when } x > 2.$$

Is $f(x)$ continuous at $x = 2$? 5

b) Find $\frac{dy}{dx}$, if $x^p y^q = (x + y)^{p+q}$. 5

c) If $y = \log \left\{ e^x \left(\frac{x-1}{x+1} \right)^{3/2} \right\}$, find $\frac{dy}{dx}$. 5

9. a) Test whether Rolles Theorem is applicable or not for the function $f(x) = 1 - x^{2/3}$ in $-1 \leq x \leq 1$. 5

b) In the mean value theorem

$$f(b) - f(a) = (b - a) f'(c), \text{ where } a < c < b,$$

find c if $f(x) = Ax^2 + Bx + C$ in $[a, b]$. 5

c) If $u = \phi(Hn)$, where Hn is a homogeneous function in x, y, z of degree n , then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = n \frac{F(u)}{F'(u)}. \quad 5$$



10. a) Integrate : $\int \frac{xdx}{\sqrt{3x^2 + 4}}$. 5

b) Evaluate : $\int_1^2 \left(\frac{x^2 - 1}{x^2} \right) e^{x + \frac{1}{x}} dx$. 5

c) Find the area between the parabolas

$y^2 = 4x$ and $x^2 = 4y$. 5

