Code No.: **5002/M**

FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (Common to all branches) Examination, May/June 2012

MATHEMATICS – I

Time : 3 Hours] [Max. Marks : 75

Answer **all** *questions from Part-A.*
Answer any **five** *questions from Part-B.*

Part A — (Marks : 25)

1. Are these vectors linearly dependent, verify (2, 1, 0), (1, 2, 5), (5,4, 5) 3

2. Find the sum of the Eigen values of the matrix 2

$$A=\begin{bmatrix} 2 & 5 & 3 & 1 \\ 1 & 6 & 3 & 2 \\ 3 & 4 & 1 & 2 \\ 2 & 0 & 0 & 2 \end{bmatrix}.$$

3. Test the convergence of $\sum\left[\frac{(-1)^n}{n(n^2+1)}\right]$. 3

4. Discuss the convergence of $\sum\frac{(n^3+4)n}{(n^2+1)(n^2+4)}$. 2

5. Expand $f(x)$ = tan x about $x=\frac{\pi}{4}$ upto x^4. 3

6. Find the radius of curvature of $x^2 + y^2 = 16$ at any point on this curve. 2

7. Determine $\underset{(x,\,y)\to(1,\,1)}{Lt}\frac{x(y-1}{y(x-1)}$. 3

8. If $Z = \log\,[x^2 + xy + y^2]$ then find $x\frac{jz}{jx}+y\frac{jz}{jy}$. 2

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9. Evaluate $\iint xy^2\, dx$ over the first quadrant of $x^2 + 2^3y^2 = 4$. 3

10. Find the directional derivative of $f(x, y) = x^2 + y^3$ at (1, 1) in the direction of $3i + 4j$. 2

Part B — (Marks : 50)

11. (a) Using Cayley - Hamilton theorem, find the inverse of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -1 \\ 2 & -4 & -4 \end{bmatrix}$. 5

(b) Reduce the quadratic form $2xy + 2yz + 2zx$ to canonical form. 5

12. (a) Discuss the convergence of $\sum_{n=1}^{\infty} \left[\frac{n^3 + \alpha}{2^n + \alpha}\right]$.

(b) Text the series $\sum \frac{[(n+1)x]^n}{n^{n+1}}$ for convergence

13. (a) Verify Lagrange's mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in [0, 4].

(b) Find the envelope of the family of straight lines $x \cos x + y \sin x = p$ where α is α parameter.

14. (a) Find the radius of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{\alpha}$ at $\left(\frac{\alpha}{4}, \frac{\alpha}{4}\right)$.

(b) Find maximum and minimum values of $f(x \cap y) = x^3 + y^3 - 3xy$.

15. (a) Use Green's theorem to evaluate

$\int_c [(2x^2 - y^2)\, dx + (x^2 + y^2)\, dy]$ where c is the boundary of the area enclosed by x-axis and the upper half of circle $x^2 + y^2 = \alpha^2$.

(b) If $\overline{A}$ is a constant vector and $\overline{R} = xi + yi + 3k$ then find $\nabla x (\overline{R} \times A)$.

16. (a) Reduce the matrix $\begin{bmatrix} 2 & 1 & -6 & -3 \\ 3 & -3 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$ to normal form and find its rank.

(b) Discuss the convergence of the series $\sum \frac{1}{n^p}$, $p > 0$.

17. (a) If $x = r\cos\theta$, $y = r\sin\theta$, find $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2$.

(b) Evaluate $\iint_s \overline{F} \cdot \overline{n}\, ds$.

where $\overline{F} = (2x + 3z)\, i - (xz + y) i + (y^2 + 2z)\, K$ and s is the surface of the sphere having centre at (3, −1, 2) and radius 3.