Name : ....

Roll No. : ....

Invigilator's Signature : ....

2012

DISCRETE MATHEMATICAL STRUCTURES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) Out of the following the singleton set is

a) $A = \{ x : 3x - 2 = 0, x \in Q \}$

b) $B = \{ x : x^2 - 1 = 0, x \in R \}$

c) $C = \{ x : 30x - 59 = 0, x \in N \}$

d) $D = \{ x : x^2 - 1 = 0, x \in Z \}$

where *Q, R, N, Z* is the set of all rational number, real number, natural number and integers respectively.

ii) If A, B & C are any three arbitrary sets, then $A-(B\cap C)$ is

a) $(A-B)\cup(A-C)$ b) $(A-B)\cap(A-C)$

c) $(A-B)\cap(C-A)$ d) $(B-A)\cup(A-C)$.

iii) The number of arrangements of 25 objects where 7 are of the first kind, 2 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

a) $\dfrac{25!}{7!2!3!4!}$ b) $\dfrac{25!}{7!2!}$

c) $\dfrac{25!}{3!4!}$ d) none of these.

iv) Out of the following statements the formula for tautology is

a) $(p\vee q)\to q$ b) $p\vee(q\to p)$

c) $p\vee(p\to q)$ d) $p\to(p\to q)$.

v) The solution of the recurrence relation $a_r-7a_{r-1}+10a_{r-2}=0$ given $a_0=0, a_1=3$ is

a) $5^{-r}-2^r$ b) 5^r+2^r

c) 5^r-2^r d) none of these.

vi) The type of the grammar, which consists of the productions $s\to aA,\ A\to aAB,\ B\to b,\ A\to a$ is

a) Type-0 b) Type-1

c) Type-2 d) Type-3.

vii) Let L be a language given by $L = \{ a^n b^n : n \geq 0 \}$, then L^2 is equal to

a) $\{ a^n b^n a^m b^m : n \geq 0, m \geq 0 \}$

b) $\{ a^n b^n : n \geq 0 \}$

c) $\{ a^n b^n a^n b^n : n \geq 0 \}$

d) none of these.

viii) The coefficient of X^{25} in $(X^3 + X^4 + X^5 + \ldots)^5$ is

a) $C (9, 5)$ b) $C (5, 9)$

c) $C (5, 5)$ d) $C (9, 9)$.

ix) For the mapping $g : [-3, 2] \to R$ defined by $g (x) = 3x + 4$ for any $x \in [-3, 2]$ then image set of g is

a) [–5, 10] b) [0, 10]

c) [2, – 3] d) none of these.

x) A spanning tree of a connected graph contains

a) all the vertices of the graph

b) all the vertices and edges of the graph

c) a few vertices of the graph

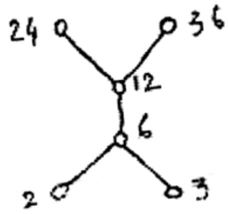
d) none of these.

xi) If a binary tree has 20 pendant vertices then the number of internal vertices of the tree is

a) 20 b) 21

c) 23 d) 19.

xii) Haase diagram is given below :



This is a

a) Poset b) Toset

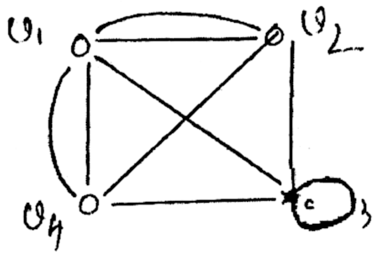
c) Lattice d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Define adjacency matrix of a simple graph $G = (V, E)$. Write down the adjacency matrix for the following undirected graph :



3. By using Principle of Mathematical Induction, prove that $4^{2n+1} + 3^{n+2}$ is an integer multiple of 13 for all positive integers *n*.

4. Let $A = \{ x \in R : x \neq 2 \}$ & $B = \{ x \in R : x \neq 1 \}$, and let the two functions $f : A \to B$ & $g : B \to A$ are defined by $f(x) = \dfrac{x}{x-2}, \forall x \in A$ and $g(x) = \dfrac{2x}{x-1}, \forall x \in B$, then find $f_o g$. Are the two functions *f* and *g* invertible ? 2 + 3

5. Over the alphabet $\sum = \{a, b\}$ design a DFA which accepts the language $L = \{\ w : \ w$ has both an even number of a's and an even number of b's.

6. Find an explicit formula for the sequence defined by $a_n = a_{n-1} + 4 \ \ \forall n \geq 2$ with $a_1 = 2\ldots$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Determine the intersection of the following two fuzzy sets :

$$A = \left\{ \frac{4}{01}, \frac{6}{0 \cdot 5}, \frac{8}{0 \cdot 6}, \frac{10}{0 \cdot 7} \right\} \text{ and}$$

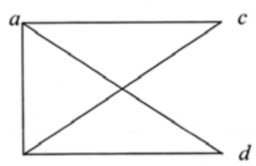
$$B = \left\{ \frac{0}{0 \cdot 4}, \frac{2}{0 \cdot 6}, \frac{4}{1}, \frac{6}{1}, \frac{8}{0 \cdot 6}, \frac{10}{0 \cdot 5} \right\} .$$

b) For each of the following mappings determine whether it is (i) injective, (ii) surjective. Find the inverse mapping of the mapping which is bijective.

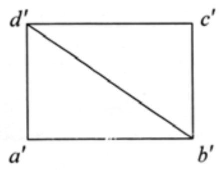
$$K : R \to R \text{ defined by } k(x) = \begin{cases} x^2 - 1, & x \geq 0 \\ -x^2 - 1, & x < 0 \end{cases}$$

c) Examine if the following graphs are isomorphic :

G :



G′



3 + 7 + 5

8. a) Solve the following recurrence relation using generating function :

$$a_n - 9a_{n-1} + 20a_{n-2} = 0 \text{ for } n \geq 2 \text{ and } a_0 = -3, a_1 = -10 .$$

b) Show that $n^2 > 2n + 1$ for $n \geq 3$ using mathematical induction.

c) Show that $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

7 + 4 + 4

9. a) Write DNF of the following statement :

$\neg \{ \neg (p \leftrightarrow q) \wedge r \}$

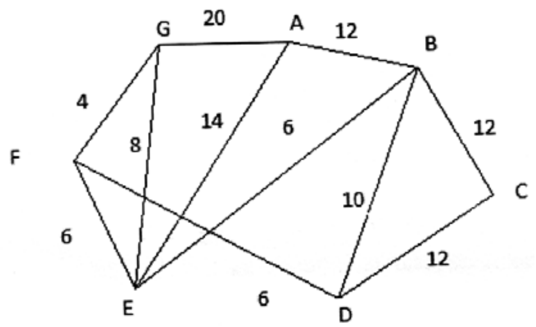
b) Verify whether the argument given below is valid or not :

All mammals are animals. Some mammals are two-legged. Therefore, some animals are two-legged.

c) Prove the following equivalence :

$\neg p \wedge q \Leftrightarrow \neg (p \vee (\neg p \wedge q))$ 5 + 5 + 5

10. a) Find by Prim's algorithm a spanning tree with minimum weight from the graph given below. Also calculate total weight of spanning tree :



b) Prove that a connected graph n with $n - 1$ vertices and edges is a tree.

c) Determine the value of n if $4 \times {}^{n}P_3 = {}^{n+1}P_3$. 6 + 6 + 3

11. a) Prove that in a bounded distributive lattice (L, $\cap$, U) an element cannot have more than one complement.

b) Find the sum of all four digits for even numbers that can be made with the digits 0, 1, 2, 3, 5, 6 & 8.

c) Define Mealy and Moore machine. Construct a Moore machine from the following Mealy machine :

	Next State			
Present State	$a = 0$		$a = 1$	
	State	Output	State	Output
s_0	s_0	1	s_1	0
s_1	s_3	1	s_3	1
s_2	s_1	1	s_4	1
s_3	s_2	0	s_0	1

4 + 6 + 5

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