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06MAT41

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Employ Taylor's series method to find an approximate solution correct to fourth decimal places for the following initial value problem at $x = 0.1$, $dy/dx = x - y^2$, $y(0)=1$. **(06 Marks)**

b. Using modified Euler's method to find $y(0.1)$ given $dy/dx = x^2 + y$, $y(0) = 1$ by taking $h=0.05$. Perform two iterations in each step. **(07 Marks)**

c. If $dy/dx = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3)=2.09$ find $y(0.4)$ correct to four decimal places. By using Milne's predictor-corrector method (Use corrector formula twice). **(07 Marks)**

2 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Find the analytic function $f(z) = u+iv$ whose real part is $e^{-x}(x\cos y + y\sin y)$. **(07 Marks)**

c. Find the bilinear transformation which maps the points $Z=0, i, \infty$ onto the points $w = 1, -i, -1$ respectively. Find the invariant points. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in terms of Laurent's series valid in the regions i) $|z-1| < 1$ ii) $|z-1| > 1$. **(07 Marks)**

c. Evaluate $\int_c \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ using Cauchy's Residues theorem where c is the circle $|z| = 3$. **(07 Marks)**

4 a. Solve in series the equation $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$ **(06 Marks)**

b. Solve Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

PART – B

5 a. The pressure and volume of a gas are related by the equation $PV^v = K$, where v and K being constants. Fit this equation to the following set of observations. **(06 Marks)**

P (kg/cm^2)	0.5	1.0	1.5	2.0	2.5	3.0
V (litre)	1.62	1.00	0.75	0.62	0.52	0.46

b. Find the correlation coefficient and the regression lines of y on x and x on y for the following data: **(07 Marks)**

x	1	2	3	4	5
y	2	5	3	8	7

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability density function of a variate X is

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find i) k ii) $P(X \geq 5)$ iii) $P(3 < X \leq 6)$ **(06 Marks)**

b. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are choosen at random, what is the probability that i) no line is busy ii) at least 5 lines are busy iii) at most 3 lines are busy. **(07 Marks)**

c. Obtain the mean and standard deviation of the normal distribution. **(07 Marks)**

7 a. Explain the following terms:
   i) Null hypothesis
   ii) Confidence limits
   iii) Type I & Type II errors. **(06 Marks)**

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate that the die is biased? **(07 Marks)**

c. The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? (Given $t_{0.05}$ for 8 df = 2.31). **(07 Marks)**

8 a. The joint probability distribution of two random variables X and Y are given below.

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine i) E(X) and E(Y) ii) E(XY) iii) COU (X,Y) **(06 Marks)**

b. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade if for a new Santro as to trade if for Maruti or an Ambassador. In 2000, he bought his first car, which was Santro. Find the probability that he has i) 2002 Santro ii) 2002 Maruti. **(07 Marks)**

c. Define stochastic matrix. Find the unique fixed probability vector for the regular stochastic matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$ **(07 Marks)**

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