401

BTS – IV – 04.13 – 0275

B.Tech. Degree IV Semester Examination April 2013

CE/ME/EC/CS/SE/IT/EB/EI/EE/FT 401 ENGINEERING MATHEMATICS III
(2006 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART A
(Answer *ALL* questions)

(8 × 5 = 40)

I. (a) If $f(z)$ is a regular function of z such that $f'(z) \neq 0$, show that

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] \log|f'(z)| = 0.$$

(b) Show that a bilinear transformation preserves cross ratio of four points.

(c) Evaluate $\oint_{|z|=1} \frac{\cos z}{z^{2n}} dz$.

(d) Find the residue of $f(z) = \cot z$ at its poles.

(e) Find the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2 + z^2)$.

(f) Solve $\left(\frac{1}{z} - \frac{1}{y}\right)p + \left(\frac{1}{x} - \frac{1}{z}\right)q = \frac{1}{y} - \frac{1}{x}$.

(g) Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(n, 0) = 6e^{-3x}$.

(h) Solve Laplace's equation over a circular region by the method of separation of variables.

PART B

(4 × 15 = 60)

II. (a) Find the necessary and sufficient condition for a function $w = f(z) = u + iv$ is analytic in a domain D. (10)

(b) Find the analytic function $f(z) = u + iv$ if $u = (1 + \cos\theta)$. (5)

OR

III. (a) Discuss the mapping $w = z + \frac{1}{z}$. (10)

(b) Find the condition where the transformation $w = \frac{az + b}{cz + d}$ transform the unit circle in the w plane into a straight line. (5)

(P.T.O.)

IV. (a) Obtain the expansion for $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ which are valid when (6)

(i) $|z|<1$

(ii) $1<|z|<4$

(iii) $|z|>4$

(b) Evaluate $\int_0^\infty \frac{1}{x^3+a^4}dx\ (a>0)$. (9)

OR

V. (a) Find the nature and location of singularities of the following: (6)

(i) $\frac{z-\sin z}{z^2}$

(ii) $(z+1)\sin\frac{1}{z-2}$

(iii) $\frac{1}{\cos z-\sin z}$

(b) Evaluate $\int_0^\infty \frac{x\sin x}{x^2+a^2}(a>0)$ using contour integration. (9)

VI. (i) Solve $yp=2yx+\log q$. (15)

(ii) Solve $(x-2z)p+(2z-y)q=y-x$.

(iii) Solve $r-4s+4t=e^{2x+y}$.

OR

VII. (i) Solve $p^2+pq=z^2$. (15)

(ii) Solve $p-q=\log(x+y)$.

(iii) Solve $(D^2+3DD'+2D'^2)z=x+y$.

VIII. (a) Derive $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$. (7)

(b) The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid point of the string always remains at rest. (8)

OR

IX. (a) Solve the Laplace's equation over a rectangular region by the method of separation of variables. (7)

(b) Find the solution of $\frac{\partial V}{\partial t}=k\frac{\partial^2 V}{\partial x^2}$, given that $V=V_0\sin nt$, when $x=0$ for every t and $V=0$ when x is very large. (8)

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