Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(CHE)/SEM-7/CHE-701/2012-13

2012

MATHEMATICAL METHODS IN CHEMICAL ENGINEERING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Two matrices A and B are conformable for product AB if the number of

      a) columns of A equal to the No. of rows of B

      b) rows of A is equal to the No. of columns of B

      c) none of these

      d) both of these.

   ii) The rank of a null matrix is

      a) 0 b) 1

      c) 2 d) 3.

iii) $\left(\frac{du}{dt} \right)^2 + u = 0$ is

a) 1st order linear difference equation

b) 1st order non-linear difference equation

c) 2nd order linear difference equation

d) 2nd order non-linear difference equation.

iv) What is the P.I of $\frac{dy}{dx} - 4y = e^{2x}$?

a) $-\frac{1}{2}e^{2x}$ b) $-e^{x}$

c) $2e^{2x}$ d) none of these.

v) When $k > 0$ the Bessel Function $J_k(0)$ s

a) 0 b) 1

c) ∞ d) $-\infty$.

vi) The IF of the differential equation $\frac{dy}{dx} + y = x$, is

a) e^{x} b) e^{-x}

c) e^{2x} d) none of these.

vii) What is the value of L (sin at) ?

a) $\frac{s}{s^2 + a^2}$ b) $\frac{a^2}{s^2 + a^2}$

c) $\frac{s^2}{s^2 + a^2}$ d) $\frac{a}{s^2 + a^2}$

viii) If $y = 5x^3 + 7z^3$ then the value of $\frac{\partial^3 y}{\partial x^3}$ and $\frac{\partial^3 y}{\partial z^3}$ are respectively

a) 30, 42 b) 42, 30

c) 15, 21 d) 10, 14.

ix) Δ^{p+q} can be written as

a) $\Delta^p . \Delta^q$ b) $\Delta^q . \Delta^p$

c) $\Delta^p + \Delta^q$ d) either (a) or (b).

x) The relationship between the differential operator and the finite difference operator is

a) $E = 1 + \Delta^2$ b) $E = 1 - \frac{1}{\Delta}$

c) $E = 1 + \Delta$ d) $E\Delta = 1$.

xi) $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$ can be called

a) Parabolic equation

b) Elliptical equation

c) Hyperbolic equation

d) Both (a) and (c).

xii) What is the rank of following matrix ?

$$\begin{bmatrix} 2 & 4 & -2 \\ 3 & 0 & 4 \\ 5 & 4 & 2 \end{bmatrix}$$

a) 1 b) 2

c) 3 d) 4.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. A closed kettle of total surface area A m^2 is heated through its surface by condensing steam at temperature T_S °C. The kettle is charged with M kg of liquid of specific heat C J/kg °C at a temperature of T_0 °C. If the process is controlled by a heat transfer coefficient h W/m^2 °C, how does the temperature of the liquid vary with time ?

3. Determine the value of α, β, γ where

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

is orthogonal.

4. Solve : $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 6xe^{4x}$.

5. Define orthogonal functions and prove that the functions sin mx and sin nx are orthogonal with respect to unity in the range $0 \le x \le \pi$, for integer values of m and n.

6. Solve the equation :

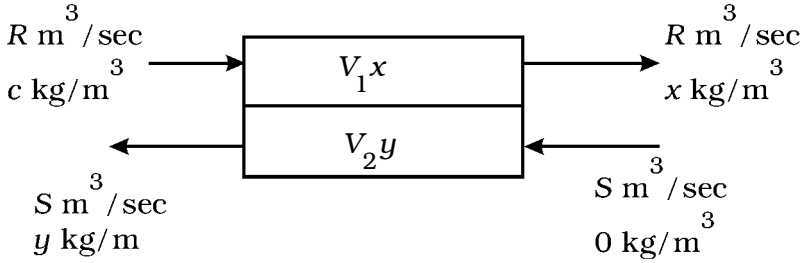
$$y_{n+2}\, y_n = \left(y_{n+1} \right)^2.$$

GROUP – C

(Long Answer Type Questions)

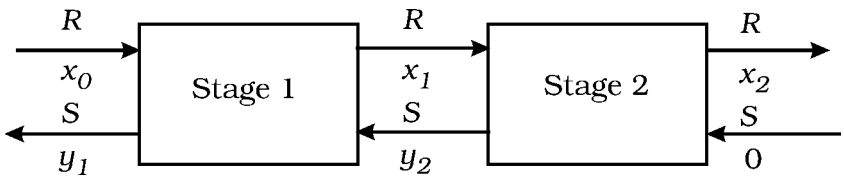
Answer any *three* of the following. 3 × 15 = 45

7. Solve the unsteady state problem (time dependent single stage solvent extraction) using he figure given below :



State assumptions used clearly. Derive expressions for x, y as a function of θ (time) [$y = mx$ where m = distribution coefficient]. Also single stage has V_1 m^3 of toluene, $V_2 m^3$ of water and no benzoic acid. [Here R = flow rate of toluene, S = flow rate of water, x = concentration of benzoic acid in exit toluene, y = concentration of benzoic acid in exit water, c = concentration of benzoic acid in feed toluene].

8 A two-stage extraction unit is used to extract benzoic acid from toluene, using water as the extracting solvent. Derive a mathematical expression to determine the final extract and raffinate composition of a two-stage solvent extraction unit.



Where, R = flow rate of toluene, m^3/s; S = flow rate of water, m^3/s; x = concentration of benzoic acid in toluene phase, kg/m^3; y = concentration of benzoic acid in water phase, kg/m^3.

If $S = 12R$, $m = 1/8$ and $x_0 = 1$ then what are the concentrations of benzoic acid in exit streams and what is the proportion of acid extracted ?

9. Consider the series reaction in a CSTR

$$A \xrightarrow{2s^{-1}} B \xrightarrow{3s^{-1}} C$$

a) We restrict the reactor operation to be isothermal and reactions to be first order. The feed to the reactor is pure A at a concentration of 1 gmol/cc. Develop the model equations for the system. Determine the transient as well as steady state concentrations of A, B, C in the CSTR. The initial concentrations of A, B, C are 1, 0, 0 gmol/cc and $\tau = 4$ s. 6

b) Solve the following differential equation

$$\frac{d^2y}{dx^2} - y = x \sin 3x + \cos x.$$ 4

c) Prove that eigenvectors corresponding to distinct eigen-values for a real matrix possess biorthogonality property. 5

10. Triangular fins of 2·5 cm thickness at base and 10 cm long are made from stainless steel (Thermal conductivity = 17·7 W/m K and density = 7850 kg/m^3) are to be fitted to an air cooled cylinder wall.

    a) If the wall temperature is 6000°C and the convective heat transfer coefficient h between solid and air = 20 W/m^2K, the ambient temperature is 400°C, derive the temperature distribution along the fin and estimate the rate of heat flow per unit mass of fin. The Bessel function values are provided below :

    10 (1·9) = 2·1782, 11 (1·9) = 1·48871 8

    b) Liquid benzene is to be chlorinated batchwise by sparging chlorine into a reactor containing benzene. If the reactor contains an agitator and all chlorine undergoes chemical reaction and HCL gas escapes from the vessel, estimate how much chlorine is to be added for maximum yield of benzene. The reaction takes place at 550° C. 7

11. The reversible reaction

    $x_1 \leftrightarrow x_2$

    occurs isothermally is a batch reactor. The forward and reverse reactions are both first order with rate constants $1s^{-1}$ and $2s^{-1}$, respectively. The initial concentrations of x_1 and x_2 are 2 gmol/cc and 3 gmol/cc. Determine the equilibrium concentration in the reactor.