B. Tech Degree IV Semester Examination, April 2008

IT/CS/EC/CE/ME/SE/EB/EI/EE 401 ENGINEERING MATHEMATICS IV

(Common for 1999 and 2002 Schemes)

Time : 3 Hours Maximum Marks : 100

I. (a) Prove that the function $f(z) = Sin\, z$ is analytic and also find its derivative. (4)

(b) Show that an analytic function with constant modulus is constant. (6)

(c) State and prove the necessary and sufficient conditions that will ensure the analyticity of a function $w = f(z) = u(x,y) + i\,v(x,y)$. (10)

OR

II. (a) Discuss the transformation about $w = e^z$. (7)

(b) Find the image of the triangular region in the z-plane bounded by the lines $x = 0, y = 0$ and $x + y = 1$ under the transformation $w = e^{i\pi/4}.z$. (6)

(c) Prove that the cross ratio of four points is in variant under bilinear transformation. (7)

III. (a) From the integral $\int_c \frac{dz}{z+2}$ where c is the circle $|z| = 1$, show that $\int_0^\pi \frac{1+2Cos\,\theta}{5+4Cos\theta} d\theta = 0.$ (7)

(b) Evaluate $\int_c \frac{zdz}{(z-1)(z-2)^2}$ where c is the circle $|z-2| = \frac{1}{2}$ using Cauchy's integral formula. (6)

(c) Expand the following functions

(i) $f(z) = Cos\, z$ about $z = -\pi/2$ as Taylor's series.

(ii) $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent's series in the region $1 < |z| < 2$. (7)

OR

IV. (a) Evaluate $\int_c \frac{z+1}{z^2+2z+4}$ where c is $|z+1+i| = 2$, using Cauchy's residue theorem. (5)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 \quad dx}{(x^2+1)(x^2+4)}$ using contour integration. (7)

(c) Use contour integration to prove that $\int_0^\infty \frac{Sin\, mx}{x} dx = \pi/2$ where $m > 0$. (8)

V. (a) Explain about Regula Falsi Method and use this method to find an approximate root of $x \ \log_{10} x - 1.2 = 0.$ (10)

(b) (i) Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$. (5)

(ii) Find the parabola of the form $y = ax^2 + bx + c$ passing through the points $(0,0), (1,1)$ and $(2,20)$. (5)

OR

(Turn Over)

VI. (a) Solve the following system of equations by Gauss elimination method.

$$2x_1 + 5x_2 + 2x_3 - 3x_4 = 3$$
$$3x_1 + 6x_2 + 5x_3 + 2x_4 = 2$$
$$4x_1 + 5x_2 + 14x_3 + 14x_4 = 11$$
$$5x_1 + 10x_2 + 8x_3 + 4x_4 = 4.$$

(10)

(b) Find Newton's divided differences polynomial for the data given below also find $f(2.5)$

x	-3	-1	0	3	5
$f(x)$	-30	-22	-12	330	3458

(10)

VII. (a) Prove that $Sin(hD) = \mu\delta$. (4)

(b) Construct a polynomial for the data given below. Also find $y(x=5)$

x	4	6	8	10
y	1	3	8	16

(8)

(c) Using Stirling's formula compute $f(1.22)$ from the following data

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	0.841	0.891	0.932	0.963	0.985

(8)

OR

VIII. (a) The table given below reveals the velocity, V of a body during the time 't' specified. Find its acceleration at $t = 1.1$.

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

(10)

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence obtain an appropriate value of π. Verify the answer with actual integration. (10)

IX. (a) Use Taylor's series method to compute $y(0.2)$ and $y(0.4)$ correct to 4 decimal places given $\frac{dy}{dx} = 1 - 2xy$ and $y(0) = 0$. (10)

(b) Using modified Euler's method, find $y(o.1), y(0.2)$. Given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$. (10)

OR

X. (a) Using Rung-Kutta method of fourth order. Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$. Given $y(0) = 1$ at $n = 0.2, 0.4$. (10)

(b) Solve $\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ given $u(0,t) = 0, u(4,t) = 0, u(x,0) = x(4-x)$

Assume $h = 1$. Find the value of u upto $t = 5$. (10)

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