SE | CMPN | IV CBSD 16/5/12
A. maths - IV

21 : 1st half.12-AM(w)

Con. 3577-12. **GN-5543**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1 is compulsory.**
(2) Attempt any **four** questions out of **remaining** questions.

1. (a) Construct dual of the following LPP : **5**

Max $z = 8x_1 + 3x_2$
Subject to $x_1 - 6x_2 \geq 2$
$5x_1 + 7x_2 = -4$ $x_1, x_2 \geq 0$

(b) Find the orthogonal trajectory of the family of curves given by : **5**

$2x - x^3 + 3xy^2 = a$

(c) Evaluate $\int_c \frac{1}{z} dz$ where c is the upper half of $|z|=1$. **5**

(d) Show that every Skew-Hermitian matrix can be expressed in the form P + i Q Where P is real Skew-Symmetric and Q is real Symmetric matrix. **5**

2. (a) Determine the analytic function f(z) where $u = -r^3 \sin 3\theta$ **6**

(b) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory **6**

(c) Use simplex method to solve the LPP Max $z = 3x_1 + 5x_2 + 4x_3$ subject to the constraints **8**

$2x_1 + 3x_2 \leq 8, \quad 2x_2 + 5x_3 \leq 10$
$3x_1 + 2x_2 + 4x_3 \leq 15, \quad x_1, x_2, x_3 \geq 0$

3. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$ using contour integration. **6**

(b) Find Eigen values and Eigen vectors of A^3 where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ **6**

(c) Use Penalty method (Big M) to solve Mini $z = 4x + y$ subject to $3x + y = 3$, $4x + 3y \geq 6$, $x + 2y \leq 4$ $x, y \geq 0$ **8**

4. (a) If $w = f(z)$ then prove that $\frac{dw}{dz} = (\cos\theta - i\sin\theta)\frac{\partial w}{\partial r}$ **6**

(b) If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 3a \\ 2 & 1 & 3b \\ 2 & -2 & 3c \end{bmatrix}$ is orthogonal find a, b, c. Also find A^{-1} **6**

(c) Use Dual Simplex method to solve the LPP **8**

Minimise $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

subject to $5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

$x_1, x_2, x_3, x_4 \geq 0$

5. (a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \cdot \tan A = A \cdot \tan 3$. 6

(b) Find the image of the line $y - x + 1 = 0$ under the transformation $w = \frac{1}{z}$. Also find the image of the line $y = x$ under the same transformation. Draw rough sketches. 6

(c) Solve the NLPP using the method of Lagrangian Multipliers. 8

Minimize $z = x_1^2 + x_2^2 + x_3^2$
subject to $x_1 + x_2 + 3x_3 = 2$, $5x_1 + 2x_2 + x_3 = 5$,
$x_1, x_2 x_3 \geq 0$

6. (a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and hence find A^{-1}, $A^3 - 5A^2$ 6

(b) Find the bilinear transformation that maps the points ∞, i, 0 onto the points 0, i, ∞. Find the fixed points. 6

(c) State and prove Cauchy's integral formula and hence evaluate $\int_c \cot z \, dz$ where c is $\left|z + \frac{1}{z}\right| = \frac{1}{3}$ 8

7. (a) Obtain two Laurents series for : 6

$$\frac{(z-2)(z+2)}{(z+1)(z+4)}$$

(b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a\cos\theta + a^2} d\theta$ using residues. 6

(c) Solve the following N. L. P. P. using Kuhn-Tucker conditions 8

Minimise $z = 7x_1^2 + 5x_2^2 - 6x_1$
Subject to $x_1 + 2x_2 \leq 10$
$x_1 + 3x_2 \leq 9$
$x_1, x_2 \geq 0$