SE/IV/MECH/Applied Maths-IV

VT-F.H.Exam. April.-12-122

Con. 3580-12.

16/05/12

GN-5480

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. 1 is **compulsory.**
(2) Answer any **four** other questions from the remaining **six** questions.

1. (a) Solve the partial differential equation $x^2 \frac{\partial^2 u}{\partial x \partial y} + 3y^2 u = 0$ by the method of separation of variables. 5
   (b) Obtain the complex form of Fourier series of $f(x) = \cos h\, 2x + \sin h 2x$ in the interval $(-2, 2)$. 5
   (c) The distribution function of a random variable X is given by – $F(x) = 1 - (1 + x)\, e^{-x}, \geq 0$. Find the density function, mean and variance of X. 5
   (d) A normal population has mean 0·1 and S.D. 2·1. Find the probability that mean of the sample of size 900 will be negative. 5

2. (a) For a binomial distribution mean is 6 and S.D. $\sqrt{2}$. Find the first two terms of the distribution. 6
   (b) Find the half-range Fourier sine series for $f(x) = x^2$ in the interval $0 \leq x \leq 3$. 6
   (c) A rod of length 30 cm has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduces to 0°C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A. 8

3. (a) If X and Y are independent random variables following N(8, 2) and N(12, $4\sqrt{3}$) respectively. Find the value of λ such that :– 6
   $P(2X - Y \leq \lambda 2) = P(X + 2Y \geq \lambda)$.
   (b) Obtain Fourier series for $f(x) = e^x$ in $(-\pi, \pi)$. Hence derive the series for $\frac{\pi}{\sin h\, \pi}$. 6
   (c) Find the coefficient of correlation and obtain the lines of regression for the date :– 8

x :	62	64	65	69	70	71	72	74
y :	126	125	139	145	165	152	180	208

4. (a) The marks obtained by number of students in a certain subject are approximatly normally distributed with mean 65 and S.D 5. If 3 students are selected at random from this group, what is the probability that atleast one of them would have scored above 75%. 6
   (b) Find the Fourier series for f(x) in $(0, 2\pi)$ 6

$$f(x) = \begin{cases} x & 0 < x \leq \pi \\ 2\pi - x & \pi < x \leq 2\pi \end{cases}$$

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(c) The bounding diameter of semi circular plate of radius 10 cm is kept at 0°C and the temperature along the boundary is given by 8

$$u(10,\theta) = \begin{cases} 50\,\theta & 0 < \theta \leq \pi/2 \\ 50\,(\pi - \theta) & \pi/2 < \theta < \pi \end{cases}$$

Find the steady state temperature function u(r, θ).

5. (a) The mean height and S.D. height of 8 randomly chosen soldiers are 166·9 and 8·29 cms. respectively. The corresponding values of 6 randomly chosen sailors are 170·3 and 8·5 cms respectively. Based on this data, can we conclude that soldiers are in general shorter than sailors ? 6

(b) Find the rank correlation coefficient for the indices of supply and price of an article. 6

Supply index	124	100	105	112	102	93	99	115	123	104	99	113	121	103	101
Price index	80	100	102	91	100	111	109	100	89	104	111	102	98	111	123

(c) If the independent random variables X and Y have the variances 36 and 16 respectively. Find the correlation coefficient betwen (X + Y) and (X – Y). 8

6. (a) Fit a Poisson distribution for the following data :– 6

x :	0	1	2	3	4	5
f :	142	156	69	27	5	1

(b) Fit a first degree curve to the following data and estimate the value of y when x = 73. 6

x :	10	20	30	40	50	60	70	80
y :	1	3	5	10	6	4	2	1

(c) Express $e^{-x}\cos x$ as Fourier cosine integral and show that :– 8

$$e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{w^2+2}{w^2+4}\cdot\cos wx\cdot dx$$

7. (a) Show that the polynomials $P_0(x) = 1$, $P_1(x) = x$; $P_2(x) = \frac{1}{2}(3x^2 - 1)$ 6

form an orthogonal set over the interval [–1, 1]. Hence find the corresponding orthonormal set.

(b) A total numer of 3759 individuals were interviewed in a public opinion survey on a political survey. Of them 1872 were men and rest women. A total 2257 individuals were in favour of the proposal and 917 were opposed to it. A total of 243 mean were undecided and 442 women were opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude. 6

(c) A tightly stretched string with fixed end points x = 0 and x = *l* in the shape defined by y = kx (*l* – x) where k is a constant is released from this position of rest. Find 8

y(x, t), the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$.

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