B. Tech Second Year : 3rd Semester

Engineering Mathematics-3, Jan., 2012

(FOR 3EC1 BRANCH OF ENGINEERING)

Times : 3 Hours **Min. Passing Marks : 24** **Total Marks : 80**

Unit-'I'

1. (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence find the Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$. **[8]**

(b) Solve : $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=t$; given that $y(0)=-3, y(1)=-1$. **[8]**

OR

(a) Find the inverse Laplace transform with the help of convolution theorem of $\frac{s}{(s^2+a^2)^2}$. **[8]**

(b) Solve: $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ where $u = u(x, t)$. **[8]**
B.C.: $u(0, t) = 0 = u(5, t)$ and $u(x, 0) = 10 \sin 4\pi x$.

Unit-'II'

2. (a) Find the Fourier Series for the function defined as:

$$f(x) = \begin{cases} -1, & \text{for } -\pi \le x < 0 \\ 0, & \text{for } x = 0 \\ 1, & \text{for } 0 < x \le \pi \end{cases}$$

Hence, prove that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...$ **[8]**

(b) For z transform prove that $z(nu_u) = -z\frac{d}{dz}z(u_n)$ with the help of this find the z-transform of ne^{-an}, $n \ge 0$. **[8]**

OR

(a) Obtain the constant term and the coefficients of first sine and cosine terms in the Fourier expansion of y as given in the followng table:

x	0	1	2	3	4	5
y	9	18	24	28	26	20

[8]

(b) Find the inverse z-transform of

$$f(z)=\frac{1}{(z-3)(z-2)};$$

If ROC is (i) $|z| < 2$, (ii) $2 < |z| < 3$, (iii) $|z| > 3$. **[8]**

Unit-'III'

3. (a) Find the Fourier cosine transform of e^{-x^2}. **[8]**

(b) Solve $\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial x^2}$ if (i) $V_x(0, t) = 0$, (ii) $V(x, 0) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x > 1 \end{cases}$ and (iii) $V(x, t)$ is bounded for $x > 0$, $t > 0$. **[8]**

OR

(a) Find f(x) if its Fourier cosine transform is $\frac{1}{1+s^2}$. **[8]**

(b) Solve $\frac{\partial \theta}{\partial t}=k\frac{\partial^2\theta}{\partial x^2}, x>0, t>0$
with B.C.: $\theta = \theta_0$ or when $x = 0, t > 0$
with I.C.: $\theta = 0$ or when $t = 0, x > 0$. **[8]**

Unit-'IV'

4. (a) Define analytic function and derive Cauchy-Riemann conditions for analytic function and examine the nature of the function

$$f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}}, z\ne 0,\ f(0) = 0$$ in the region including the origin. **[8]**

(b) If $(u - v) = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$ find f(z) in terms of z. **[8]**

OR

(a) Find the bilinear transform action which maps the points $z = 1, i, -1$ respectively on to the points $w = i, 0, -i$. For this transformation find the image of concentric circles $|z| = r$, $(r > 1)$. **[8]**

(b) Verify Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$ if C is the circle $|z - 1| = 2$. **[8]**

Unit-'V'

5. (a) Obtain expansion for $\frac{z^2-4}{(z+1)(z+4)}$, which are valid, for the regions:
(i) $|z| < 1$, (ii) $1 < |z| < 4$ and (iii) $|z| > 4$. **[8]**

(b) Evaluate $\int_0^\infty \frac{1-\cos x}{x^2}dx$ by contour integration. **[8]**

OR

(a) Evaluate $\int_C \frac{z^2e^{zt}}{z^2+1}dz$ where C is the circle $|z| = 2$ and t is a quantity independent of z. **[8]**

(b) Use method of contour integration to evaluate

$$\int_0^{2\pi}\frac{d\theta}{1+a^2-2\cos\theta}, 0<a<1.$$ **[8]**