B. Tech Degree III Semester Examination November 2010

IT/CS 303 DISCRETE COMPUTATIONAL STRUCTURES

(2006 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART – A

(Answer ***ALL*** questions)

(8 x 5=40)

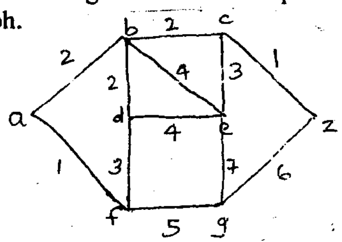
I. (a) Use mathematical induction to show that $n! \geq 2^{n-1}$ for $n = 1, 2, ...$

(b) Let R and S be the relations on
$A = \{1, 2, 3, 4\}$ defined by
$R = \{(1,1), (3,1), (3,4), (4,2), (4,3)\}$
$S = \{(1,3), (2,1), (3,1), (3,2), (4,4)\}$. Find the composition relation RoS

(c) What is a recursive algorithm? Write a recursive algorithm to find the maximum of a finite sequence of numbers.

(d) State Pigeon Hole Principle. Show that if we select ISI distinct computer science courses numbered between 1 and 300 inclusive, atleast two are consecutively numbered.

(e) Find the shortest path and the length of the shortest path from vertex 'a' to vertex 'z' in the connected, weighted graph.



(f) Draw the undirected graph represented by the incidence matrix as shown below.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
V_1	1	1	1	0	0	0	0
V_2	0	0	1	1	1	0	1
V_3	0	0	0	0	0	1	0
V_4	1	1	0	1	0	0	0
V_5	0	0	0	0	1	1	0

(g) Consider the set $A = \{4, 5, 6, 7\}$. Let R be the relation $\leq$ on A. Draw the directed graph and the Hasse diagram of R given by
$R = \{(4,5), (4,6), (4,7), (5,6), (5,7), (6,7), (4,4), (5,5), (6,6), (7,7)\}$.

(h) Consider an algebraic system $(G, *)$ where G is the set of all non-zero real numbers and $*$ is a binary operation defined by $a * b = \frac{ab}{4}$. Show that $(G, *)$ is an abelian group.

PART B

(4 x 15 = 60)

II. (a) Consider the relation
$R = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)\}$
on $\{1, 2, 3, 4, 5\}$. Determine whether the given relation is an equivalence relation. If the relation is an equivalence relation list the equivalence classes. (8)

(b) Determine whether the given arguments are valid or not

(i) $p \rightarrow q$
$\underline{p \vee q}$
$\therefore q$

(ii) $p \rightarrow q$
$\underline{\sim q}$
$\therefore \sim p$

(7)

OR

III. (a) Let f and g be functions from the positive integers to the positive integers defined by the equations $f(n) = 2n + 1$, $g(n) = 3n - 1$

Find the compositions

(i) fof (ii) gog (iii) fog (iv) gof

(b) State and prove the generalized De Morgan's laws for logic

IV. (a) Solve the recurrence relation

$a_n = 2^n a_{n-1}, n > 0$ with initial condition $a_0 = 1$.

(b) What is Time complexity of an algorithm? Differentiate Big oh and Theta notations.

OR

V. (a) Five microprocessors are randomly selected from a lot of 1000 microprocessors among which 20 are defective. Find the probability of obtaining no defective microprocessors.

(b) How many permutations can be made out of the letters of the word 'COMPUTER'? How many of these

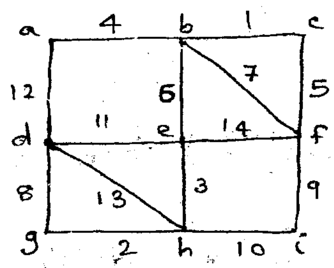
(i) begin with C? (ii) end with R?

(iii) begin with C and end with R? (iv) C & R occupy the end places?

VI. (a) What is a minimal spanning tree?

(b) Explain how Prim's algorithm finds a minimal spanning tree.

(c) Use Prim's algorithm to find a minimal spanning tree in the following graph.



OR

VII. Give an example of a graph that has a Hamiltonian cycle and an Euler cycle. Prove that the graph has the specified properties.

VIII. (a) Define a lattice. Let $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ and let the relation $\leq$ be relation/(divides) be a partial ordering on D_{100}.

(i) Determine the GLB of B, where $B = \{10, 20\}$

(ii) Determine the LUB of B, where $B = \{10, 20\}$

(iii) Determine the GLB of B, where $B = \{5, 10, 20, 25\}$

(iv) Determine the LUB of B, where $B = \{5, 10, 20, 25\}$.

(b) Let $(A, *)$ be a semigroup. For every a and b in A, if $a \neq b$ then $a * b \neq b * a$ and $a * a = a$,

(i) Show that for every a, b in A $a * b * a = a$

(ii) Show that for every a, b, c in A $a * b * c = a * c$

OR

IX. (a) Define homomorphism and isomorphism.

(b) Consider a ring $(R, +, *)$ defined by $a * a = a$

Determine whether the ring is commutative or not.

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