Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.TECH(CSE)(N)/SEM-5/CS-503/2012-13

2012

DISCRETE MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

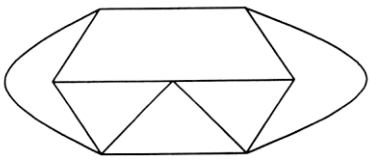
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

   i) What is the chromatic number of the following graph with 7 vertices ?



   a) 6 b) 5

   c) 4 d) 3.

ii) If there are n^r arrangements of r objects and n bins, then

a) the objects and bins are all distinguishable

b) the objects are distinguishable and bins are indistinguishable

c) the objects are indistinguishable and bins are distinguishable

d) the objects and bins are indistinguishable.

iii) Consider the set A of all integers greater than 1. Let D be a relation defined on A by $(x, y) \varepsilon D$ iff x divides y. Then which of the following is true ?

a) D is both a lattice and a partial ordering

b) D is a lattice but not a partial ordering

c) D is neither a lattice nor a partial ordering

d) D is a partial ordering but not a lattice.

iv) If 12 distinct points are placed on the circumference of a circle and all the chords connecting these points are drawn, at how many points do the chords intersect ? Assume that no three chords intersect at the same point.

a) $C(12, 2)$ b) $C(12, 4)$

c) 2^{12} d) $12! / 2$.

v) The set of natural numbers N with the relation ship '|' (divides) is a poset. How many minimal and maximal elements does it have ?

a) 1 minimal and 1 maximal

b) 1 minimal and 0 maximal

c) 1 minimal and more than 1 maximal

d) 0 minimal and 0 maximal.

vi) What is the result of $(-3)X_8\,5 +_8 (-3)X_8(-5)$ in $[Z_8, +_8, X_8]$, where Z_8 is the set of integers modulo 8, $+_8$ is the modulo 8 addition operation and X_8 is the modulo 8 multiplication operation ?

a) 0 b) 7

c) 8 d) 2.

vii) How many ways are there to travel in *xyz* space from the origin (0, 0, 0) to the point (4, 3, 5) by taking unit steps in positive *x, y, z* directions only ?

a) 4!.3!.5!

b) 60

c) 12!/(5!4!3!)

d) 3^{12}.

viii) $A \wedge B$ is equivalent to which of the following ?

a) $\neg A \rightarrow \neg B$ b) $\neg A \rightarrow B$

c) $\neg B \rightarrow A$ d) $\neg(A \rightarrow \neg B)$.

ix) A sequence $d = (d_1, d_2, d_3, \ldots, d_n)$ is graphic if there is a simple undirected graph with degree sequence d. Which of the following degree sequences are graphic ? Why ?

P : (2, 3, 3, 4, 4, 5)

Q : (2, 3, 4, 4, 5)

a) Neither P or Q b) Both P and Q

c) P only d) Q only.

x) A complemented, distributive lattice is also called a Boolean Algebra. Consider a set $S = \{a, b, c\}$ and let $M = \wp(S)$ be the power set of S. Consider the inclusion (subset) relation '$\subseteq$'. Then $(M, \subseteq)$ is

a) not a partial ordering

b) a partial ordering but not a lattice

c) a lattice but not a boolean algebra

d) a boolean algebra.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. C_9 is a cycle (*i.e.,* a circular chain) with the nine vertices *a, b, c, d, e, f, g, h, i.* How many distinct maximal matchings of size four in C_9 contain the edge *ab* ?

3. Consider K_6, the complete graph on the six vertices *a, b, c, d, e, f*. The graph G_1 is obtained from K_6 by deleting the edge *ab*. The graph G_2 is obtained from G_1 by deleting the edge *cd*. What are the chromatic numbers of G_1 and G_2 ?

4. A new flag is to be designed with 6 vertical stripes using 4 colours. In how many ways can this be done so that no 2 adjacent stripes have the same colour ?

5. Give the sequence whose generating function is $g(z) = 5(z^5 - 1)/(z - 1)$.

6. Consider the poset $S = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$ under the relation '|' (i.e. 'divides'). Find the following : Maximum element, Minimal element, Greatest element, Least element, lub (2, 9), glb (60, 72).

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that *s* is a valid conclusion from the premises $p \to \sim q,\ q \vee r, \sim s \to p$.

   b) How many 10 bit binary strings are there none of which contains the patters '110' ?

   c) Use theory of congruence to prove that for $n \geq 1$, $17 \mid (2^{3n+1} + 3 \cdot 5^{2n+1})$. 5 + 5 + 5

8. a) Show that t is a valid conclusion from the premises $p \Rightarrow q,\ q \Rightarrow r,\ r \Rightarrow s$ and $p \vee t$.

   b) For any integer n, prove that the integer $8n+3$ and $5n+2$ are relatively prime. Hence find integers $x,\ y$ such that $(8n+3x)+(5n+2) = \gcd(8n+3, 5n+2)$.

   c) Define CRS (mode m) (complete residue system modulo m). Find all CRS (mod 5). 5 + 5 + 5

9. a) Solve the recurrence relation :

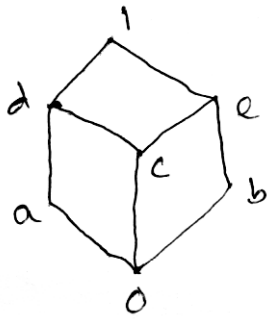
   $$a_{n+2} - 4a_{n+1} + 4a_{n-2} = (r+1)2^r$$

   b) Show that every bipartite graph is 2-chromatic.

   c) A positive integer n is expressed in the form $10b + b$. Prove that n is divisible by 17 if $a - 5b$ is divisible by 17.

   3 + 5 + 7

10. a) Show that the poset given in the following Hasse diagram is a lattice. Is it distributive and complemented ? Justify your answer.



   b) Show that in a complemented distributive lattice $< L, \wedge, \vee >$

   i) $(a \wedge b)' = a' \vee b'$

   ii) $(a \vee b)' = a' \wedge b'$

c) Solve the following recurrence relation using generating function :

$a_n = 4(a_{n-1} - a_{n-2}) + 2^n (n \geq 2); a_0 = 1, a_1 = 4.$ 6 + 4 + 5

11. a) Check the validity of the following arguments :

"If my program runs successfully then I will submit my project. I can appear the examination only if I submit my project. Either my program runs successfully or the computer crashes then I can not appear in examination."

b) Define SDR of a family of finite sets. What is Hall's Marriage Condition ? Consider the family of finite sets $S = \{A_1, A_2, A_3, A_4\}$ where $A_1 = \{a, b, d, e\}$, $A_2 = \{b, c, d, e, f\}$, $A_3 = \{c, f\}$ and $A_4 = \{b, c, f\}$. Show whether *S* satisfies the marriage condition. If yes, find two valid SDR of *S*.

c) Write down the truth table for conditional and bi-conditional proposition. 5 + 5 + 5

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