AC Unit - 1

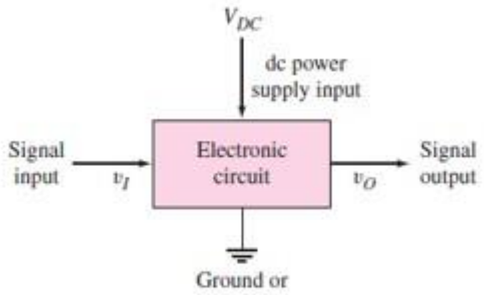
- Introduction - Recap of Small Signal Amplifiers
- Multistage Amplifiers, Cascode amplifier
- Darlington pair
- the MOS Differential Pair
- Small-Signal Operation of the MOS Differential Pair
- The BJT Differential Pair
- Nonideal Characteristics of the Differential Amplifier.

AMPLIFIER:

A circuit that increases amplitude of given signal is an amplifier. The amplifier is an electronic circuit, which **amplifies or increases strength of weak signal**.

- A small AC signal fed into amplifier is obtained as large AC signal of same frequency.
- A amplifier is essential part in radios, TV's and other communications

A linear amplifier magnifies an input signal and produces an output signal whose magnitude is larger and directly proportional to the input signal.



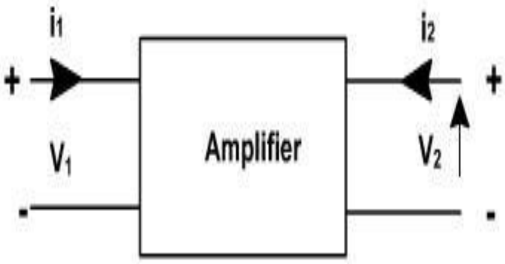
The ac analysis, called a small-signal analysis, can be performed with the dc source set to zero.

Small-Signal Hybrid-π Equivalent Circuit of the Bipolar Transistor

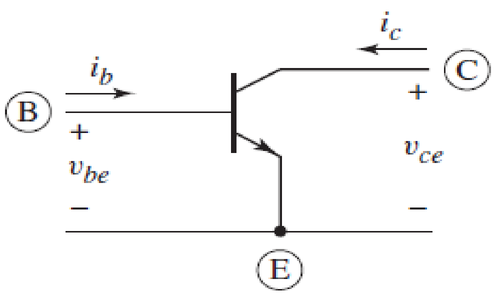
All the transistor amplifiers are two port networks having two voltages and two currents. The positive directions of voltages and currents.Out of four variables two can be selected as are independent variables and two are dependent variables.

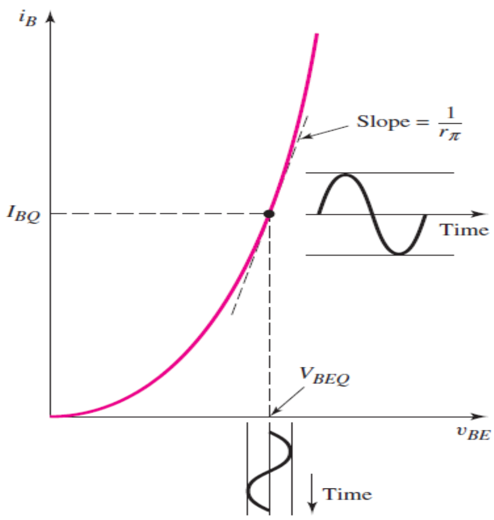
In h- parameters

- $(V_1$ & $I_2)$ – Dependent Variables
- $(I_1$ & $V_2)$ – Independent Variables



Assuming the transistor as two port network. The i_B vs v_{BE} graph the small time varying signal is superimposed on Q-point. The slope of Q-point is constant with units of conductance.

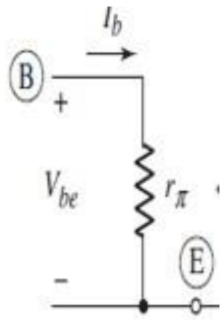




The inverse of this conductance is the small-signal resistance defined as r_π

$$V_{be} = i_b\, r_\pi$$

By using above equation we have to draw input port



$$\frac{1}{r_\pi} = \frac{\partial i_B}{\partial v_{BE}}\Big|_{Q-pt} = \frac{\partial}{\partial v_{BE}}\left[\frac{I_S}{\beta}.\exp\left(\frac{v_{BE}}{V_T}\right)\right]\Big|_{Q-pt} \qquad \therefore i_B = \frac{i_c}{\beta} \quad \therefore i_c = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$\frac{1}{r_\pi} = \frac{1}{V_T}\left[\frac{I_S}{\beta}.\exp\left(\frac{v_{BE}}{V_T}\right)\right]\Big|_{Q-pt} = \frac{I_{BQ}}{V_T}$$

then

$$\frac{v_{be}}{i_b} = r_\pi = \frac{V_T}{I_{BQ}} = \frac{\beta V_T}{I_{CQ}} \qquad \therefore i_B = \frac{i_C}{\beta}$$

- The resistance r_π is called the **diffusion resistance** or base–emitter input resistance

For Output Collector–Emitter Port

Assuming the collector current is independent of V_{CE}

$$\Delta i_C = \frac{\partial i_C}{\partial v_{BE}}\Big|_{Q-pt}.\Delta v_{BE}$$

$$or\ i_C = \frac{\partial i_C}{\partial v_{BE}}\Big|_{Q-pt}.v_{BE} \qquad ---1$$

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$\frac{\partial i_C}{\partial v_{BE}}\Big|_{Q-pt} = \frac{1}{V_T}\left[I_S.\exp\left(\frac{v_{BE}}{V_T}\right)\right]\Big|_{Q-pt} = \frac{I_{CQ}}{V_T}$$

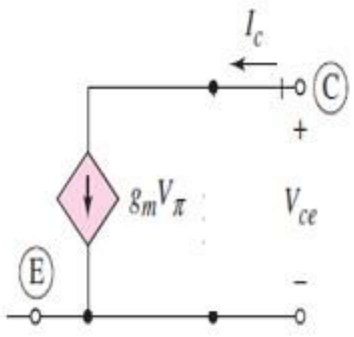
$$g_m = \frac{I_{CQ}}{V_T}$$ $\therefore$ Which is called Transconductance

$$\frac{\partial i_C}{\partial v_{BE}}\Big|_{Q-pt} = g_m \qquad ---2$$

Substitute equation 2 in 1

We can then write the small-signal collector current as $$\boldsymbol{i_c = g_m V_{be}}$$

By using above equation we have to draw output port



Combine input port an output port, we will get small signal Hybrid-π Equivalent Circuit of the Bipolar Transistor

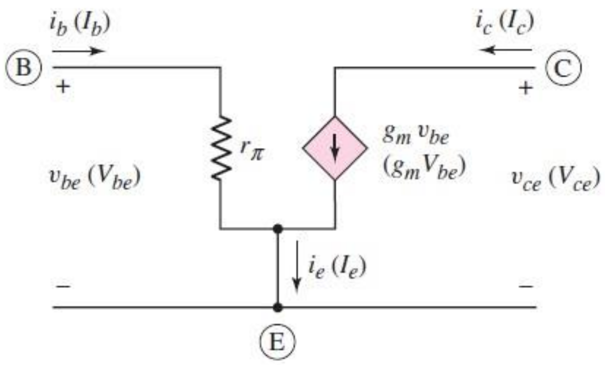


Fig: A simplified small-signal hybrid-π equivalent circuit

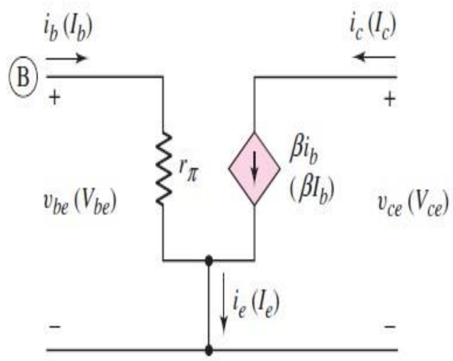
Alternative Form of Equivalent Circuit

$$i_C = \frac{\partial\, i_C}{\partial\, i_B}\bigg|_{Q-pt} .\, i_B$$

$$where\ \frac{\partial\, i_C}{\partial\, i_B}\bigg|_{Q-pt} \equiv \beta$$

We can then write as $\boldsymbol{i_c = \text{Q}\ i_b}$

$$\boldsymbol{r_\pi g_m} = \left[\frac{QV_T}{I_{CQ}}\right]\left[\frac{I_{CQ}}{V_T}\right] = \text{Q}$$



Small-Signal Equivalent-Circuit Models:

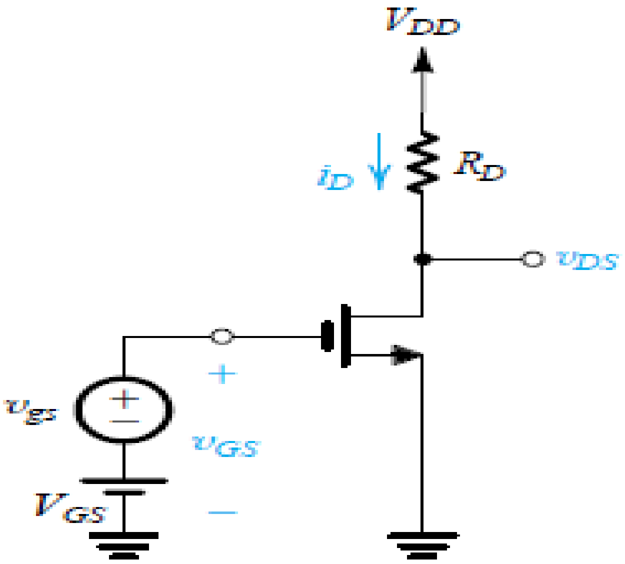


Fig. MOSFET

- The FET behaves as a voltage-controlled current source.
- It accepts a signal v_{gs} between gate and source and provides a current $g_m v_{gs}$ at the drain terminal.
- The input resistance of this controlled source is very high and ideally infinite.
- The output resistance looking into the drain also is high, and we have assumed it to be infinite.

The DC bias current is $I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{gs} - V_t)^2$

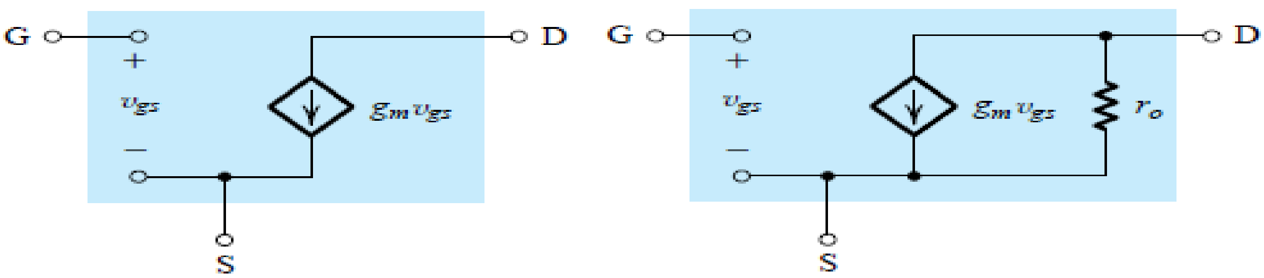
$$k'_n = \mu_n C_{OX}$$

$V_{OV} = V_{GS} - V_t$ is the overdrive voltage at which the MOSFET is biased to operate.

$$g_m = k'_n \frac{W}{L}(V_{gs} - V_t) = k'_n \frac{W}{L} V_{OV}$$

$$g_m = \frac{2 I_D}{V_{gs} - V_t} = \frac{2 I_D}{V_{OV}}$$

The small-signal model of the MOSFET

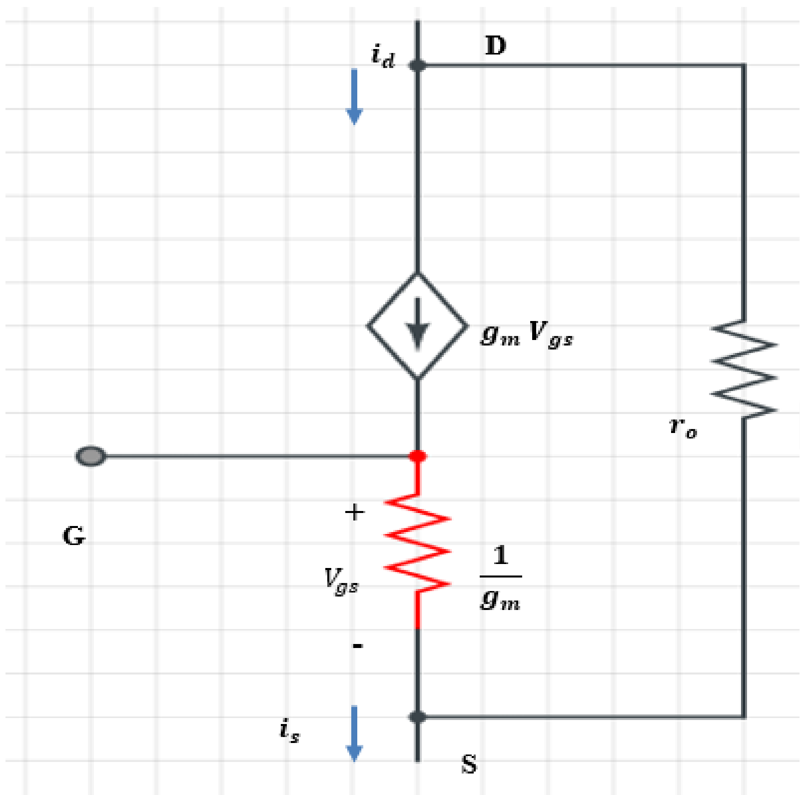


(a) (b)

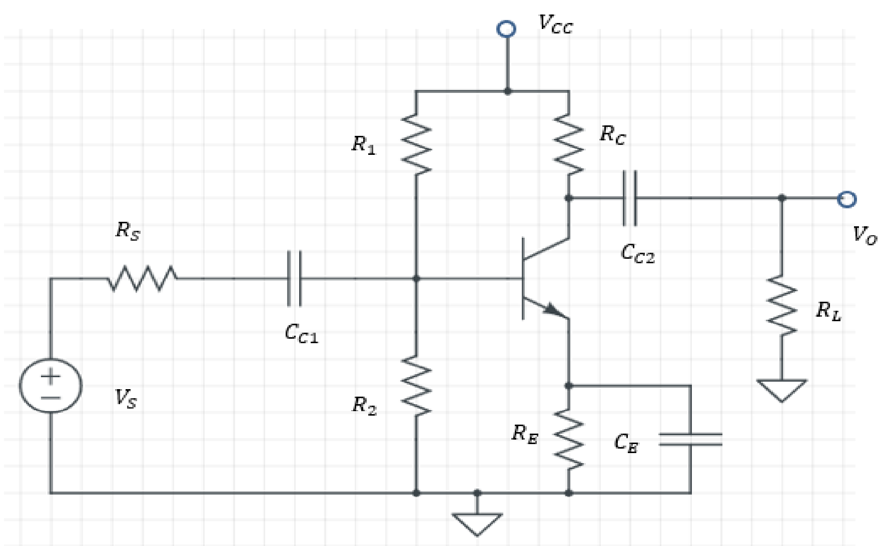
Fig. Small-signal models for the MOSFET: (a) neglecting the dependence of i_D on v_{DS} in saturation (b) including the effect of channel-length modulation, modeled by output resistance

$$g_m = k'_n \frac{W}{L}(V_{gs} - V_t) = k'_n \frac{W}{L} V_{OV} \qquad r_o = \frac{|V_A|}{I_D}$$

MOSFET T equivalent –circuit model

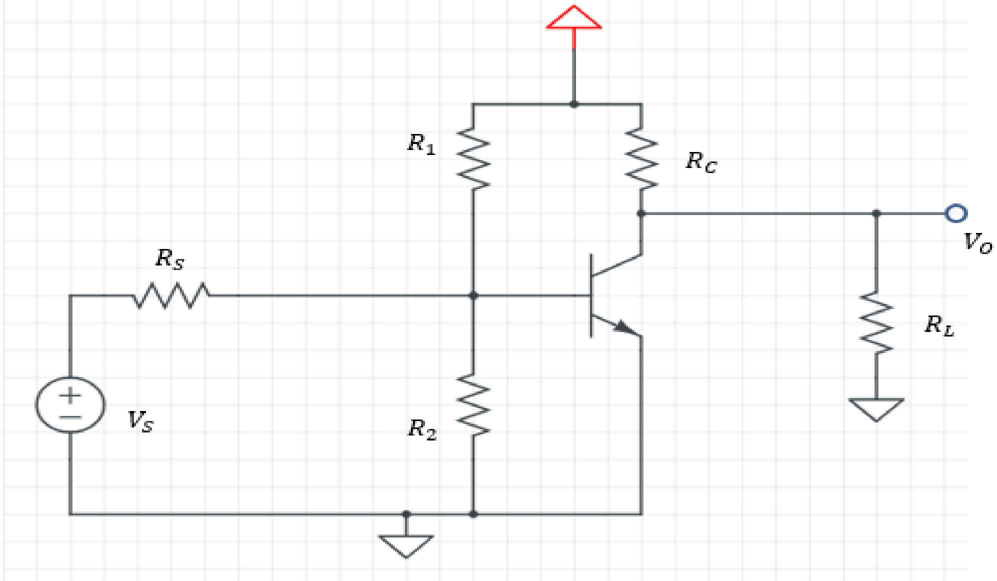


Common Emitter Amplifier: (CE Amplifier)

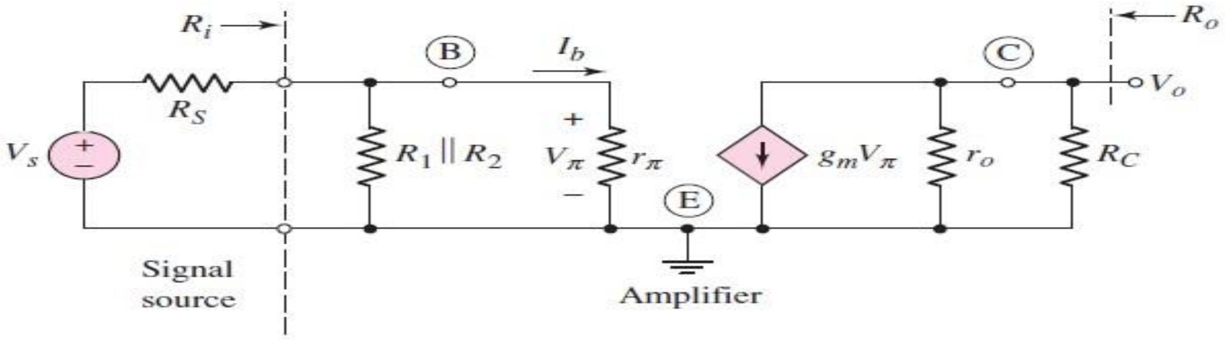


- V_s = Signal Source
- R_s = Source Resistance
- C_C = Coupling capacitor
- R_1 & R_2 = Self bias
- R_c= Load
- V_{cc} = DC Supply Voltage
- V_o = output voltage

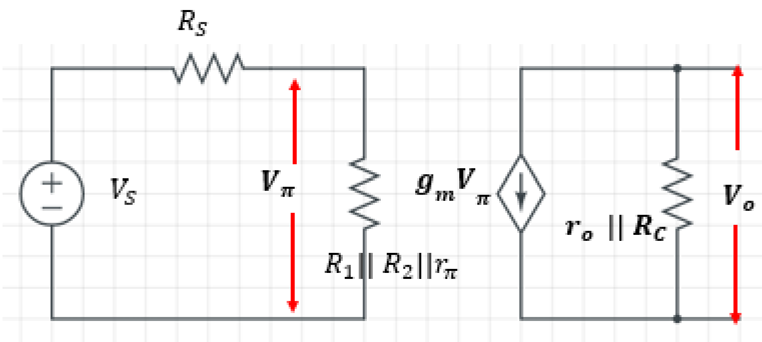
For AC(mid frequency) analysis all external capacitors are short circuit.



g_mV_π



$R_1|| R_2||r_\pi$



$$V_O = -g_mV_\pi[r_o||R_C] \qquad \text{---1}$$

And the control voltage V_π is found to be

$$V_\pi = \frac{R_1||R_2||r_\pi}{(R_1||R_2||r_\pi)+R_s}.V_S \qquad \text{---2}$$

Substitute eq2 in eq1

$$V_O = -g_m\frac{R_1||R_2||r_\pi}{(R_1||R_2||r_\pi)+R_s}.V_S\,[r_o||R_C]$$

$$A_V = \frac{V_O}{V_S} = -g_m\,[r_o||R_C]\,[\frac{R_1||R_2||r_\pi}{(R_1||R_2||r_\pi)+R_s}]$$

Assume $R_1||R_2||r_\pi = R_B$ and $r_o||R_C = R_C$

$$A_V = \frac{V_O}{V_S} = -g_m R_C \frac{R_B}{R_B + R_S} \qquad \therefore R_S \ll R_B$$

$$A_V = -g_m R_C \frac{R_B}{R_B}$$

$$A_V = -g_m R_C$$

Multistage amplifiers:

- In practical applications, the output of a single state amplifier is usually insufficient, though it is a voltage or power amplifier. Hence they are replaced by Multistage amplifiers.
- A generalized three-stage amplifier is shown in below figure.

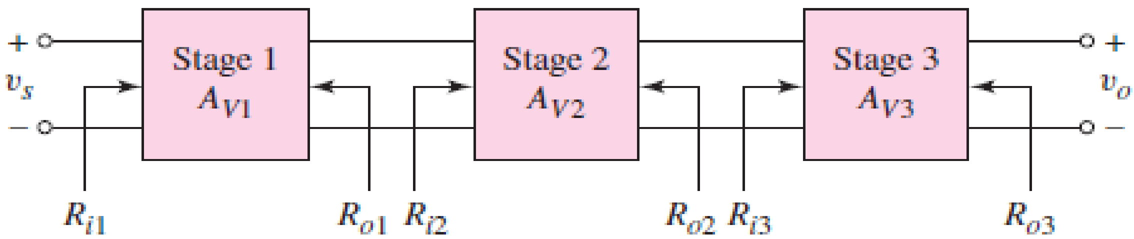


Fig. A generalized three-stage amplifier

- In Multi-stage amplifiers, the output of first stage is coupled to the input of next stage using a coupling device. These coupling devices can usually be a capacitor or a transformer. This process of joining two amplifier stages using a coupling device can be called as Cascading.
- In general, the input impedance of the first stage must be high and the output impedance of the last must be low.
- The overall gain is the product of the individual gains.

Two stage RC coupled CE-CE cascade amplifier:

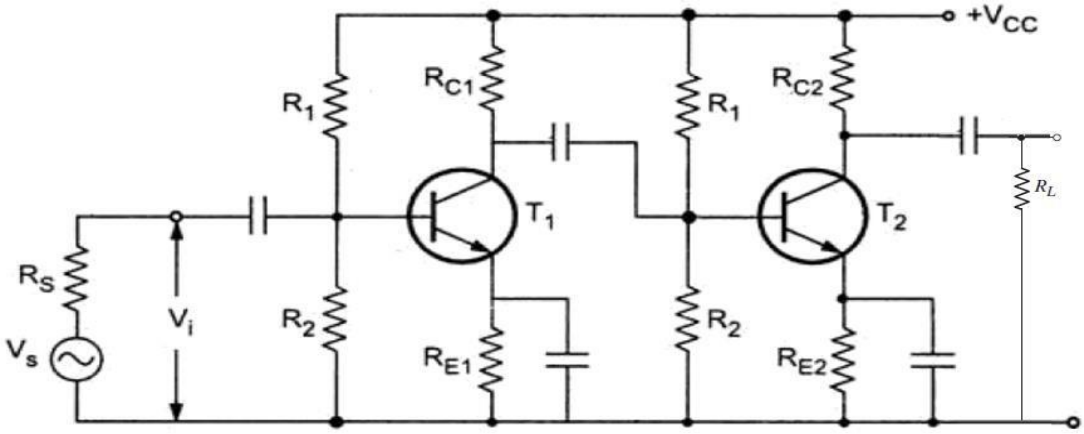


Fig. Two stage RC coupled CE-CE cascade amplifier

AC analysis of the amplifier circuit:

- Fig shows the small-signal equivalent circuit, assuming all capacitors act as short circuits, DC sources set to zero and each transistor output resistance r_o is infinite.

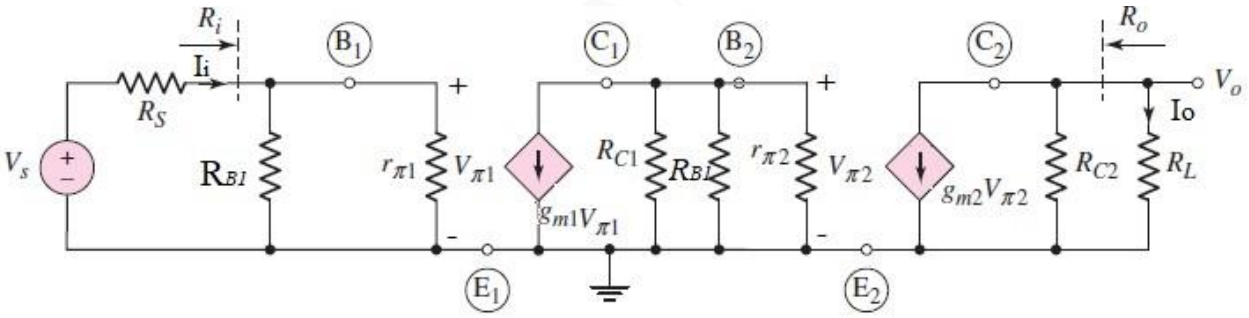


Fig. Small-signal equivalent circuit of the cascade circuit

- The bias resistance of the amplifier is $R_{B1} = R_1 \parallel R_2$
- The input resistance of the amplifier is $R_i = R_{B1} \parallel r_{\pi1}$
- The out resistance of the amplifier is $R_o = R_{C2}$

The Voltage gain of the amplifier is

$$A_{VS} = \frac{V_O}{V_s} = g_{m1} g_{m2} (R_{C2} \parallel R_L)(R_{C1} \parallel R_{B1} \parallel r_{\pi2}) \frac{R_i}{R_i + R_s}$$

The current gain of the amplifier is

$$A_{IS} = \frac{I_O}{I_i} = g_{m1} g_{m2} (R_{C1} \parallel R_{B1} \parallel r_{\pi2}) r_{\pi1} \left(\frac{R_{C2}}{R_{C2} + R_L}\right) \left(\frac{R_{B1}}{R_{B1} + r_{\pi1}}\right)$$

Cascode Amplifier :

- Cascode amplifier is a type of multistage amplifier. The CE amplifier followed by CB amplifier is called cascode amplifier. The high bandwidth and high gain are major advantages of cascode amplifier. The input is into a common-emitter amplifier (Q_1), which drives a common-base amplifier (Q_2). The output signal current of Q_1 is the input signal of Q_2.

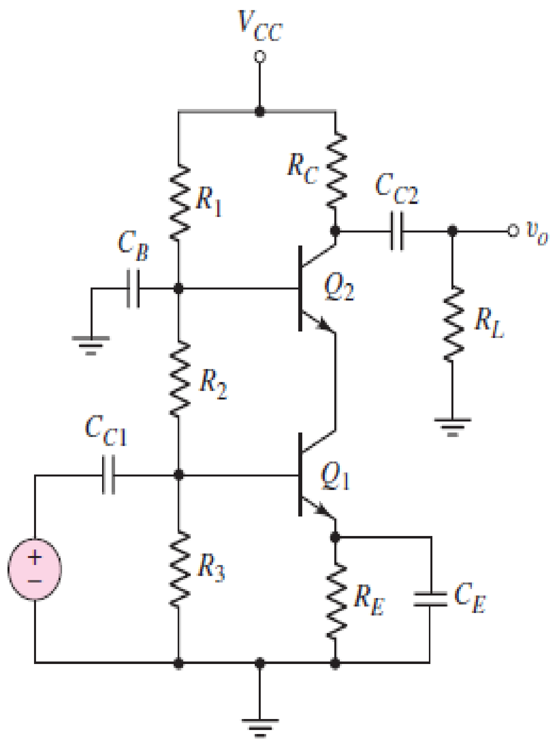
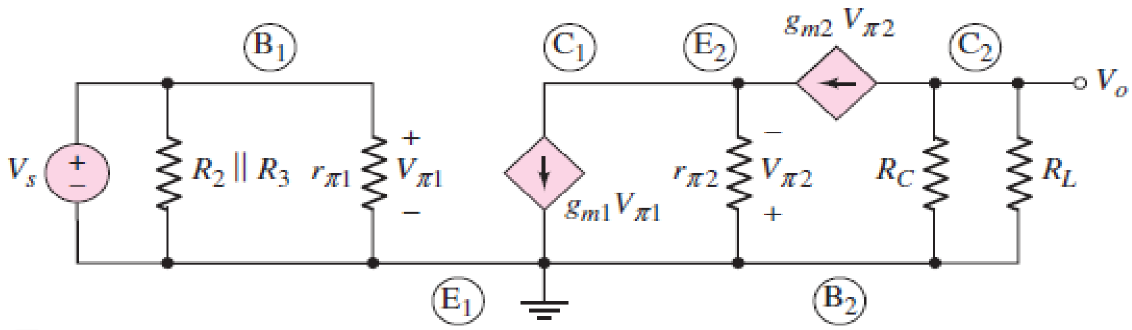


Fig. Cascode amplifier



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. Small-signal equivalent circuit of the cascode amplifier

The bias resistance of the amplifier is $\boldsymbol{R_{B1} = R_2 \parallel R_3}$

The input resistance of the amplifier is $\boldsymbol{R_i = R_{B1} \parallel r_{\pi 1}}$

The out resistance of the amplifier is $\boldsymbol{R_o = R_C}$

$V_{\pi 1} = V_s$ since we are assuming an ideal signal voltage source.

The Voltage gain of the amplifier is

$$A_{VS} = \frac{V_O}{V_s} \qquad ---1$$

The output voltage is

$$V_o = -g_{m2}V_{\pi2}[R_c \parallel \boldsymbol{R_L}] \qquad ---2$$

Writing a KCL equation at E_2, we have

$$g_{m1}V_{\pi1} = \frac{V_{\pi2}}{r_{\pi2}} + g_{m2}V_{\pi2}$$

The control voltage $V_{\pi2}$ (noting that $V_{\pi1} = V_s$), *we find*

$$V_{\pi2} = g_{m1}V_s r_{\pi2} - g_{m2}V_{\pi2}\, r_{\pi2}$$

$$V_{\pi2} + g_{m2}V_{\pi2}\, r_{\pi2} = g_{m1}V_s r_{\pi2}$$

$$V_{\pi2}(1 + g_{m2}\, r_{\pi2}) = g_{m1}V_s r_{\pi2}$$

$$V_{\pi2} = \frac{g_{m1}V_s r_{\pi2}}{(1+g_{m2}\, r_{\pi2})} \qquad \therefore \beta_2 = g_{m2}\, r_{\pi2}$$

$$V_{\pi2} = \left(\frac{r_{\pi2}}{1+\beta_2}\right) g_{m1}V_s \qquad ---3$$

Substitute eq. (3) in eq. (2), we get output voltage as

$$V_o = -g_{m1}g_{m2}\left(\frac{r_{\pi2}}{1+\beta_2}\right) g_{m1}V_s[R_c \parallel \boldsymbol{R_L}]$$

Therefore, the small-signal voltage gain is

$$\boldsymbol{A_{vs} = \frac{V_o}{V_s} = -g_{m1}g_{m2}\left(\frac{r_{\pi2}}{1+Q_2}\right) g_{m1}\,[R_c \parallel R_L]}$$

We know that $g_{m2}\left(\frac{r_{\pi2}}{1+\beta_2}\right) = \frac{\beta_2}{1+\beta_2} \cong 1$

The gain of the cascode amplifier is then approximately

$$\boldsymbol{A_{vs} \cong = -g_{m1}\,[R_c \parallel R_L]}$$

Which is the same as for a single-stage common-emitter amplifier.

The current gain of the amplifier is $\boldsymbol{A_{IS}} = \frac{I_O}{I_s}$

The output current is

$$I_O = -g_{m2} V_{\pi 2} \left(\frac{R_C}{R_C + R_L}\right) \qquad ------(4)$$

But we know that

$$V_{\pi 2} = \frac{r_{\pi 2}}{1+\beta_2} g_{m1} V_s \qquad ----(5)$$

Substitute eq.(5) in eq.(4), we get

$$I_O = -g_{m2} \left(\frac{R_C}{R_C + R_L}\right) \frac{r_{\pi 2}}{1+\beta_2} g_{m1} V_s \quad ---(6)$$

We know that

$$V_s = I_s R_i \quad ---(7)$$

Substitute eq.(7) in eq.(6) we get

$$I_O = -g_{m2} \left(\frac{R_C}{R_C + R_L}\right) \frac{r_{\pi 2}}{1+\beta_2} g_{m1} I_s R_i$$

The current gain of the amplifier is

$$A_{IS} = \frac{I_O}{I_i} = -g_{m1} g_{m2} \left(\frac{r_{\pi 2}}{1 + Q_2}\right) \left(\frac{R_C}{R_C + R_L}\right) R_i$$

We know that $g_{m2} \left(\frac{r_{\pi 2}}{1+\beta_2}\right) = \frac{\beta_2}{1+\beta_2} \cong 1$

$$A_{IS} = -g_{m1} \left(\frac{R_C}{R_C + R_L}\right) R_i$$

Darlington Pair amplifier:

- The Darlington pair invented at Bell by the engineer Darlington in 1953
- The main advantage of Darlington pair is the composite transistor act as a single transistor with high current gain.
- The darlington pair made up with two transistors of current gain β1, β2

- The overall current gain of darrlington pair is multiplication of individual current gains.
    - If matched transistors then β1=β2 then $\beta_d = \beta 1^2$

- Generally darlington pair available in package contain 3 terminals i.e. base ,emitter ,and collector
- Darlington pair can be used as emitter follower its equivalent is cascading of two emitter followers.

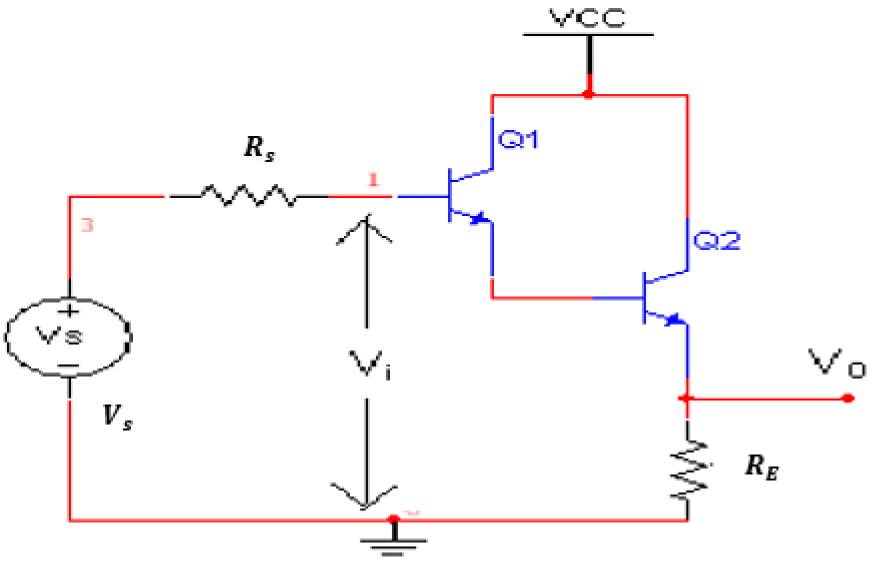


Fig: A Darlington pair amplifier or configuration

The Darlington pair basically consisting of two bipolar transistors with the emitter of one transistor connected to the base of the other, such that the current amplified by the first transistor is amplified further by the second one.

The collectors of both transistors are connected together. It is often packaged as a single transistor. The high current gain and high input impedance are the advantages of Darlington pair. One drawback is doubling of the base–emitter voltage.

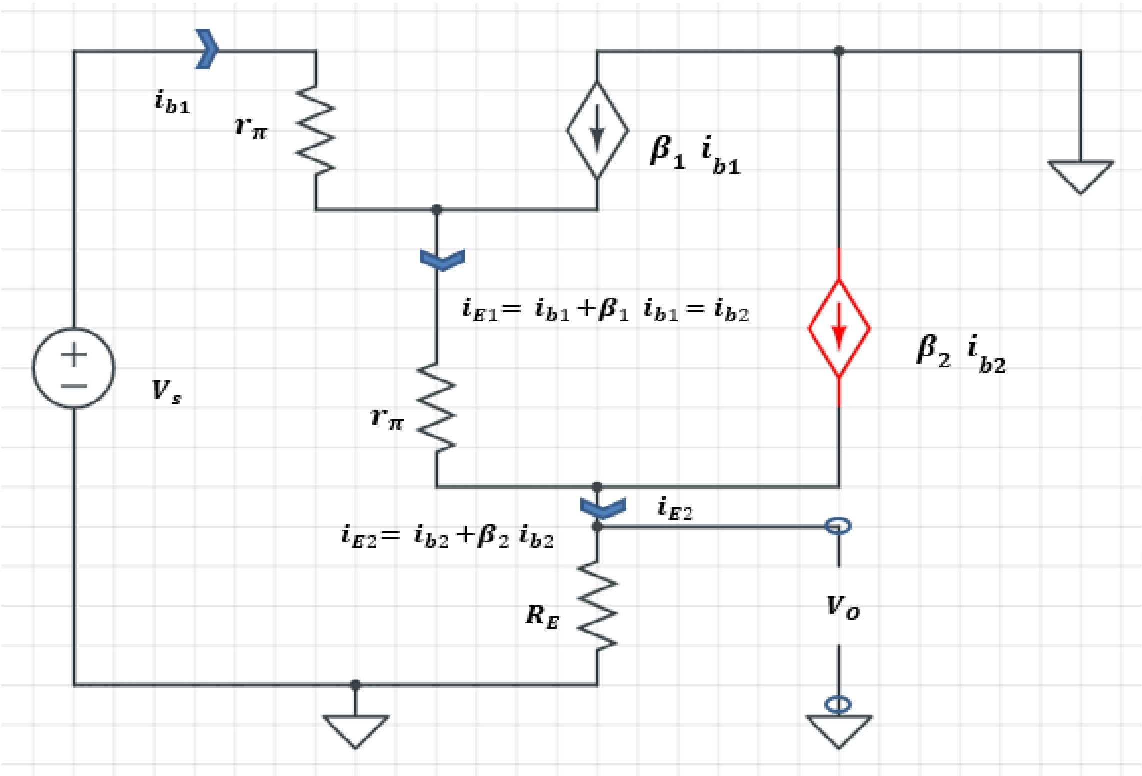


Fig: Small-signal equivalent circuit of Darlington pair

$i_{E1} = i_{b1} + \beta_1 i_{b1} = i_{b1}[1 + \beta_1] = i_{b2}$ ---1

$i_{E2} = i_{b2} + \beta_1 i_{b2} = i_{b2}[1 + \beta_2]$ ---2

Substitute equation 1 in 2

$i_{E2} = i_{b2}[1 + \beta_2] = i_{b1}[1 + \beta_1][1 + \beta_2]$ ---3

Current gain

$$A_I = \frac{i_{E2}}{i_{b1}} = \frac{i_{b1}[1+\beta_1][1+\beta_2]}{i_{b1}}$$

$A_I = [1 + \beta_1][1 + \beta_2]$ $\quad\therefore \beta_1 \gg 1, \beta_2 \gg 1$

$A_I = \beta_1\ \beta_2$ $\quad\therefore$ identical transistors $\beta_1 = \beta_2$

$$\mathbf{A_I = Q_1^2 = Q_2^2 = Q_D}$$

Voltage gain

$$A_V = \frac{V_O}{V_S} = \frac{i_{E2} R_E}{V_S} = \frac{i_{b1}[1+\beta_1][1+\beta_2]\ R_E}{V_S}$$ ---4

Apply KVL for input loop

$V_S = i_{b1}\ r_{\pi 1} + i_{b2}\ r_{\pi 2} + i_{E2} R_E$ ---5

Substitute equation 1, 2 in 5

$$V_S = i_{b1}\, r_{\pi 1} + i_{b1}[1+\beta_1]\, r_{\pi 2} + i_{b1}[1+\beta_1][1+\beta_2]\, R_E$$

$$V_S = i_{b1}[r_{\pi 1} + [1+\beta_1]\, r_{\pi 2} + [1+\beta_1][1+\beta_2]\, R_E] \qquad \therefore \beta_1 \gg 1, \beta_2 \gg 1$$

$$V_S = i_{b1}[r_{\pi 1} + \beta_1 r_{\pi 2} + \beta_1 \beta_2 R_E] \qquad \therefore r_{\pi 1} \ll \beta_1 r_{\pi 2} \qquad \therefore r_{\pi 2} = \frac{\beta_2}{g_m}$$

$$V_S = i_{b1}[\beta_1 \frac{\beta_2}{g_m} + \beta_1 \beta_2 R_E] \qquad \therefore \beta_1 \beta_2 = \beta_D$$

$$V_S = i_{b1}[\frac{\beta_D}{g_m} + \beta_D R_E]$$

$$V_S = i_{b1}\beta_D [\frac{1}{g_m} + R_E] \qquad \text{---6}$$

Substitute equation 6 in 4

$$A_V = \frac{i_{b1}[1+\beta_1][1+\beta_2]\, R_E}{i_{b1}\beta_D[\frac{1}{g_m}+R_E]} \qquad \therefore \beta_1 \beta_2 = \beta_D$$

$$A_V = \frac{i_{b1}\beta_D\, R_E}{i_{b1}\beta_D[\frac{1}{g_m}+R_E]}$$

$$A_V = \frac{i_{b1}\beta_D\, R_E}{i_{b1}\beta_D[\frac{1}{g_m}+R_E]}$$

$$\boxed{A_V = \frac{R_E}{[\frac{1}{g_m}+R_E]}} \qquad \therefore \frac{1}{g_m} \ll R_E$$

$$A_V = \frac{R_E}{R_E} \cong 1$$

Input impedance

$$Z_i = \frac{V_S}{I_{b1}} = \frac{i_{b1}\beta_D[\frac{1}{g_m}+R_E]}{I_{b1}}$$

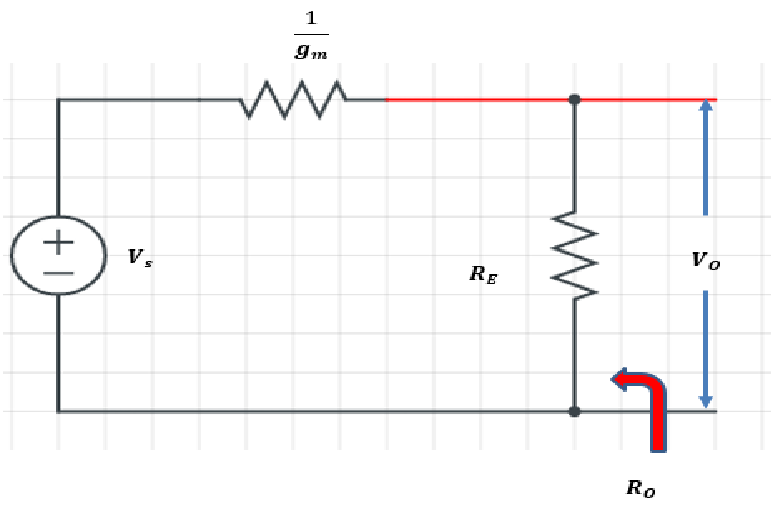
$$Z_i = Q_D\, [\frac{1}{g_m} + R_E]$$

Output impedance

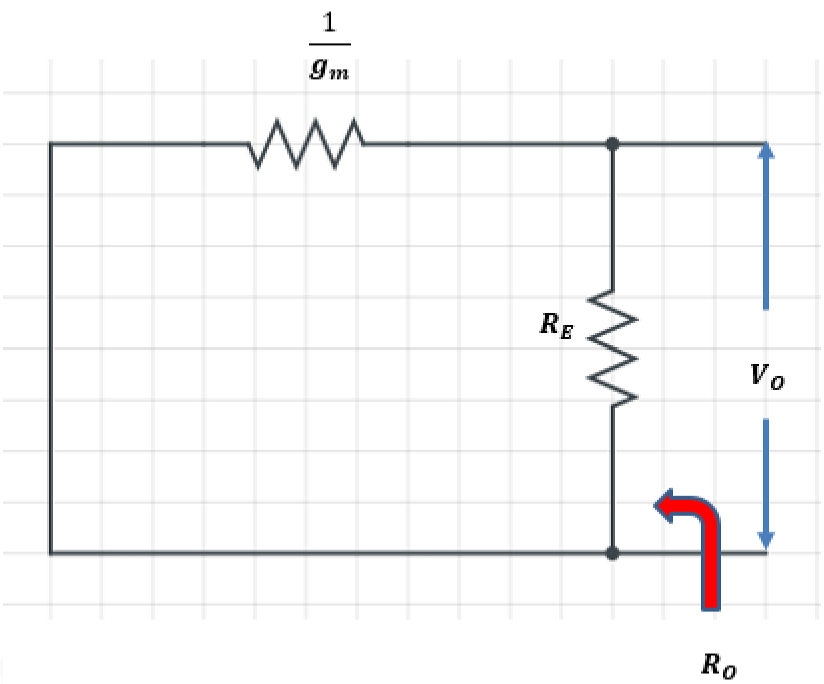
$$A_V = \frac{V_O}{V_S} = \frac{R_E}{[\frac{1}{g_m}+R_E]}$$

$$V_O = \frac{R_E}{[\frac{1}{g_m}+R_E]} V_S \qquad \text{---1}$$

Equation 1 convert to equivalent circuit



For calculating output impedance $V_S = 0$, the above circuit simplified to



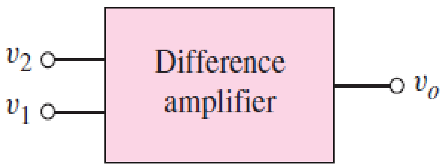
$Z_O = \frac{1}{g_m} || R_E$ ∴Low resistance || high resistance ≅ Low resistance $\therefore \frac{1}{g_m} \ll R_E$

$$Z_O \cong \frac{1}{g_m}$$

Differential pair or Amplifier:

- The differential-pair or differential-amplifier is the most widely used building block in analog integrated-circuit design. For instance, the input stage of every op-amp is a differential amplifier.
- The BJT differential amplifier is the basis of a very-high-speed logic circuit family like emitter-coupled logic (ECL).
- Differential Amplifiers is an amplifier in which the output voltage is directly proportional to the difference of input signal. This amplifier, also called a diff-amp

  There are two input terminals and one output terminal



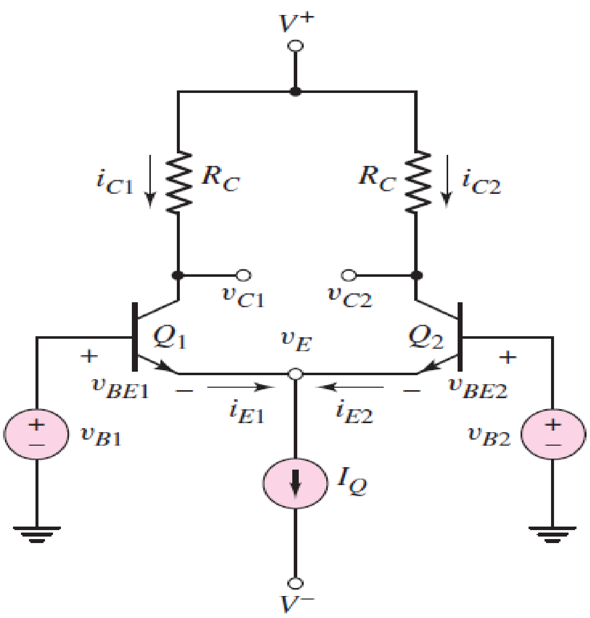
$$V_o = A_{VO}(V_1 - V_2)$$

Where A_{vo} is called the open-loop voltage gain.

The **differential-mode input voltage** $V_d = V_1 - V_2$

The **common-mode input voltage** $V_{cm} = \frac{V_1+V_2}{2}$

The BJT Differential Pair:



- The basic BJT differential pair is shown in fig.

Fig. The basic BJT differential pair

- It consists of two identical transistors Q_1 and Q_2, whose emitters are joined together and biased by a constant-current source I_Q. Each collector is connected to the V_{CC} through a resistance R_C. The two transistors must be operated in active region.

Basic Operation of the BJT differential pair: case (I): $v_{B1} = v_{B2} = V_{CM}$

- Let us consider the first case where the two bases joined together and connected to a common-mode voltage V_{CM} as shown in fig.(a) and $v_{B1} = v_{B2} = V_{CM}$.
- Since Q_1 and Q_2 are matched, the current I will divide equally between the two transistors. Thus, $i_{E1} = i_{E2} = I/2$. The voltage at the emitters will be $V_{CM} - V_{BE}$
- Where $V_{BE} = 0.7$ V
- The voltage at each collector will be $V_{CC} - \frac{\alpha I}{2} R_C$

$$I_{E1} = I_{E2} = \frac{\alpha I_Q}{2}$$

$$V_{C1} = V_{CC} - \frac{\alpha I}{2} R_C = V_{C2}$$

∴ If the base currents are negligible $I_{C1} = I_{E1}$, $I_{C2} = I_{E2}$

$$\frac{\alpha I_Q}{2} \cong \frac{I_Q}{2}$$

$$\boldsymbol{V_o = V_{C2} - V_{C1} = 0}$$

- The difference in voltage between the two collectors will be zero. Thus the differential pair does not respond to (i.e., it rejects) to the common-mode input voltage.

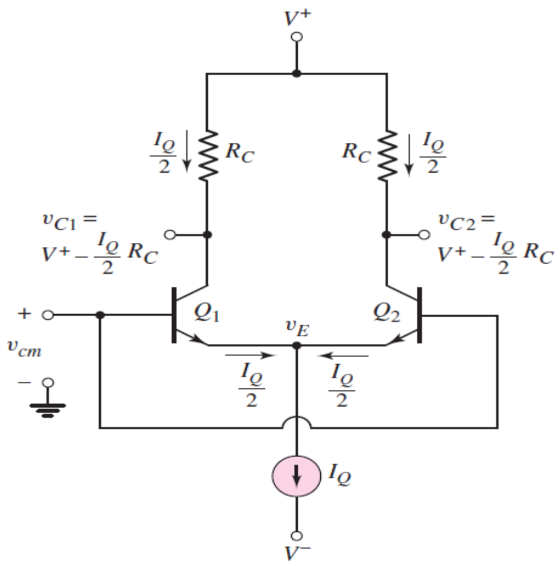


Fig: The differential pair with a common-mode input voltage

Case II: if V_{B1} increases by a few millivolts and V_{B2} decreases by the same amount.

$V_{BE1} > V_{BE2}$, which means that i_{C1} increases by ΔI above its quiescent value and i_{C2} decreases by ΔI *times*

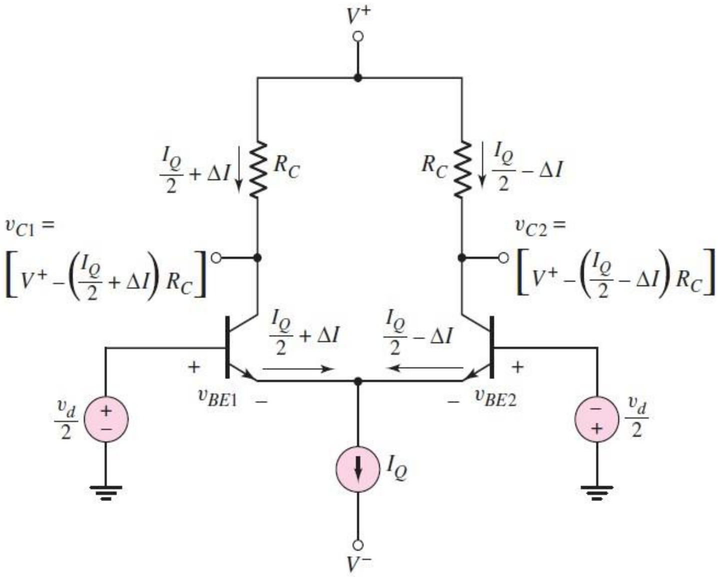
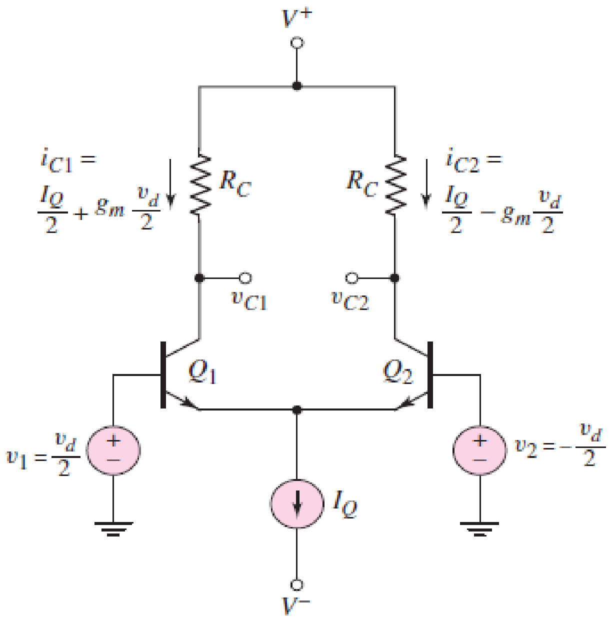


Fig: Differential pair with a small differential input signal

- ➢ From Fig, It can be seen that we are able to steer the entire bias current from one side of the pair to the other with small difference voltages. This current steering property of the differential pair allows it to be used in logic circuits.
- ➢ Apply a very small differential signal, which will result in one of the transistors conducting a current of $\frac{I}{2} + \boldsymbol{\Delta I}$; the current in the other transistor will be $\frac{I}{2} - \boldsymbol{\Delta I}$ . .

$$\boldsymbol{V_o = V_{C2} - V_{C1} = [V_{CC} - (\frac{I_Q}{2} - \Delta I)\, R_C] - [V_{CC} - (\frac{I_Q}{2} + \Delta I)\, R_C] = 2\Delta I R_C}$$

A voltage difference is created between v_{C2} and v_{C1} when a differential-mode input voltage is applied.



The magnitude of the small-signal collector current in each transistor is then $(g_m v_d)/2$.

$V_o = V_{C2} - V_{C1}$

$V_o = [V_{CC} - I_{C2}R_C] - [V_{CC} - I_{C1}R_C] = (I_{C1} - I_{C2})R_C$

$V_o = [(\frac{I_Q}{2} + \frac{g_m V_d}{2}) - (\frac{I_Q}{2} - \frac{g_m V_d}{2})]\ R_C$

$$V_o = g_m V_d R_C$$

Differential mode gain:

The ratio of the output signal voltage to the differential-mode input signal is called the differential-mode gain, *Ad*, which is

$V_o = g_m V_d R_C$

$$A_d = \frac{V_o}{V_d} = g_m R_c$$

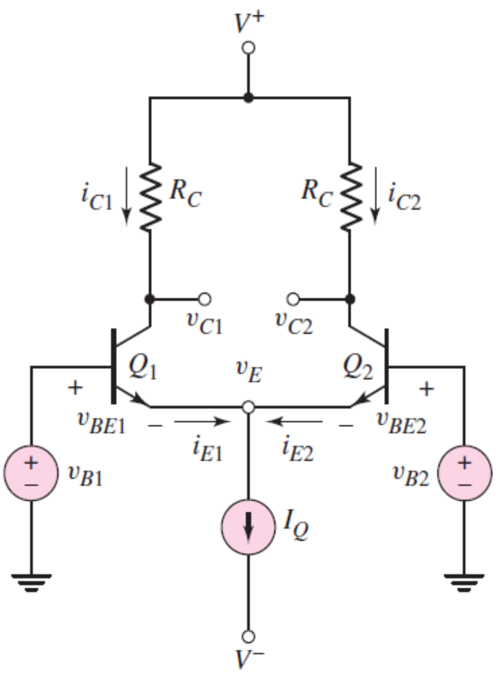
$$\therefore g_m = \frac{I_C}{V_T} = \frac{\frac{I_Q}{2}}{V_T} = \frac{I_Q}{2V_T}$$

In many cases output is taken at one collect terminal which is called a **one-sided output**

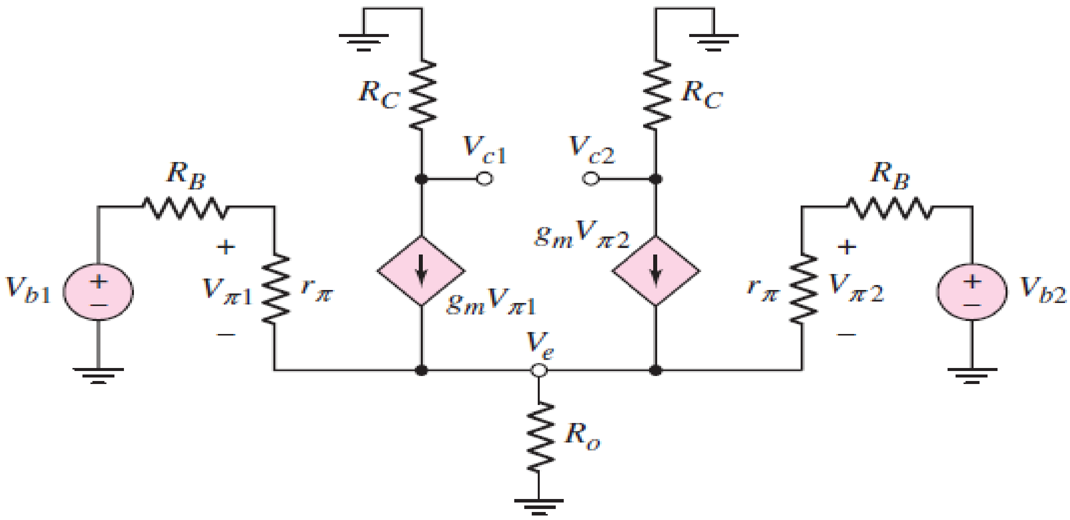
$V_o = \frac{g_m V_d R_C}{2}$

$A_d = \frac{g_m R_C}{2}$ ∴ For Unbalanced output /Single output

Small signal analysis of BJT differential pair



The small signal equivalent circuit of differential amplifier is given as



Assuming the transistors are identical , so $r_{\pi1} = r_{\pi2} \equiv r_\pi$ and $g_{m1} = g_{m2} \equiv g_m$

Apply KCL at node V_e

$$\frac{V_{\pi1}}{r_\pi} + g_m V_{\pi1} + g_m V_{\pi2} + \frac{V_{\pi2}}{r_\pi} = \frac{V_e}{R_o} \qquad \text{---1}$$

$$V_{\pi1}\left[\frac{1}{r_\pi} + g_m\right] + V_{\pi1}\left[\frac{1}{r_\pi} + g_m\right] = \frac{V_e}{R_o}$$

$$V_{\pi1}\left[\frac{1+g_m r_\pi}{r_\pi}\right] + V_{\pi1}\left[\frac{1+g_m r_\pi}{r_\pi}\right] = \frac{V_e}{R_o} \qquad \therefore g_m r_\pi = \beta$$

$$V_{\pi1}\left[\frac{1+\beta}{r_\pi}\right] + V_{\pi1}\left[\frac{1+\beta}{r_\pi}\right] = \frac{V_e}{R_o} \qquad \text{---2}$$

From the circuit

$$\frac{V_{\pi1}}{r_\pi} = \frac{V_{b1}-V_e}{r_\pi+R_B} \quad and \quad \frac{V_{\pi2}}{r_\pi} = \frac{V_{b2}-V_e}{r_\pi+R_B}$$

$$V_{\pi1} = r_\pi\left[\frac{V_{b1}-V_e}{r_\pi+R_B}\right] \; and \; V_{\pi2} = r_\pi\left[\frac{V_{b2}-V_e}{r_\pi+R_B}\right]$$

Substituting $V_{\pi1}$, $V_{\pi2}$ in equation 2

$$r_\pi\left[\frac{V_{b1}-V_e}{r_\pi+R_B}\right]\left[\frac{1+\beta}{r_\pi}\right] + r_\pi\left[\frac{V_{b2}-V_e}{r_\pi+R_B}\right]\left[\frac{1+\beta}{r_\pi}\right] = \frac{V_e}{R_o}$$

$$\left[\frac{1+\beta}{r_\pi+R_B}\right][V_{b1} + V_{b2} - 2V_e] = \frac{V_e}{R_o}$$

$$[V_{b1} + V_{b2} - 2V_e] = \frac{V_e}{R_o}\left[\frac{r_\pi+R_B}{1+\beta}\right]$$

$$[V_{b1} + V_{b2}] = \frac{V_e}{R_o}\left[\frac{r_\pi+R_B}{1+\beta}\right] + 2V_e$$

$$[V_{b1} + V_{b2}] = [[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2]\, V_e$$

$$V_e = \frac{[V_{b1}+V_{b2}]}{[[\frac{r_\pi+R_B}{(1+\beta)R_o}]+2]} \qquad \text{---3}$$

If we consider a one-sided output at the collector of Q_2, then

$$V_o = V_{C2} = -(g_m V_{\pi 2})R_C = -\beta i_{b2} R_C = -\beta R_C\, [\frac{V_{b2}-V_e}{r_\pi+R_B}] \qquad \text{---4}$$

Substitute eq3 in eq4 $\quad \therefore g_m V_\pi = I_c = \beta I_b \quad \therefore i_{b2} = [\frac{V_{b2}-V_e}{r_\pi+R_B}]$

$$V_o = \frac{-\beta R_C}{r_\pi+R_B}\, [V_{b2} - \frac{[V_{b1}+V_{b2}]}{[[\frac{r_\pi+R_B}{(1+\beta)R_o}]+2]}]$$

$$V_o = \frac{-\beta R_C}{r_\pi+R_B}\, [\frac{2V_{b2} + V_{b2}[\frac{r_\pi+R_B}{(1+\beta)R_o}] - V_{b1} - V_{b2}}{[[\frac{r_\pi+R_B}{(1+\beta)R_o}]+2]}]$$

$$\boldsymbol{V_o = \frac{-\beta R_C}{r_\pi+R_B}\, \{\frac{V_{b2}[[\frac{r_\pi+R_B}{(1+\beta)R_o}]+1] - V_{b1}}{[\frac{r_\pi+R_B}{(1+\beta)R_o}]+2}\}} \qquad \text{---5}$$

In an ideal constant-current source, the output resistance is $R_o = \infty$, and above Equation reduces to

$$V_o = \frac{-\beta R_C [V_{b2} - V_{b1}]}{2(r_\pi+R_B)}$$

The differential-mode input is $V_d = V_{b2} - V_{b1}$

$$V_o = \frac{-\beta R_C V_d}{2(r_\pi+R_B)}$$

The differential-mode gain is

$$\boldsymbol{A_d = \frac{V_o}{V_d} = \frac{-\beta R_C}{2(r_\pi+R_B)}}$$

The two input signals can be written as the sum of a differential-mode input signal component and a common-mode input signal component

V_{b1} and V_{b2} in terms of V_d and V_{cm}

$$V_{b1} = V_{cm} - \frac{v_d}{2} \qquad and \;\; V_{b2} = V_{cm} - \frac{v_d}{2}$$

Substitute above values in equation 5

$$V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{(V_{cm} - \frac{v_d}{2})[[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 1] - (V_{cm} + \frac{v_d}{2})}{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2}\right\}$$

$$V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{V_{Cm}[[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 1 - 1] - \frac{v_d}{2}[[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 1 + 1]}{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2}\right\}$$

$$V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{V_{Cm}[[\frac{r_\pi + R_B}{(1+\beta)R_o}]] - \frac{v_d}{2}[[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2]}{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2}\right\}$$

$$V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{V_{Cm}[[\frac{r_\pi + R_B}{(1+\beta)R_o}]]}{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2} - \frac{v_d}{2}\frac{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2}{[\frac{r_\pi + R_B}{(1+\beta)R_o}] + 2}\right\}$$

$$V_o = \left(\frac{-\beta R_C}{r_\pi + R_B}\right)\frac{V_{Cm}[[\frac{r_\pi + R_B}{(1+\beta)R_o}]]}{\frac{(r_\pi + R_B) + 2(1+\beta)R_o}{(1+\beta)R_o}} + \frac{\beta R_C}{r_\pi + R_B}\frac{v_d}{2}$$

$$V_o = \left[\frac{-\beta R_C}{(r_\pi + R_B) + 2(1+\beta)R_o}\right]V_{Cm} + \left[\frac{\beta R_C}{r_\pi + R_B}\right]\frac{v_d}{2} \qquad \text{---6}$$

We can write the output voltage in the general form

$$V_o = A_{cm}V_{Cm} + A_d V_d \qquad \text{---7}$$

Compare equations 6 and 7

$$A_{cm} = \frac{-QR_C}{(r_\pi + R_B) + 2(1 + Q)R_o} \quad and \quad A_d = \frac{QR_C}{(r_\pi + R_B)2}$$

The common-mode gain goes to zero for an ideal current source in which *Ro* =∞.

For Two sided output (Balanced output)

$V_o = V_{C2} - V_{C1}$

The differential mode voltage gain is $A_d = \frac{QR_C}{r_\pi + R_B}$ and the common-mode voltage gain is given by

, $A_{cm} = 0$

Common-Mode Rejection Ratio (CMRR): The ability of a differential amplifier to reject a common-mode signal is described in terms of the common-mode rejection ratio (CMRR

The CMRR is a figure of merit for the diff-amp and is defined as

$$CMRR = \frac{A_d}{A_{cm}}$$

For an ideal diff-amp, $A_{cm} = 0$ and CMRR = ∞.

Usually, the CMRR is expressed in decibels, as follows:

$$CMRR_{dB} = 20\ \log_{10} \frac{A_d}{A_{cm}}$$

Form the differential amplifier analysis we can substitute the A_{cm} & A_d and CMRR Can be expressed as,

$$CMRR = \frac{A_d}{A_{cm}} = \frac{\frac{\beta R_C}{(r_\pi + R_B)2}}{\frac{-\beta R_C}{(r_\pi + R_B) + 2(1+\beta)R_o}}$$

$$CMRR = \frac{A_d}{A_{cm}} = \frac{1}{2}\frac{(r_\pi + R_B) + 2(1+\beta)R_o}{(r_\pi + R_B)}$$

$$CMRR = \frac{A_d}{A_{cm}} = \frac{1}{2}[1 + \frac{2(1+Q)R_o}{(r_\pi + R_B)}]$$

Determine the differential- and common-mode gains and the common mode rejection ratio of a diff-amp$V+$ = 10 V, $V-$ = −10 V, I_Q = 0.8 mA, and R_C = 12 k.The transistor parameters are β = 100 and R_o = 25 k. Assume the source resistors R_B are zero. Use a one-sided output at V_{C2}.

$$A_d = \frac{\beta R_C}{(r_\pi + R_B)2} \qquad \therefore R_B = 0$$

$$A_d = \frac{g_m R_C}{2} \qquad \therefore g_m r_\pi = \beta$$

$$A_d = \frac{g_m R_C}{2} = \frac{I_{CQ} R_C}{2V_T} = \frac{\frac{I_Q}{2} R_C}{2V_T} = \frac{I_Q R_C}{4V_T} = \frac{(0.8)(12)}{4\,(0.026)} = \mathbf{92.3} \qquad \therefore V_T = 26\,mv$$

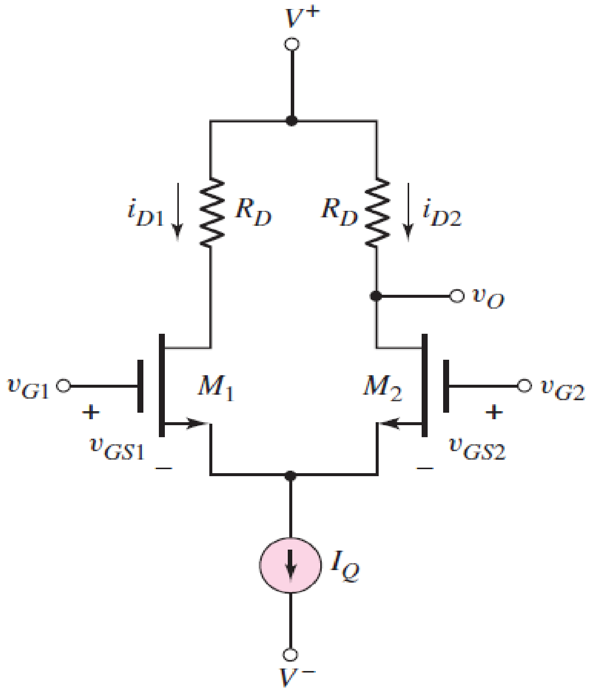
$$A_{cm} = \frac{-\beta R_C}{(r_\pi + R_B) + 2(1+\beta)R_o} = \mathbf{-0.237} \qquad \therefore r_\pi = \frac{\beta}{g_m}$$

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \left|\frac{92.3}{-0.237}\right| = \mathbf{389}$$

$$CMRR_{dB} = 20\ \log_{10} CMRR = 20\ \log_{10} 389 = \mathbf{51.8\ dB}$$

MOSFET Differential Amplifier:

- The basic MOSFET differential pair, with matched transistors M_1 and M_2 biased with a constant current I_Q.
- Assume that M_1 and M_2 are always biased in the saturation region.
- Even with $v_{G1} = v_{G2} = 0$, the transistors M_1 and M_2 can be biased in the saturation region by the current source I_Q.



This is a DC coupled diff amplifier

The ac equivalent circuit of the diff-amp configuration, Shown in fig.

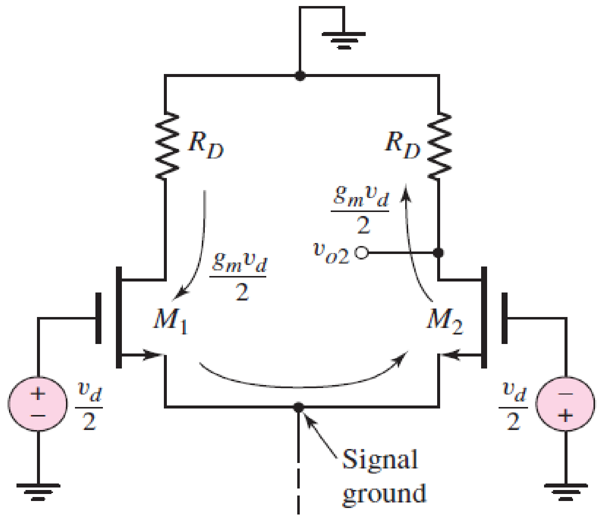


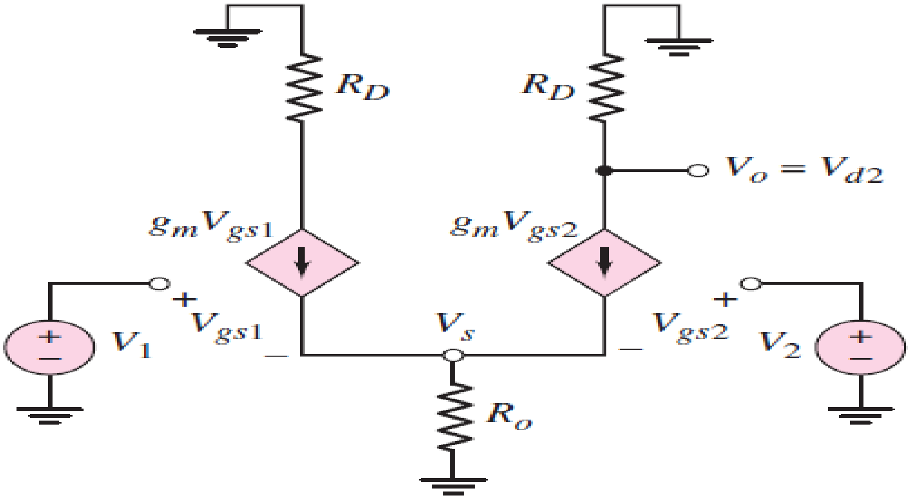
Figure AC equivalent circuit, MOSFET differential amplifier

One sided output voltage is:

$$V_O = V_{o2} = [\frac{g_m V_d}{2}] R_D$$

$$\boldsymbol{A_d = \frac{V_o}{V_d} = \frac{g_m R_D}{2}}$$

Small-Signal Equivalent Circuit Analysis (MOSFT Diff Amp)



Assuming the Transistors are identical in all parameters. (g_{m1}=g_{m2}=g_m)

Apply KCL at node V_s

$$g_m V_{gs1} + g_m V_{gs2} = \frac{V_s}{R_o} \qquad \text{---1}$$

From the circuit $V_{gs1} = V_1 - V_s$ *and* $V_{gs2} = V_2 - V_s$

Substitute above values in equation 1

$$g_m (V_1 - V_s) + g_m (V_2 - V_s) = \frac{V_s}{R_o}$$

$$g_m (V_1 + V_2 - 2V_s) = \frac{V_s}{R_o}$$

$$(V_1 + V_2 - 2V_s) = \frac{V_s}{g_m R_o}$$

$$(V_1 + V_2) = \frac{V_s}{g_m R_o} + 2V_s$$

$$(V_1 + V_2) = V_s \left(2 + \frac{1}{g_m R_o}\right)$$

$$V_s = \frac{(V_1 + V_2)}{\left(2 + \frac{1}{g_m R_o}\right)} \qquad \text{---2}$$

For a one-sided output at the drain of M_2

$$V_O = V_{d2} = -(g_m V_{gs2}) R_D = -g_m R_D (V_2 - V_s) \qquad \text{---3} \qquad \therefore V_{gs2} = V_2 - V_s$$

Substitute equation 2 in equation 3

$$V_O = -g_m R_D \left[V_2 - \frac{(V_1+V_2)}{(2+\frac{1}{g_mR_o})}\right]$$

$$V_O = -g_m R_D \left[\frac{V_2(2+\frac{1}{g_mR_o})-V_1-V_2}{(2+\frac{1}{g_mR_o})}\right]$$

$$V_O = -g_m R_D \left[\frac{V_2(2+\frac{1}{g_mR_o}-1)-V_1}{(2+\frac{1}{g_mR_o})}\right]$$

$$\boldsymbol{V_O = -g_m R_D \left[\frac{V_2(1+\frac{1}{g_mR_o})-V_1}{(2+\frac{1}{g_mR_o})}\right]} \qquad \textbf{---4}$$

V_{b1} and V_{b2} in terms of V_d and V_{cm}

$$V_1 = V_{cm} - \frac{v_d}{2} \quad and \quad V_2 = V_{cm} - \frac{v_d}{2}$$

Substitute above values in equation 4

$$V_O = -g_m R_D \left[\frac{(V_{cm}-\frac{v_d}{2})(1+\frac{1}{g_mR_o}) - (V_{cm}+\frac{v_d}{2})}{(2+\frac{1}{g_mR_o})}\right]$$

$$V_O = -g_m R_D \left[\frac{V_{cm}(1+\frac{1}{g_mR_o}-1)-\frac{v_d}{2}(1+\frac{1}{g_mR_o}+1)}{(2+\frac{1}{g_mR_o})}\right]$$

$$V_O = -g_m R_D \left[\frac{V_{cm}(\frac{1}{g_mR_o})-\frac{v_d}{2}(2+\frac{1}{g_mR_o})}{(2+\frac{1}{g_mR_o})}\right]$$

$$V_O = -g_m R_D \left[\frac{(\frac{1}{g_mR_o})}{(\frac{2g_mR_o+1}{g_mR_o})}\right] V_{cm} + \frac{g_mR_D}{2}\left[\frac{(2+\frac{1}{g_mR_o})}{(2+\frac{1}{g_mR_o})}\right] v_d$$

$$V_O = -g_m R_D \left[\frac{1}{2g_mR_o+1}\right] V_{cm} + \frac{g_mR_D}{2} v_d \qquad \text{---5}$$

We can write the output voltage in the general form

$$V_o = A_{cm}V_{Cm} + A_dV_d \qquad \text{---6}$$

Compare equations 5 & 6

$$\boldsymbol{A_d = \frac{g_mR_D}{2} \quad and \quad A_{cm} = \left[\frac{-g_mR_D}{2g_mR_o+1}\right]}$$

The Transconductance g_m of the MOSFET is $2\sqrt{K_n I_{DQ}} = \sqrt{2K_n I_Q}$ $\quad \therefore I_{DQ} = \frac{I_Q}{2}$

Or $\quad g_m = \frac{2I_D}{V_{gs}-V_t} = \frac{2\frac{I_Q}{2}}{V_{ov}} = \frac{I_Q}{V_{ov}}$ $\quad \therefore V_{ov} = V_{gs} - V_t$

$$A_d = \frac{g_m R_D}{2} = \sqrt{2K_n I_Q}\,\frac{R_D}{2} = \sqrt{\frac{K_n I_Q}{2}}\,R_D$$

$$A_{cm} = \left[\frac{-g_m R_D}{2g_m R_o+1}\right] = \left[\frac{-\sqrt{2K_n I_Q}\,R_D}{2\sqrt{2K_n I_Q}\,R_o+1}\right]$$

A_{cm} = 0; when Ro = ∞. (Ideal Source)

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{\sqrt{\frac{K_n I_Q}{2}}\,R_D}{\frac{\sqrt{2K_n I_Q}\,R_D}{2\sqrt{2K_n I_Q}\,R_o+1}} = \frac{1}{2}\,\frac{\sqrt{2K_n I_Q}\,R_D[1+2\sqrt{2K_n I_Q}\,R_o]}{\sqrt{2K_n I_Q}\,R_D}$$

$$\boldsymbol{CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{1}{2}[1+2\sqrt{2K_n I_Q}\,R_o]}$$

For Two-Sided Output (Balanced Output):

$$A_d = g_m R_D \quad and\ A_{cm} = 0$$

Other Nonideal Characteristics of the Differential Amplifier

1. Input Offset Voltage of the MOS Differential Pair:

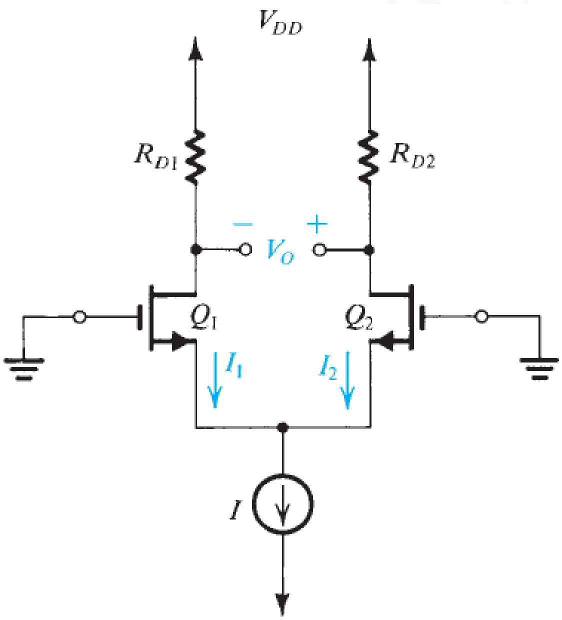


Fig.(a) both inputs grounded

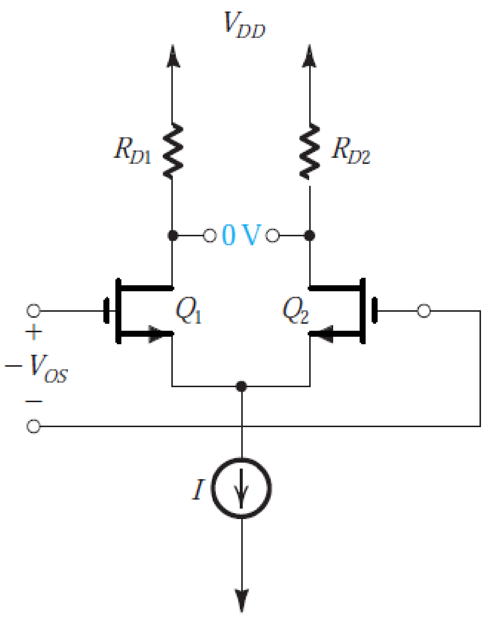


Fig.(b) Vos applied at inputs

- Consider the basic MOS differential amplifier with both inputs grounded, as shown in Fig. (a).
- If the two sides of the differential pair were perfectly matched, then current I would split equally between Q_1 and Q_2, and V_O would be zero.
- But practical circuits exhibit mismatches that result in a dc output voltage V_O even with both inputs grounded and is called output offset dc voltage.
- Input Offset Voltage V_{os} is that voltage which must be applied at the input to make output Offset Voltage zero. $\boldsymbol{V_{OS} = V_0/A_d}$
- The offset voltage is a result of device mismatches, its polarity is not known a priori.
- Three factors contribute to the dc offset voltage of the MOS differential pair:
    - Mismatch in load resistances,
    - Mismatch in W/L,
    - Mismatch in V_t.
- Consider the three contributing factors one at a time.

Consider first the case where Q_1 and Q_2 are perfectly matched but R_{D1} and R_{D2} show a mismatch ΔR_D

$$R_{D1} = R_D + \frac{\Delta R_D}{2} \; and \; R_{D2} = R_D - \frac{\Delta R_D}{2}$$

Because Q_1 *and* Q_2 are matched, the current I will split equally between them. Because of the mismatch in load resistances, the output voltages $V_{D!}$ *and* V_{D2} *will be*

$$V_{D!} = V_{DD} - \frac{I}{2}\left(R_D + \frac{\Delta R_D}{2}\right)$$

$$V_{D2} = V_{DD} - \frac{I}{2}\left(R_D - \frac{\Delta R_D}{2}\right)$$

$$V_o = V_{D2} - V_{D1} = V_{DD} - \frac{I}{2}\left(R_D + \frac{\Delta R_D}{2}\right) - \left(V_{DD} - \frac{I}{2}\left(R_D - \frac{\Delta R_D}{2}\right)\right) = \frac{I}{2}\Delta R_D$$

Input offset voltage is obtained by dividing V_o by the gain $g_m R_D$

$$V_{os} = \frac{V_o}{A_d} = \frac{V_o}{g_m R_D} \qquad \therefore g_m = \frac{I}{V_{ov}}$$

$$\boldsymbol{V_{os} = \frac{\frac{I}{2}\Delta R_D}{\frac{I}{V_{ov}} R_D} = \left(\frac{V_{ov}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right)}$$

Thus the offset voltage is directly proportional to V_{OV}.. Let V_{OV} =0.2V and the worst case mismatch will be $\Delta R_D/R_D = 0.02$. The resulting input offset voltage will be $|V_{0S}| = 0.1X\ 0.02 = 2mV$.

Similarly V_{os} due to mismatch in $\frac{W}{L}$

$$V_{os} = \left(\frac{V_{ov}}{2}\right)\left(\frac{\Delta \frac{W}{L}}{\frac{W}{L}}\right)$$

Similarly V_{os} due to mismatch in V_t

$$V_{os} = \Delta V_t$$

Total input offset voltage can be found as

$$V_{os} = \sqrt{\left(\left(\frac{V_{ov}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right)\right)^2 + \left(\left(\frac{V_{ov}}{2}\right)\left(\frac{\Delta \frac{W}{L}}{\frac{W}{L}}\right)\right)^2 + (\Delta V_t)^2}$$

Input Offset Voltage of the Bipolar Differential Amplifier:

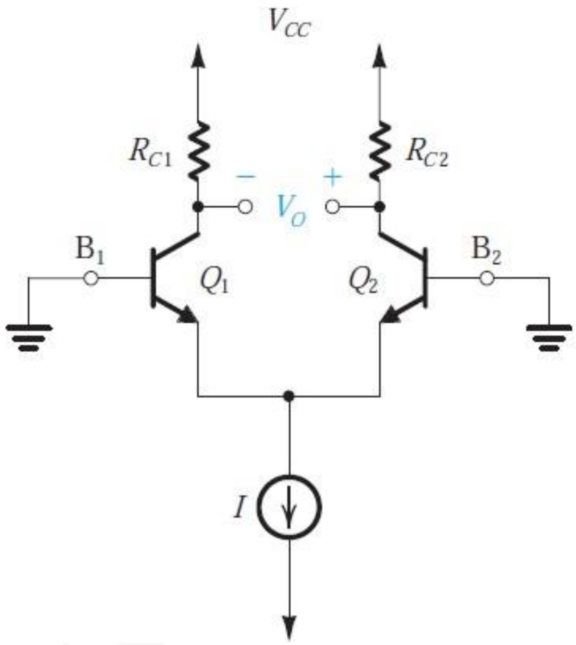


Fig.(a) both inputs grounded

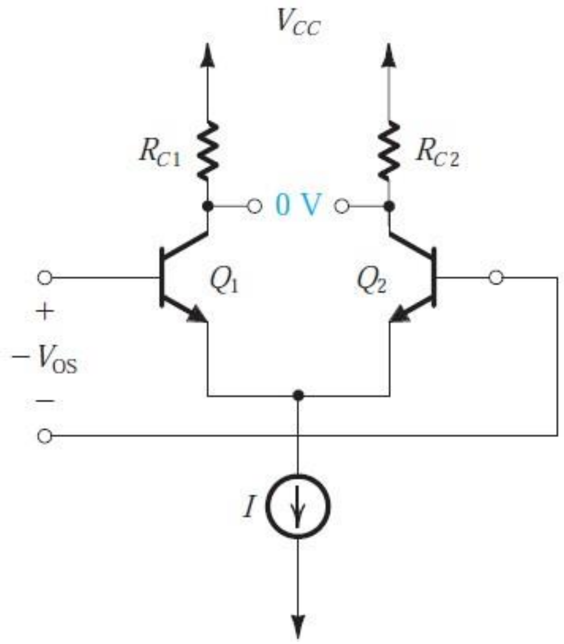


Fig.(b) Vos applied at inputs

Here the output offset results from mismatches in the load resistances in the load resistance R_{c1} *and* R_{c2} and from junction area, β, and other mismatches in Q_1 *and* Q_2

Consider first the effect of the load mismatch

$$R_{c1} = R_c + \frac{\Delta R_c}{2} \quad and\ R_{c2} = R_c - \frac{\Delta R}{2}$$

Assume that Q_1 *and* Q_2 are perfectly matched. The current I will be divide between Q_1 *and* Q_2, and thus

$$V_{c1} = V_{cc} - \left(\frac{\alpha I}{2}\right)\left(R_c + \frac{\Delta R_c}{2}\right) \ and V_{c2} = V_{cc} - \left(\frac{\alpha I}{2}\right)\left(R_c - \frac{\Delta R_c}{2}\right)$$

$$V_o = V_{c2} - V_{c1} = V_{cc} - (\frac{\alpha I}{2})(R_c - \frac{\Delta R_c}{2}) - (V_{cc} - (\frac{\alpha I}{2})(R_c + \frac{\Delta R_c}{2})) = (\frac{\alpha I}{2})\Delta R_c$$

Input offset voltage is obtained by dividing V_o by the gain $g_m R_D$

$$V_{os} = \frac{V_o}{A_d} = \frac{V_o}{g_m R_c} \qquad \therefore g_m = \frac{I_{CQ}}{V_T} = \frac{(\frac{\alpha I}{2})}{V_T}$$

$$V_{os} = \frac{V_o}{A_d} = \frac{(\frac{\alpha I}{2})\Delta R_c}{\frac{(\frac{\alpha I}{2})}{V_T} R_c} = V_T (\frac{\Delta R_c}{R_c})$$

Here the offset voltage is proportional to V_T and $V_T = 25\ mV$. The worst case mismatch will be $\Delta R_D / R_D = 0.02$. The resulting input offset voltage will be $|V_{0S}| = 25\ X\ 0.02 = 0.5mV$.

Let the transistor have a mismatch in their emitter-base junction areas. Such an area mismatch gives rise to a proportional mismatch in the scale currents I_S.

Similarly V_{os} due to mismatch in I_S

$$V_{os} = V_T (\frac{\Delta I_S}{I_S})$$

Total input offset voltage can be found as

$$V_{os} = \sqrt{((\frac{\Delta R_c}{R_c}))^2 + (V_T (\frac{\Delta I_S}{I_S}))^2}$$

$$\boldsymbol{V_{os} = V_T \sqrt{(\frac{\Delta R_c}{R_c})^2 + (\frac{\Delta I_S}{I_S})^2}}$$

Input Bias and Offset Currents of the Bipolar Differential Amplifier:

- In a perfectly symmetric differential pair the two input terminals carry equal dc currents; that is, $\boldsymbol{I_{B1} = I_{B2} = \frac{I/2}{1+Q}}$
- This is the input bias current of the differential amplifier. Mismatches in the amplifier circuit like a mismatch in β make the two input dc currents unequal. The resulting difference is the input offset current, I_{OS}, given as

$$\boldsymbol{I_{OS} = |I_{B1} - I_{B2}|}$$

Let $Q_1 = Q + \frac{\Delta Q}{2}$ **and** $Q_1 = Q - \frac{\Delta Q}{2}$

Then $I_{B1} = \frac{I}{2} \frac{1}{1+\beta+\frac{\Delta\beta}{2}}$ and $I_{B2} = \frac{I}{2} \frac{1}{1+\beta-\frac{\Delta\beta}{2}}$

$$I_{OS} = |I_{B1} - I_{B2}| = \frac{I}{2(1 + Q)}(\frac{\Delta Q}{Q})$$

Formally input bias current I_B is defined as

$$I_B = \frac{I_{B1} + I_{B2}}{2} = \frac{I}{2(1 + Q)}$$

$$I_{OS} = |I_{B1} - I_{B2}| = I_B (\frac{\Delta Q}{Q})$$

For example, a 10% β mismatch results in an offset current that is one-tenth the value of the input bias current.

Finally note that a great advantage of the MOS differential pair is that it does not suffer from a finite input bias current or from mismatches.