

97663

**B.C.A. 1st Semester (New)
Examination-December, 2013**

Mathematics

Paper-BCA-103

Time : 3 hours

Max. Marks : 80

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question no. **1** is **compulsory**.

1. (a) Define union and Intersection of set with examples. $2 \times 8 = 16$
- (b) Write the set in Roster form
 $A = \{x : |x - 3| < 8, x \text{ is an integer}\}$
- (c) Construct 2×2 Matrix $A = [a_{ij}]$, such that $a_{ij} = \frac{(i + 2j)^2}{2}$
- (d) Define equivalence relation with example

(e) Evaluate $\lim_{x \rightarrow 0} \left(\frac{e^x - e^{-x}}{x} \right)$

(f) if $y = \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$, find $\frac{dy}{dx}$

(g) $\int \frac{x}{1 + \sqrt{x}} dx$

(h) $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx$

Unit-I

2. (a) Prove $(A \cup B)' = A' \cap B'$ 8

(b) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$,
 $D = \{2, 4, 5\}$, verify that 8

$$(A \times B) \cap (C \times D) = (A \cap C) \cap (B \cap D)$$

3. (a) Prove that 8

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-z)(y-z)(z-x)$$

(b) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, verify that

$$A \cdot (\text{adj} \cdot A) = (\text{adj} \cdot A) \cdot A = (A)I_3 \quad 8$$

Unit-II

4. (a) Show that the relation R in the set R of real Numbers defined as $R = \{(a, b) : a \leq b\}$, is reflexive and Transition but not symmetric. 8

(b) If $f(x) = \log \frac{1+x}{1-x}$, show that
 $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ 8

5. (a) Evaluate :- 8

(i) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

(ii) $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x}$

- (b) Show that $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist 8

Unit-III

6. (a) If $y = \frac{x}{x+4}$, show that $\frac{xdy}{dx} = y(1-y)$ 8

(b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$ d.w.r.to x 8

7. (a) If $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, find $\frac{dy}{dx}$ 8

(b) If $y = (x + \sqrt{x^2 + a^2})^4$, then Prove that

$$\frac{dy}{dx} = \frac{xy}{\sqrt{x^2 + a^2}} \quad 8$$

Unit-IV

8. (a) Evaluate 8

(i) $\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

(ii) $\int e^x \cos x dx$

(b) (i) $\int_1^2 \frac{1}{x(1+\log x)} dx$ 8

(ii) $\int_0^{\pi/2} \sqrt{1+\sin x} dx$

9. (a) Evaluate 8

(i) $\int \frac{xe^x}{(1+x)^2} dx$

(ii) $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

(b) (i) $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}} dx$

(ii) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx$ 8