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Semester V - Computer Engineering

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Theory of Computer Science

Semester V - Computer Engineering

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EM046A

Theory of Computer Science

Semester V - Computer Engineering (MU)

Edition 2019

Printed at : 37/2, Ashtvinayak Industrial Estate, Near Pari Company,
Narhe, Pune, Maharashtra State India,
Pune – 411041

Published by

TechKnowledge Publications

Head Office : B/5, First floor, Maniratna Complex, Taware Colony, Aranyeshwar Corner,
Pune - 411 009. Maharashtra State, India
Ph : 91-20-24221234, 91-20-24225678.
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Theory of Computer Science

Chapter 1 : Basic Concepts and Finite Automata

Q. 1 Write note on Chomsky Hierarchy.

MU - Dec. 2009, Dec. 2012, May 2013, May 2014, Dec. 2014, May 2015, Dec. 2016, May 2017, Dec. 2017

Ans. : Chomsky Hierarchy

A grammar can be classified on the basis of production rules. Chomsky classified grammars into the following types :

1. Type 3 : Regular grammar
2. Type 2 : Context free grammar
3. Type 1 : Context sensitive grammar
4. Type 0 : Unrestricted grammar.

1. Type 3 or Regular Grammar

A grammar is called Type 3 or regular grammar if all its productions are of the following forms :

$A \rightarrow \varepsilon$

$A \rightarrow a$

$A \rightarrow aB$

$A \rightarrow Ba$

Where, $a \in \Sigma$ and $A, B \in V$.

A language generated by Type 3 grammar is known as regular language.

2. Type 2 or Context Free Grammar

A grammar is called Type 2 or context free grammar if all its productions are of the following form $A \rightarrow \alpha$ where $A \in V$ and $\alpha \in (V \cup T)^*$.

V is a set of variables and T is a set of terminals.

The language generated by a Type 2 grammar is called a context free language, a regular language but not the reverse.

3. Type 1 or Context Sensitive Grammar

A grammar is called a Type 1 or context sensitive grammar if all its productions are of the following form.

$\alpha \rightarrow \beta$

Where, β is atleast as long as α.

4. Type 0 or Unrestricted Grammar

Productions can be written without any restriction in a unrestricted grammar. If there is production of the $\alpha \rightarrow \beta$, then length of α could be more than length of β.

Every grammar also is a Type 0 grammar.

A Type 2 grammar is also a Type 1 grammar

A Type 3 grammar is also a Type 2 grammar.

Q. 2 State applications of Finite Automata in brief.

May 2010

Ans. :

Applications of Finite Automata

Finite automata are used for solving several common types of computer algorithms. Some of them are :

(i) Design of digital circuit

(ii) String matching

(iii) Communication protocols for information exchange.

(iv) Lexical analysis phase of a compiler.

Finite automata can work as an algorithm for regular language. It can be used for checking whether a string $w \in L$, where L is a regular language.

Q. 3 What is Finite Automata? Dec. 2012

Ans. :

Finite Automata

Finite automata are also called a finite state machine.

A finite state machine is a mathematical model for actual physical process. By considering the possible inputs on which these machines can work, one can analyse their strengths and weaknesses.

Finite automata are used for solving several common types of computer algorithms. Some of them are :

1. Design of digital circuits.
2. String matching.
3. Communication protocols for information exchange.
4. Lexical analyser of a typical compiler.

Q. 4 Define the term : Unrestricted grammar

May 2013

Ans. :

Unrestricted grammar

Productions can be written without any restriction in a unrestricted grammar. If there is production of the $\alpha \rightarrow \beta$, then length of α could be more than length of β.

Every grammar also is a Type 0 grammar.

A Type 2 grammar is also a Type 1 grammar

A Type 3 grammar is also a Type 2 grammar.

Chapter 2 : Finite Automata

Q. 1 Write short note on Mealy machine. Dec. 2005

Ans. :

Mealy Machine

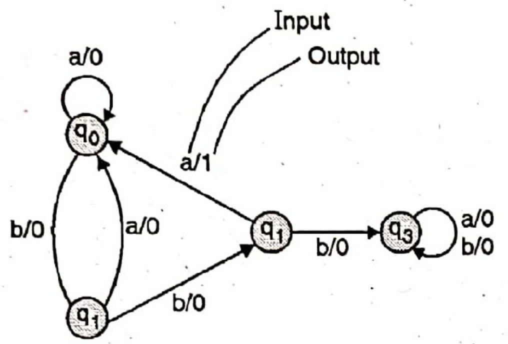


Fig. 2.1 : State diagram of a Mealy machine

State transition function (δ) (or STF) :

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

Fig. 2.2 : State transition function for Mealy machine of Fig. 2.1

Output function (λ) (or MAF) :

	a	b
$\rightarrow q_0$	0	0
q_1	0	0
q_2	1	0
q_3	0	0

Fig. 2.3 : Output function for mealy machine of Fig. 2.1

State table for both δ and λ (both STF and MAF) :

	A	b
$\rightarrow q_0$	$q_0/0$	$q_1/0$
q_1	$q_0/0$	$q_2/0$
q_2	$q_0/1$	$q_3/0$
q_3	$q_3/0$	$q_3/0$

→ Output

→ Next state

Fig. 2.4 : State table depicting both transition and output behavior of mealy machine of Fig. 2.1

An arc from state q_i in a mealy machine is associated with :

1. Input alphabet $\in \Sigma$
2. An output alphabet $\in O$.

An arc marked as 'a/0' in Fig. 2.1 implies that :

1. a is in input
2. 0 is an output.

State transition behavior and output behavior of a mealy machine can be shown separately as in Fig. 2.2 and 2.3; or they can be combined together as in Fig. 2.4.

Formal Definition of a Mealy Machine

A mealy machine M is defined as :

$$M = \{Q, \Sigma, O, \delta, \lambda, q_0\}$$

Where, Q = A finite set of states.

Σ = A finite set of input alphabet

O = A finite set of output alphabet

δ = A transition function $\Sigma \times Q \rightarrow Q$

λ = An output function $\Sigma \times Q \rightarrow O$

q_0 = $q_0 \in Q$ is an initial state.

Q. 2 Distinguish between NFA and DFA.

MU - May 2007, Dec. 2009, May 2011, May 2014, May 2015, May 2016, May 2017, Dec. 2017

Ans. :

Difference between NFA and DFA

Parameter	NFA	DFA
Transition	Non-deterministic.	Deterministic
No. of states.	NFA has fewer number of states.	More, if NFA contains Q states then the corresponding DFA will have $\leq 2^Q$ states.
Power	NFA is as powerful as a DFA	DFA is as powerful as an NFA
Design	Easy to design due to non-determinism.	Relatively, more difficult to design as transitions are deterministic.
Acceptance	It is difficult to find whether $w \in L$ as there are several paths. Backtracking is required to explore several parallel paths.	It is easy to find whether $w \in L$ as transitions are deterministic.

Q. 3 Define DFA. May 2010

Ans. :

Definition of DFA

A deterministic finite automata is a quintuple.

$M = (Q, \Sigma, \delta, q_0, F)$, where

Q is a set of states.

Σ is a set of alphabet.

$q_0 \in Q$ is the initial state,

$F \subseteq Q$ is the set of final states, and δ, the transition function, is a function from $Q \times \Sigma$ to Q.

Q. 4 Obtain a grammar to generate the language $L = \{0^n 1^{2n} \mid n \geq 0)\}$. May 2010

Ans. :

Productions for the required language are as follows.

$P = \{S \rightarrow 0\,S11 \mid \varepsilon\}$

CFG for the above language is ({S}, {0, 1}, P, S)

Q. 5 Give deterministic finite automata accepting the following languages over the alphabet {0, 1}

(a) Number of 1's is even and number of 0's is even.

(b) Number of 1's is odd and number of 0's is odd. May 2010

Ans. :

(a) Number of 1's is even and number of 0's is even.

At any instance of time, we will have following cases for number of 0's and number of 1's seen by the machine.

Situations		State
Number of 0's	Number of 1's	
Even	Even	q_0
Even	Odd	q_1
Odd	Even	q_2
Odd	Odd	q_3

An input 0 in state q_0, will make number of 0's odd.

$\delta(q_0, 0) \Rightarrow q_2$

An input 1 in state q_0, will make number of 1's odd.

$\delta(q_0, 1) \Rightarrow q_1$

An input 0 in state q_1, will make number of 0's odd.

$\delta(q_1, 0) \Rightarrow q_3$

An input 1 in state q_1, will make number of 1's even.

$\delta(q_1, 1) \Rightarrow q_0$

An input 0 in state q_2, will make number of 0's even.

$\delta(q_2, 0) \Rightarrow q_0$

An input 1 in state q_2, will make number of 1's odd.

$\delta(q_2, 1) \Rightarrow q_3$

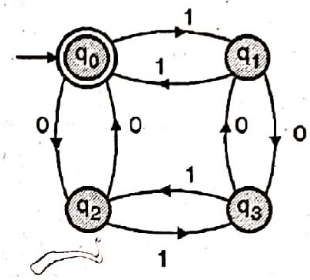
An input 0 in state q_3, will make number of 0's even.

$\delta(q_3, 0) \Rightarrow q_1$

An input 1 in state q_3, will make number of 1's even.

$\delta(q_3, 0) \Rightarrow q_2$

q_0 is the starting state. An empty string contains even number of 0's and even number of 1's. q_0 is a final state. q_0 stands for even number of 0's and even number of 1's.



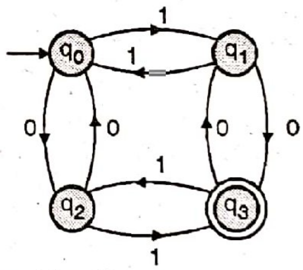
	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

(a) Transition diagram (b) Transition table

Fig. 2.5 : Final DFA for Q .5(a)

(b) Number of 1's is odd and number of 0's is odd.

In solution of Q. 5(a), the state q_3 stands for odd number of 0's should be declared as final state.



	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3^*	q_1	q_2

(c) Transition diagram (d) Transition table

Fig. 2.5 : Final DFA for for Q .5(b)

Q. 6 Give the finite automation M accepting (a,b)*(baaa). Dec. 2012

Ans. :

The R.E. = (a, b)* (baaa), represents strings ending in baaa.

The FA is given below

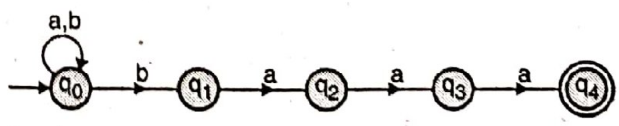


Fig. 2.6

Q. 7 Give applications of Finite Automata. May 2014

Ans. :

Applications of Finite Automata

Finite automata are used for solving several common types of computer algorithms. Some of them are :

(i) Design of digital circuit

(ii) String matching

(iii) Communication protocols for information exchange.

(iv) Lexical analysis phase of a compiler.

Finite automata can work as an algorithm for regular language. It can be used for checking whether a string w∈L, where L is a regular language.

Q. 8 Design a DFA to accept strings over the alphabet set {a, b} that begin with 'aa' but not end with 'aa'. **Dec. 2014**

Ans. :

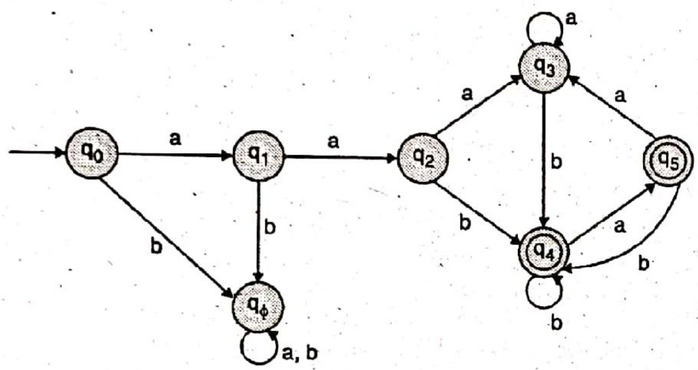


Fig. 2.7

A string not starting with aa will reach the dead state q_ϕ.

A string starting with aa will reach the state q_2.

A string starting with aa and not ending in aa will be either in q_4 or q_5.

The DFA is given by,

$$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_\phi\}, \{a, b\}, \delta, q_0, \{q_4, q_5\})$$

Q. 9 Design a MOORE and MEALY machine to decrement a binary number.

Ans. :

One can decrement a binary by adding 11...1 (all 1's is 2's complement of 1) to the given number. The addition should start from the least significant digit.

Mealy machine

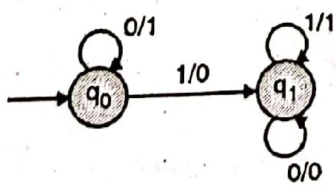


Fig. 2.8

(q_0 – Previous carry as 0, q_1 – Previous carry as 1)

i.e., all trailing 0's are written as 1 and the first 1 is written as 0.

Moore machine :

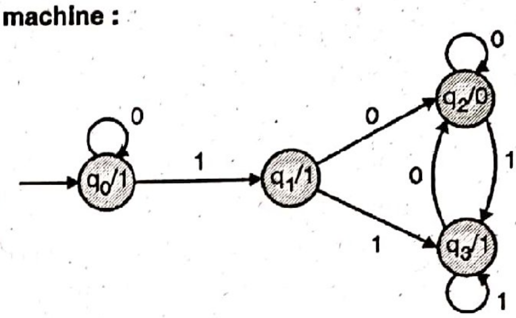


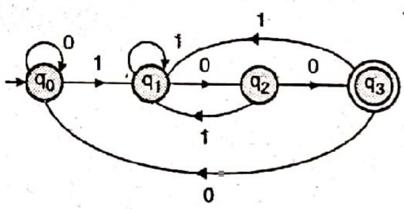
Fig. 2.9

Q. 10 Design minimized DFA for accepting strings ending with 100 over alphabet (0, 1). **May 2015**

Ans. :

All strings ending in 100 :

The substring '100' should be at the end of the string. Transitions from q_3 should be modified to handle the condition that the string has to end in '100'.



	1	0
→q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3^*	q_1	q_0

(a) State transition diagram (b) State transition table

Fig. 2.10

q_3 to q_1 on input 1 :

An input of 1 in q_3 will make the previous four characters as '1001'. Out of the four characters as '1001' only the last character '1' is relevant to '100'.

q_3 to q_0 on input 0 :

An input of 0 in q_3 will make the previous four characters '1000'. Out of the four characters '1000', nothing is relevant to '100'.

Q. 11 Design Moore Machine t generate output A if string is ending with abb, B if string ending with aba and C otherwise over alphabet (a, b). and convert it to mealy machine.

Ans. :

Design of Moore machine

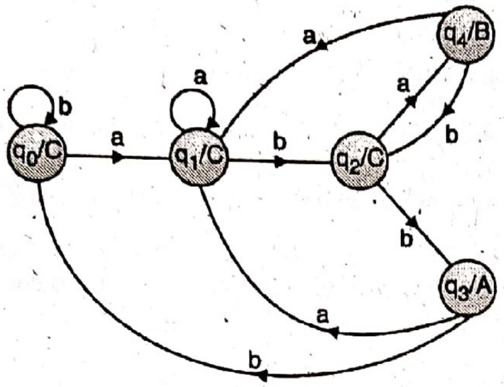


Fig. 2.11

Conversion into Mealy machine :

Step 1 : Construction of a trivial Mealy machine by moving output associated with a state to transition entering into that state.

	a	b
q_0	q_1, C	q_0, C
q_1	q_1, C	q_2, C
q_2	q_1, B	q_3, A
q_3	q_1, C	q_0, C
q_4	q_1, C	q_2, C

Step 2 : Minimization

The two states q_1 and q_4 can be merged into a single state, say q_1.

	a	b
q_0	q_1, C	q_0, C
q_1	q_1, C	q_2, C
q_2	q_1, B	q_3, A
q_3	q_1, C	q_0, C

The two state q_0, q_3 can be merged into a single state, say q_0.

	a	b
q_0	q_1, C	q_0, C
q_1	q_1, C	q_2, C
q_2	q_1, B	q_0, A

The final Mealy machine is

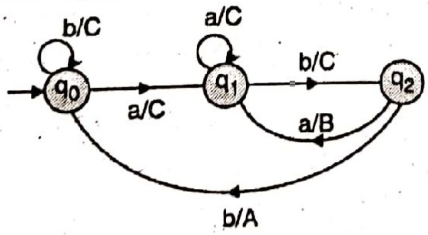


Fig. 2.12

Q. 12 Convert following ∈-NFA to NFA without ∈.

Dec. 2015

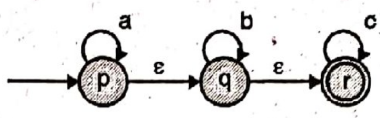


Fig. 2.13

Ans. :

To convert ∈-NFA to NFA without ∈

Step 1 : To remove ∈ transition from q state to r state, we do following

(a) Duplicate transitions of r state on q state

(b) Since r is the final state, we make q as well as the final state.

Step 2 : To remove ∈ transition from p state to q state do following :

(a) Duplicate the transitions of q state on p state

(b) Since q is a final state we make p as well as the final state.

Thus, the NFA is :

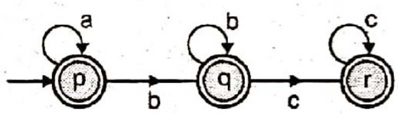


Fig. 2.14

Since all 3 states in the NFA are final states, we can merge all 3 states

∴ NFA – without ∈ is

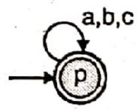


Fig. 2.15

Q. 13 Design the DFA to accept the language containing all the strings over Σ = {a, b, c} that starts and ends with different symbols.

Ans. :

$M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Sigma = \{a, b, c\}$

q_0 = initial state

$F = \{q_3, q_5, q_7\}$

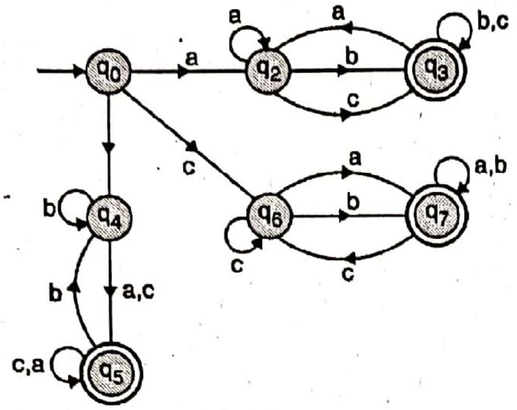


Fig. 2.16

δ = Transitions are :

$\delta(q_0, a) \Rightarrow q_2$	$\delta(q_6, c) \Rightarrow q_6$
$\delta(q_0, b) \Rightarrow q_4$	$\delta(q_3, a) \Rightarrow q_2$
$\delta(q_0, c) \Rightarrow q_6$	$\delta(q_3, b) \Rightarrow q_3$
$\delta(q_2, a) \Rightarrow q_2$	$\delta(q_3, c) \Rightarrow q_3$
$\delta(q_2, b) \Rightarrow q_3$	$\delta(q_5, a) \Rightarrow q_5$
$\delta(q_2, c) \Rightarrow q_3$	$\delta(q_5, b) \Rightarrow q_4$
$\delta(q_4, a) \Rightarrow q_5$	$\delta(q_5, c) \Rightarrow q_5$
$\delta(q_4, c) \Rightarrow q_5$	$\delta(q_7, a) \Rightarrow q_7$
$\delta(q_4, b) \Rightarrow q_4$	$\delta(q_7, b) \Rightarrow q_7$
$\delta(q_6, a) \Rightarrow q_7$	$\delta(q_7, c) \Rightarrow q_6$
$\delta(q_6, b) \Rightarrow q_7$	

Q. 14 Convert the following grammar into finite automata.

S → aX | bY | a | b

X → aS | bY | b

Y → aX | bS

Ans. :

The above grammar can be converted to FA as follows :

For every non terminating symbol we consider it as a different state

$M = \{Q, \Sigma, \delta, S, F\}$

$Q = \{S, X, Y\}$

$\Sigma = \{a, b\}$

S = initial state

$F = \{X, Y\}$

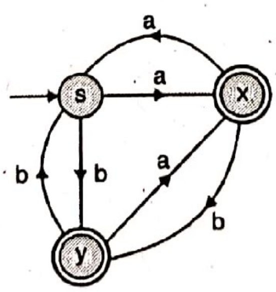


Fig. 2.17

δ : Transition functions are :

$\delta(S, a) \Rightarrow X$

$\delta(S, b) \Rightarrow Y$

$\delta(X, a) \Rightarrow S$

$\delta(X, b) \Rightarrow Y$

$\delta(Y, a) \Rightarrow X$

$\delta(Y, b) \Rightarrow S$

Q. 15 Design the DFA to accept all the binary strings over Σ = {0, 1} that are beginning with 1 and having its decimal value multiple of 5. May 2016

Ans. :

Running remained is maintained through the states q_0, q_1, q_2, q_3, q_4. If the number start with 0, it is rejected.

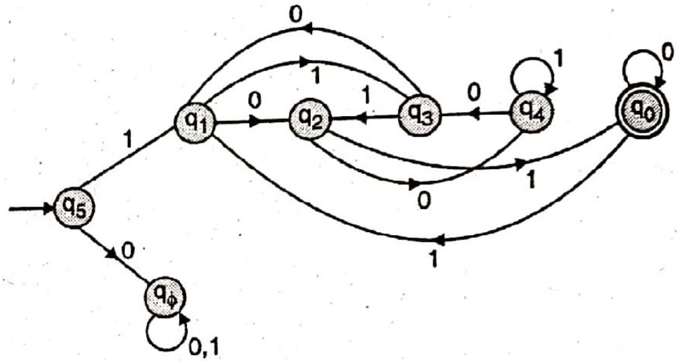


Fig. 2.18

Reminder calculation for finding the next state

State	Binary value of the state	$\delta(q_i, 0)$	$\delta(q_i, 1)$
q_0	0	$00 \div 5 = 0(q_0)$	$01 \div 5 = 1\ (q_1)$
q_1	1	$10 \div 5 = 2\ (q_2)$	$11 \div 5 = 3\ (q_3)$
q_2	10	$100 \div 5 = 4\ (q_4)$	$101 \div 5 = 0\ (q_0)$
q_3	11	$110 \div 5 = 1\ (q_1)$	$111 \div 5 = 2\ (q_2)$
q_4	100	$1000 \div 5 = 3\ (q_3)$	$1001 \div 5 = 4\ (q_4)$

The operator ÷ is for reminder.

Q. 16 Design mealy machine to find out 2's complement of a binary number.

Ans. :

2's complement of a binary number

2's complement of a binary number can be found by not changing bits from right end till the first '1' and then complementing remaining bits. For example, the 2's complement of a binary number 0101101000 is calculated as given below :

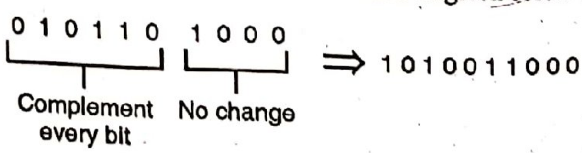


Fig. 2.19

The required mealy machine is given below.

The input is entered from right to left.

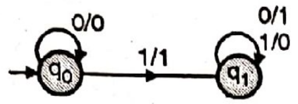


Fig. 2.20

Q. 17 Convert the following NFA to an equivalent DFA

State	a	b	∈
→ q_0	$\{q_0, q_1\}$	$\{q_1\}$	{ }
q_1	$\{q_2\}$	$\{q_1, q_2\}$	{ }
* q_2	$\{q_0\}$	$\{q_2\}$	$\{q_1\}$

Ans. :

The transition graph of the given NFA is :

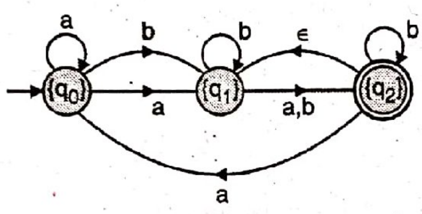


Fig. 2.21

∈-closure of states :

$q_0 \rightarrow (q_0)$

$q_1 \rightarrow (q_1)$

$q_2 \rightarrow (q_1, q_2)$

NFA to DFA using direct method.

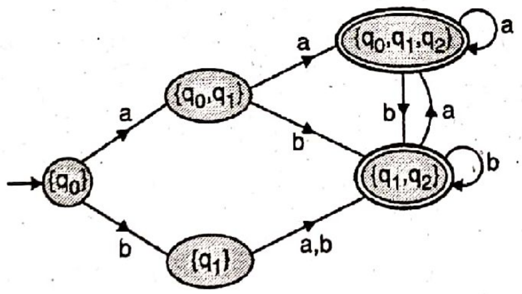


Fig. 2.22

Q. 18 Design a DFA over an alphabet Σ = {a, b} to recognize a language in which every 'a' is followed by 'b' **Dec. 2016**

Ans. :

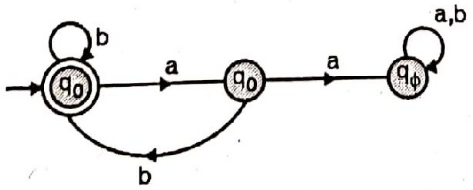


Fig. 2.23

If 'a' is followed by 'a' then the machine enters the failure state q_ϕ

A 'b' immediately after 'a' takes the machine to the accepting state q_0

Q. 19 Design a mealy machine to determine the residue mod 3 of a binary number.

Ans. :

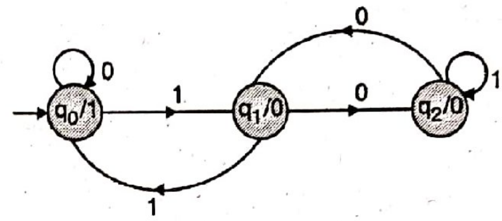


Fig. 2.24

State q_0 is for the running reminder as 0.

State q_1 is for the running reminder as 1.

State q_2 is for the running reminder as 2.

Output 1 indicates divisibility by 3

Output 0 indicate that the number is not divisible by 3.

∴ Required R.E. = $(0 + 1 (1 + 01)^* 00)^*$

Q. 20 Convert the following NFA to an equivalent DFA

State	a	b	∈
→ q_0	$\{q_0, q_1\}$	q_1	{ }
q_1	$\{q_2\}$	$\{q_1, q_2\}$	{ }
* q_2	$\{q_0\}$	$\{q_2\}$	$\{q_1\}$

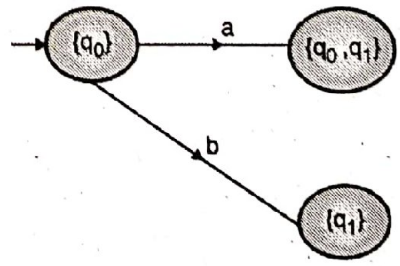
Ans. :

∈ - closure of states

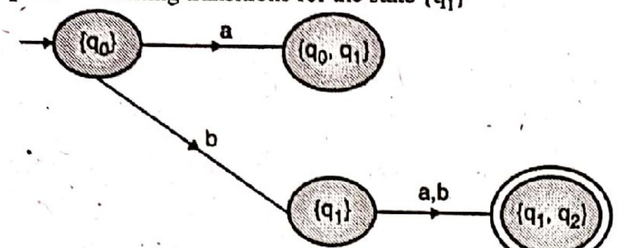
State	∈ - closure
q_0	$\{q_0\}$
q_1	$\{q_1\}$
q_2	$\{q_1, q_2\}$

Constructing DFA using the direct method

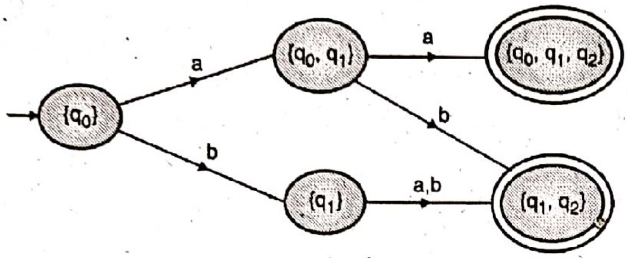
Step 1 : Transitions for the state $\{q_0\}$



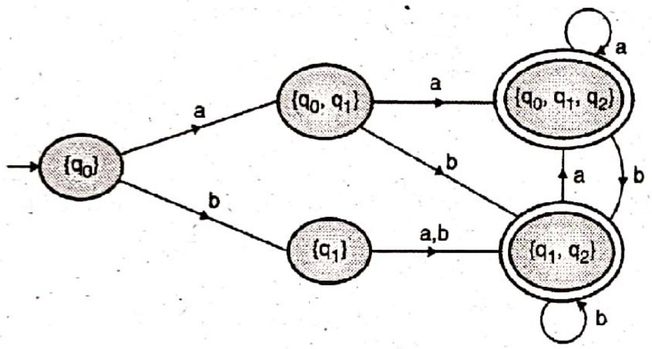
Step 2 : Writing transitions for the state $\{q_1\}$



Step 3 : Writing transitions for the state $\{q_0, q_1\}$



Step 4 : Writing transitions for the states $\{q_1, q_2\}$ and $\{q_0, q_1, q_2\}$



Q. 21 Draw DFA for the following language over {a, b} :

(a) All strings starting with abb.

(b) All strings with abb as a substring i.e., abb anywhere in the string.

(c) All strings ending in abb. **May 2017**

Ans. :

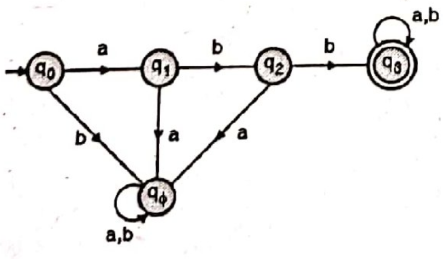
(a) All strings starting with abb

First input as 'b' will take the machine to a failure state.

First two inputs as 'aa' will take the machine to a failure state.

First three inputs as 'aba' will take the machine to a failure state.

First three inputs as 'abb' will take the machine to a final state.

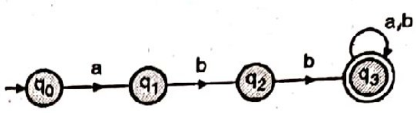


	a	b
$\rightarrow q_0$	q_1	q_ϕ
q_1	q_ϕ	q_2
q_2	q_ϕ	q_3
q_3^*	q_3	q_3
q_ϕ	q_ϕ	q_ϕ

(a) State transition diagram (b) State transition table

Fig. 2.25 : Final DFA for Q. 21(a)

A DFA without explicit failure state is given in Fig. 2.25(a)



	a	b
$\rightarrow q_0$	q_1	ϕ
q_1	ϕ	q_2
q_2	ϕ	q_3
q_3^*	q_3	q_3

(a) State transition diagram (b) State transition table

Fig. 2.26 : Final DFA for Q. 21(a), without a failure / dead state

(b) All strings with abb as a substring

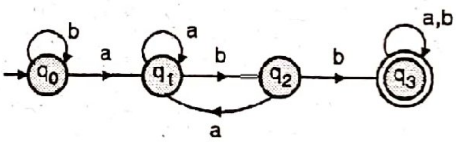
The machine will have fours states :

State q_0 – It is the starting state and indicates that nothing of relevance to complete 'abb' has been seen.

State q_1 – preceding character is 'a' and 'bb' is required to complete 'abb'.

State q_2, – Preceding characters are 'ab' and 'b' is required to complete 'abb.'

State q_3, – Preceding characters are 'abb' and the substring 'abb' has been seen by the machine.



	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3^*	q_3	q_3

(a) State transition diagram (b) State transition table

Fig. 2.27 : Final DFA for Q. 21(b)

q_0 to q_0 on input 'b' :

First character in 'abb' is a.

q_0 to q_1 on input 'a' :

q_1 is for preceding characters as 'a', first character of abb.

q_1 to q_1 on input 'a' :

An input of 'a' in state q_1 will make the preceding two characters as 'aa'. Last 'a' will still constitute the first 'a' of abb.

q_1 to q_2 on input 'b' :

q_2 is for preceding two characters as 'ab' of 'abb'.

q_2 to q_1 on input 'a' :

An input 'a' in q_2 will make the preceding three characters as 'aba'. Out of the three characters 'aba', only the last character 'a' is relevant to 'abb'.

q_2 to q_3 on input b :

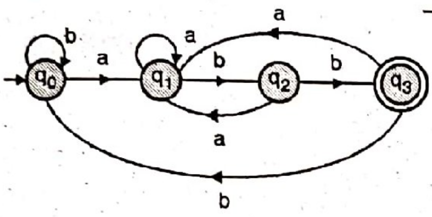
∴ q_3 is for preceding three characters as 'abb'.

q_3 to q_3 on input a or b :

The substring 'abb' has been seen by the machine and a new input will not change this status.

(c) All strings ending in abb

As the substring 'abb' should be at the end of the string. Transitions from q_3 should be modified to handle the condition that the string has to end in 'abb'.



(a) State transition diagram

	a	b
→ q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3^*	q_1	q_0

(b) State transition table

Fig. 2.28 : Final DFA for Q. 21(c)

q_3 to q_1 on input a :

An input of a in q_3 will make the previous four characters as 'abba'. Out of the four characters as 'abba' only the last character 'a' is relevant to 'abb'.

q_3 to q_0 on input b :

An input of b in q_3 will make the previous four characters 'abbb'. Out of the four characters 'abbb', nothing is relevant to 'abb'.

Q. 22 Design a DFA which can accept a binary number divisible by 3

Or

Design of a divisibility – by – 3 – tester for a binary number. **Dec. 2005, May 2014, May 2017**

Ans. :

A binary number is divisible by 3, if the remainder when divided by 3 will work out to be zero. We must device a mechanism for finding the final remainder.

We can calculate the running remainder based on previous remainder and the next input.

The running remainder could be :

0 → associated state, q_0

1 → associated state, q_1

2 → associated state, q_2

Starting with the most significant bit, input is taken one bit at a time. Running remainder is calculated after every input. The process of finding the running remainder is being explained with the help of an example.

Number to be divided : 101101.

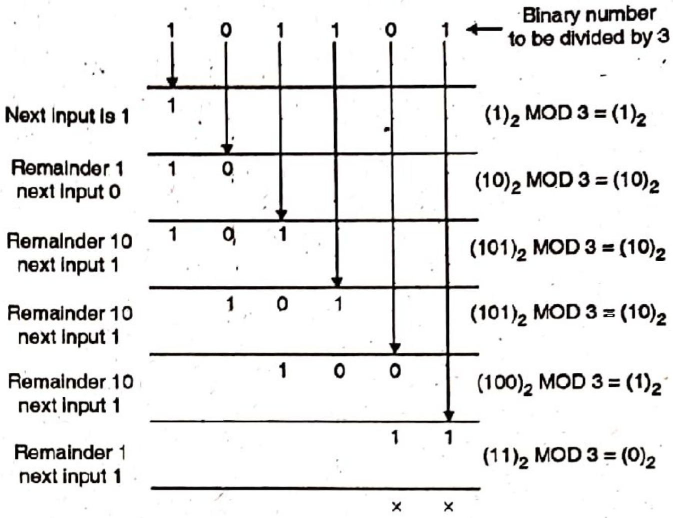
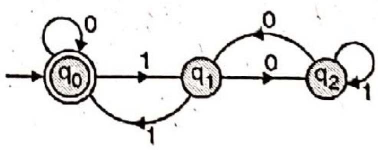


Fig. 2.29

The calculation of next remainder is shown below.

Previous remainder	Next input	Calculation of remainder		Next remainder
0 (q_0)	0	00 % 3	⇒	0 (q_0)
0 (q_0)	1	01 % 3	⇒	1 (q_1)
1 (q_1)	0	10 % 3	⇒	10 (q_2)
1 (q_1)	1	11 % 3	⇒	0 (q_0)
10 (q_2)	0	100 % 3	⇒	1 (q_1)
10 (q_2)	1	101 % 3	⇒	10 (q_2)
↑ Binary		↑ Binary ↖ decimal		↑ Binary



(b) State transition diagram

	0	1
→ q_0^*	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

(c) State transition table

Fig. 2.30 : DFA for Q. 22

Q. 23 Design a DFA for a mod 5 tester for ternary input. **Dec. 2017**

Ans. :

A ternary system has three alphabets

$\Sigma = \{0, 1, 2\}$

Base of a ternary number is 3.

The running remainder could be :

$(0)_3 = 0 \rightarrow$ associated state, q_0

$(1)_3 = 1 \rightarrow$ associated state, q_1

$(2)_3 = 2 \rightarrow$ associated state, q_2

$(10)_3 = 3 \rightarrow$ associated state, q_3

$(11)_3 = 4 \rightarrow$ associated state, q_4

↑ ↑

Ternary Decimal

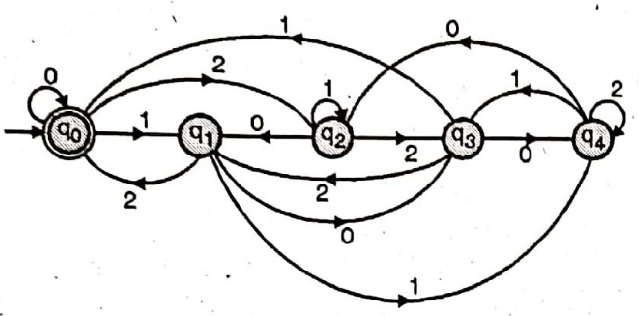


Fig. 2.31

Q. 24 Design DFA that accepts the following language :

(i) Set of all strings with odd number of 1's followed by even number of 0's Σ = {0, 1}.

(ii) Set of all strings which begin and end with different letters Σ = {x, y, z}.

(iii) Strings ending with 110 or 111.

Dec. 2006, Dec. 2009, Dec. 2010

Ans. :

(i)

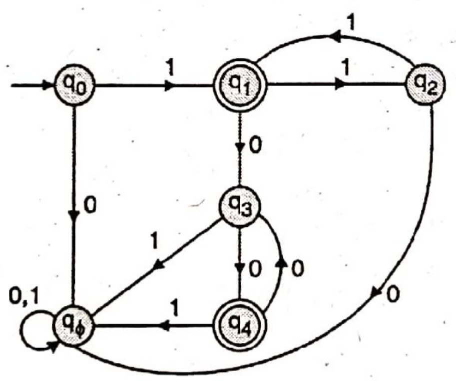


Fig. 2.32(a)

(ii)

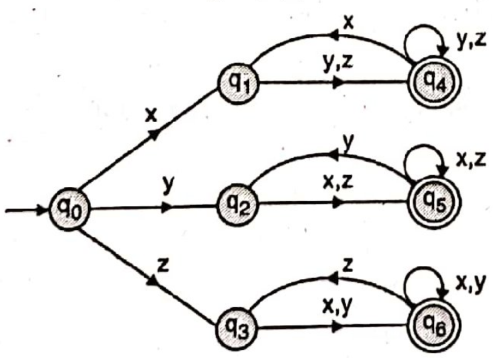


Fig. 2.32(b)

(iii)

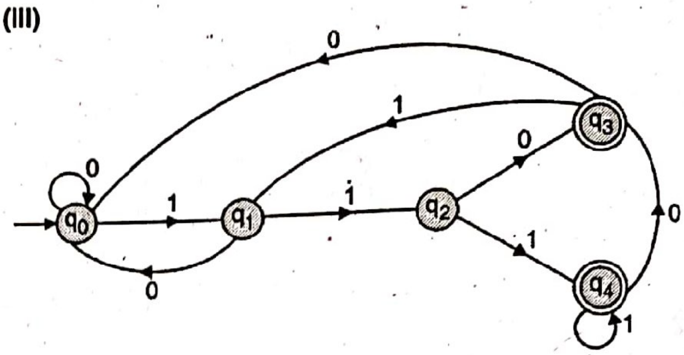


Fig. 2.33(c)

Q. 25 Construct the minimum state automata equivalent to given DFA.

May 2011

	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
q_3^*	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

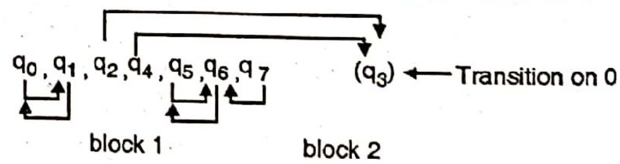
Ans. :

Step 1 : Finding 0-equivalence partitioning of states by putting final and non-final states into independent block.

$P_0 = (q_0, q_1, q_2, q_4, q_5, q_6, q_7) \quad (q_3)$

block 1 block 2

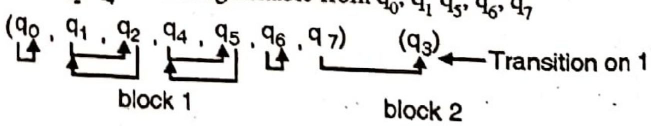
Step 2 : Finding 1-equivalence partitioning of states by considering transition on '0' and transition on '1'.



On input 0, block 1 is successor of q_0, q_1, q_5, q_6, q_7.

On input 0, block 2 is successor of q_2, q_4.

∴ q_2, q_4 are distinguishable from q_0, q_1, q_5, q_6, q_7



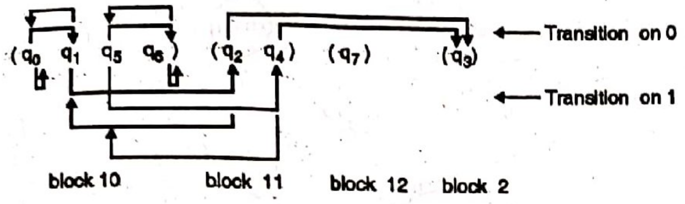
On input 1, block 2 is successor of q_7.

On input 1, block 1 is successor of $q_0, q_1, q_2, q_4, q_5, q_6$.

q_7 is distinguishable from $q_0, q_1, q_2, q_4, q_5, q_6$.

$P_1 = (q_0, q_1, q_5, q_6)\ (q_2, q_4)\ (q_7)\ (q_3)$

Step 3 : Finding 2-equivalence partitioning of states by considering transition on '0' and transition on '1'.



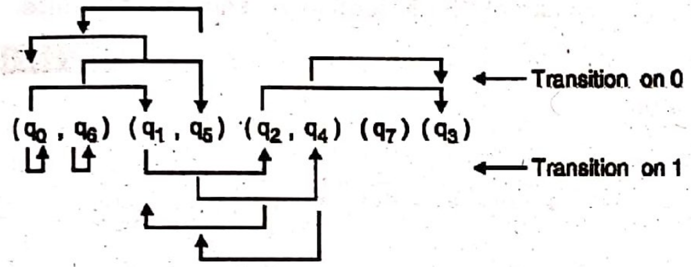
On input 1, block 11 is successor of q_1, q_5.

On input 1, block 10 is successor of q_0, q_6.

q_1, q_5 is distinguishable from q_0, q_6.

$P_2 = (q_0, q_6)(q_1, q_5)(q_2, q_4)(q_7)(q_3)$

Step 4 : Finding 3-equivalence partitioning of states by considering transition on 0 and 1.



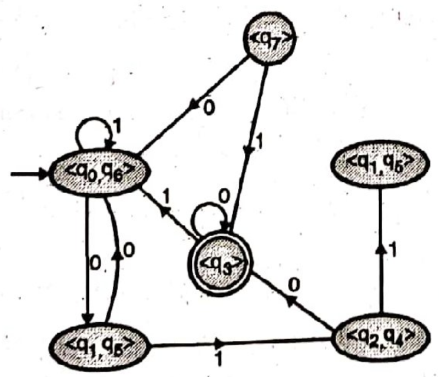
Blocks can not be divided further.

$\therefore P_3 = P_2 = (q_0, q_6)(q_1, q_5)(q_2, q_4)(q_7)(q_3)$ which is final set of blocks of equivalent classes.

Step 5 : Construction of minimum state DFA.

	0	1
$\rightarrow(q_0, q_6)$	(q_1, q_5)	(q_0, q_6)
(q_1, q_5)	(q_0, q_6)	(q_2, q_4)
(q_2, q_4)	(q_3)	(q_1, q_5)
$(q_3)^*$	(q_3)	(q_0, q_6)
(q_7)	(q_0, q_6)	(q_3)

(a) State transition diagram for minimum-



(b) State transition diagram for minimum-state DFA state DFA

Fig. 2.34

Q. 26 A language L is accepted by some NFA if and only if it is accepted by some DFA.

OR

For every NFA, there exists an equivalent DFA.

Dec. 2014

Ans. :

Proof

Given theorem has two parts :

1. If L is accepted by a DFA M_2, then L is accepted by some NFA M_1.
2. If L is accepted by an NFA M_1, then L is accepted by some DFA M_2.

First part can be proved trivially. Determinism is a case of non-determinism. Thus a DFA is also an NFA.

Second part of the theorem is proved below :

Construct M_2 from M_1 using subset generation algorithm as explained earlier. We can prove the theorem using induction on the length of ω.

Base case : Let $\omega = \varepsilon$ with $|\omega| = 0$, where $|\omega|$ is length of ω.

Starting state for both NFA and DFA are taken as q_0. When $\omega = \varepsilon$, both DFA and NFA will be in q_0. Hence, the base case is proved.

Assumption : Let us assume that both NFA and DFA are equivalent for every string of length n. We must show that the machines M_1 (NFA) and M_2 (DFA) are equivalent for strings of length (n + 1). Let $\omega_{n+1} = \omega_n a$, where ω_n is a string of length n and ω_{n+1} is a string of length (n + 1). 'a' is an arbitrary alphabet from Σ.

$\delta_2(q_2, \omega_n) = \delta_2(q_0, \omega_n)$, where δ_2 is transition function of DFA (M_2) and δ, is transition function of NFA (M_1).

If the subset reached by NFA is given by

$\{p_1, p_2, \cdots p_k\}$

$$\text{then,} \quad \delta_2(q_0, \omega_{n+1}) = \bigcup_{i=1}^{k} \delta_2(p_i, a) \quad \text{...(i)}$$

$$\text{or } \delta_2(\{p_1, p_2, \ldots p_k\}, a) = \bigcup_{i=1}^{k} \delta_1(p_i, a) \quad \text{...(ii)}$$

$$\text{also,} \quad \delta_2(q_0, \omega_n) = \{p_1, p_2, \ldots p_k\} \quad \text{...(iii)}$$

from (i), (ii) and (iii) we get,

$$\begin{aligned} \delta_2(q_0, \omega_{n+1}) &= \delta_2(\delta_2(q_0, \omega_n), a) \\ &= \delta_2(\{p_1, p_2, \ldots p_k\}, a) \\ &= \bigcup_{i=1}^{k} \delta_1(p_i, a) = \delta_1(q_0, s_{n+1}) \end{aligned}$$

Thus, the result is true for $|\omega| = n + 1$, hence it is always true.

Q. 27 Convert the following NFA to a DFA and informally describe the language it accepts.

Dec. 2011

	0	1
→p	{p, q}	{p}
q	{r, s}	{t}
r	{p, r}	{t}
s*	ϕ	ϕ
t*	ϕ	ϕ

Ans. :

Step 1 : {p} is taken as the first subset.

0-Successor of {p} $= \delta(\{p\}, 0) = \{p, q\}$

1-Successor of {p} $= \delta(\{p\}, 1) = \{p\}$

Step 2 : The new subsets {p, q} is generated. Successors of {p, q} are calculated.

$$\delta(\{p, q\}, 0) = \delta(p, 0) \cup \delta(q, 0) = \{p, q\} \cup \{r, s\} = \{p, q, r, s\}$$

$$\delta(\{p, q\}, 1) = \delta(p, 1) \cup \delta(q, 1) = \{p\} \cup \{t\} = \{p, t\}$$

Step 3 : Two new subsets {p, q, r, s} and {p, t} are generated. Their successors are calculated.

$$\delta(\{p, q, r, s\}, 0) = \delta(p, 0) \cup \delta(q, 0) \cup \delta(r, 0) \cup \delta(s, 0) = \{p, q\} \cup \{r, s\} \cup \{p, r\} \cup \phi = \{p, q, r, s\}$$

$$\delta(\{p, q, r, s\}, 1) = \delta(p, 1) \cup \delta(q, 1) \cup \delta(r, 1) \cup \delta(s, 1) = \{q\} \cup \{t\} \cup \{t\} \cup \phi = \{p, t\}$$

$$\delta(\{p, t\}, 0) = \delta(p, 0) \cup \delta(t, 0) = \{p, q\} \cup \phi = \{p, q\}$$

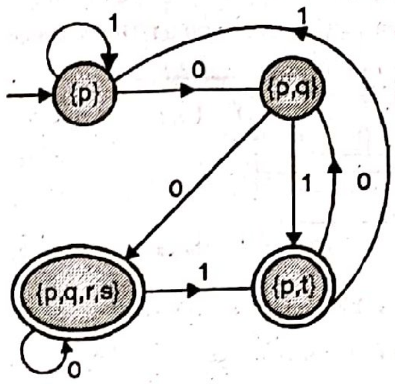
$$\delta(\{p, t\}, 1) = \delta(p, 1) \cup \delta(t, 1) = \{p\} \cup \phi = \{p\}$$

No, new subset is generated. Every subset containing either s or t is marked as a final state.

Informal Description: Strings over {0, 1} with second digit from the end is 0.

	0	1
→{p}	{p, q}	{p}
{p, q}	{p, q, r, s}	{p, t}
{p, q, r, s}*	{p, q, r, s}	{p, t}
{p, t}*	{p, q}	{p}

a) State table



(b) State diagram

Fig. 2.35 : Final DFA for Q. 27

Q. 28 Construct a NFA that accepts a set of all strings over {a, b} ending in aba. Use this NFA to construct DFA accepting the same set of strings.

May 2014

Ans. :

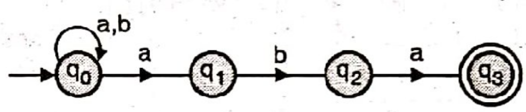


Fig. 2.36 (a) : Non-deterministic finite automata

Non-determinism should be utilized to full extent while designing an NFA. A string of length n, ending in aba can be recognized by the NFA given in Fig. 2.36(a). First n-3 characters can be absorbed by the state q_0 by making a guess. On guessing the last three characters as aba, the machine can make a transition from q_0 to q_3.

NFA to DFA conversion :

Step 1 : $\{q_0\}$ is taken as first subset

a-successor of $\{q_0\} = \delta(q_0, a) = \{q_0, q_1\}$

b-successor of $\{q_0\} = \delta(q_0, b) = \{q_0\}$

Step 2 : A new subset $\{q_0, q_1\}$ is generated. Successors of $\{q_0, q_1\}$ are calculated.

$$\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a) = \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$$

$$\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(q_1, b) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$$

Step 3 : A new subset $\{q_0, q_2\}$ is generated. Successors of $\{q_0, q_2\}$ are calculated.

$$\delta(\{q_0, q_2\}, a) = \delta(q_0, a) \cup \delta(q_2, a) = \{q_0, q_1\} \cup \{q_3\} = \{q_0, q_1, q_3\}$$

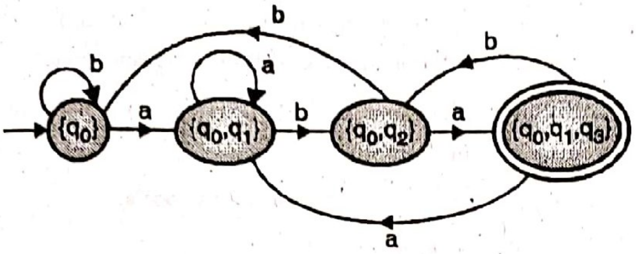
$$\delta(\{q_0, q_2\}, b) = \delta(q_0, b) \cup \delta(q_2, b) = \{q_0\} \cup \phi = \{q_0\}$$

Step 4 : A new subset $\{q_0, q_1 q_3\}$ is generated. Successors of $\{q_0, q_1 q_3\}$ are calculated.

$$\delta(\{q_0, q_1 q_3\}, a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a) = \{q_0, q_1\} \cup \phi \cup \phi = \{q_0, q_1\}$$

$$\delta(\{q_0, q_1 q_3\}, b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b)$$
$$= \{q_0\} \cup \{q_2\} \cup \phi = \{q_0, q_2\}$$

No, new subset is generated. Every subset containing q_3 is marked as a final state.



(b) State diagram of the DFA

	a	b
$\rightarrow\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}^*$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

(c) State table of the DFA

Fig. 2.36

Q. 29 Give Mealy and Moore machine for the following : From input Σ*, where Σ = (0, 1, 2) print the residue modulo 5 of the input treated as ternary (base 3).

May 2006, Dec. 2015

Ans. :

(a) Mealy machine

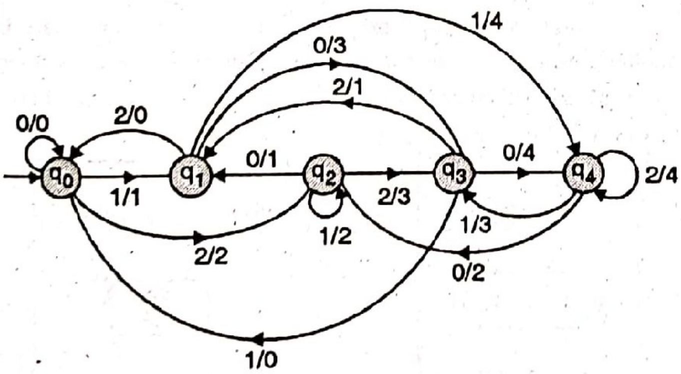


Fig. 2.37(a) : Mealy machine

Meaning of various states is :

q_0 – Running remainder is 0

q_1 – Running remainder is 1

q_2 – Running remainder is 2.

q_3 – Running remainder is $3 = (10)_3$

q_4 – Running remainder is $4 = (11)_3$

(4: In decimal system; $(11)_3$: In ternary system)

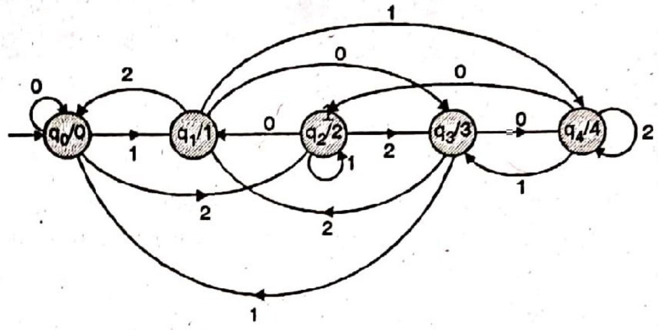
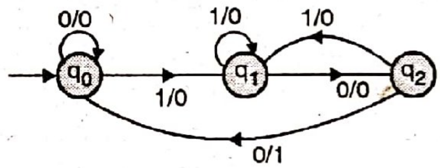


Fig. 2.37(b) : Moore machine

Q. 30 Design a mealy machine for a binary input sequence such that if the sequence ends with 100 the output is 1 otherwise output is 0.

Dec. 2006, May 2008, Dec. 2008

Ans. :



(a) State diagram

	0	1
$\rightarrow q_0$	q_0, 0	q_1, 0
q_1	q_2, 0	q_1, 0
q_2	q_0, 1	q_1, 0

(b) State table

Fig. 2.38

Meaning of various states :

q_0 – start state

q_1 – previous symbol is 1

q_2 – preceding two symbols are 10

A transition from q_2 to q_0 will make the preceding three symbol as 100 and hence the output 1.

Chapter 3 : Regular Expressions and Languages

Q. 1 Write short note on Myhill-Nerode theorem.

Dec. 2005, May 2006, Dec. 2006, May 2007, May 2008, Dec. 2008, Dec. 2012, May 2013

Ans. :

Myhill-Nerode theorem

Given a language L, two strings x and y are said to be in the same class if for all possible strings z either both xz and yz are in L or both are not.

The Myhill-Nerode theorem says :

1. A language L divides the set of all possible strings into mutually exclusive classes.
2. If L is regular, the number of classes created by L is finite.
3. If the number of classes L creates is finite, then L is regular.

In finite automata, each state can be thought of as creating a class of strings. Two strings are said to be in the same class if they both trace a path from starting state q_0 to some state q_i (say).

Number of strings is infinite.

Number of states in an FA is finite.

Many strings when applied to the FA will end up in the same state. Each state of FA can stand for a class of strings.

Q. 2 Show that

$(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1) = 0^*1(0 + 10^*1)^*$ **May 2006**

Ans. :

$$\begin{aligned} \text{L.H.S.} &= (1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1) \\ &= (1+00^*1)[\varepsilon+(0+10^*1)^*(0+10^*1)] \\ &= (1+00^*1)(0+10^*1)^* \\ &= [(\varepsilon+00^*)1](0+10^*1)^* = 0^*1(0+10^*1)^* \\ &= \text{R.H.S.} \end{aligned}$$

Q. 3 Prove $L = \{(ab)^n a^k : n > k, k \geq 0\}$ is not regular. **May 2006**

Ans. :

Step 1 : Let us assume that L is regular and L is accepted by an FA with n states.

Step 2 : Let us choose a string

$$\omega = (ab)^{n+1}a^n$$

$$|\omega| = 2(n+1)+n = 3n+2 \geq n$$

Let us write ω as xyz, with

$$|y| > 0 \quad \text{and} \quad |xy| \leq n$$

The string xy will contain a maximum of n symbols from $(ab)^n$.

Step 3 : In the string xy^iz with i = 0, at least one 'a' or atleast one 'b' will be erased from $(ab)^{n+1}$ of $(ab)^{n+1}a^n$. This will lead to one of the following situations :

1. Number of a's in $(ab)^n$ is equal to number of a's in a^k of $(ab)^n a^k$.
2. xy^0z will not be of the form $(ab)^n a^k$.

Therefore, $xy^0z \in L$.

Hence, this is proved by contradiction.

Q. 4 Write short notes on closure properties of regular language. **Dec. 2006, May 2013. Dec. 2014**

Ans. :

Closure properties of regular language

If an operation on regular languages generates a regular language then we say that the class of regular languages is closed under the above operation. Some of the important closure properties for regular languages are given below.

1.	Union	2.	Difference
3.	Concatenation	4.	Intersection
5.	Complementation	6.	Kleene star
7.	Transpose or reversal.		

1. Regular Language is Closed under Union

Let $M_1 = (S, \Sigma, \delta_1, s_0, F)$ and

$M_2 = (Q, \Sigma, \delta_2, q_0, G)$ be two given automata.

To prove the closure property; we must show that there is another machine M_3 which accepts every string accepted by either M_1 or M_2 and no other string. The construction M_3 is quite simple as shown in Fig. 3.1.

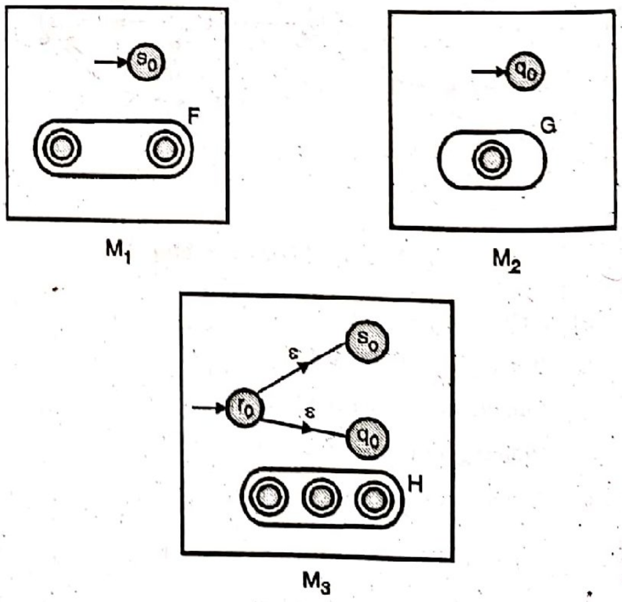


Fig. 3.1 : M_3 is constructed such the $L(M_3) = L(M_1) \cup L(M_2)$

Machine M_3 is constructed to accept $L(M_1) \cup L(M_2)$.

$M_3 = (R, \Sigma, \delta_3, r_0, H)$ where r_0 is a new start state. Two ε-moves, one from r_0 to s_0 and another from r_0 to q_0 are added.

$$R = S \cup Q \cup \{r_0\}$$

$$H = F \cup G$$

$$\delta_3 = \delta_1 \cup \delta_2 \cup \{(r_0, \varepsilon, s_0), (r_0, \varepsilon, q_0)\}$$

Machine M_3 can non-deterministically choose either M_1 or M_2. Therefore,

$$L(M_3) = L(M_1) \cup L(M_2)$$

2. Regular Language is Closed under Concatenation

Let $M_1 = (S, \Sigma, \delta_1, s_0, F)$

and $M_2 = (Q, \Sigma, \delta_2, q_0, G)$ be two given automata.

To prove that closure property under concatenation, we must show that there is another machine M_3 such that $L(M_3) = L(M_1) \cdot L(M_2)$. The construction of M_3 is shown in Fig. 3.2.

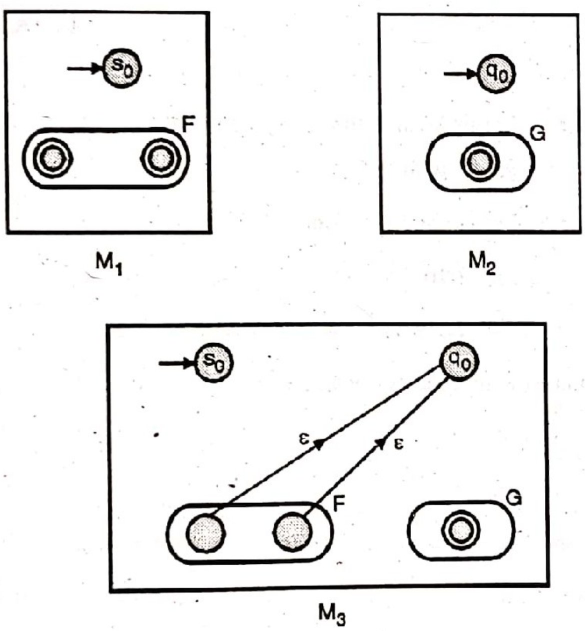


Fig. 3.2 : M_3 is constructed such that $L(M_3) = L(M_1) \cdot L(M_2)$

M_3 is constructed by adding ε-move from every final state of M_1 to start state of M_2.

Machine M_3 is given by :

$$M_3 = (R, \Sigma, \delta_3, s_0, G) \text{ where}$$

$$\delta_3 = \delta_1 \cup \delta_2 \cup \{\varepsilon\text{-move from every final state of } M_1 \text{ to start state of } M_2\}$$

Machine M_3 recognizes $L(M_1) \cdot L(M_2)$ by going non-deterministically from the final state of M_1 to start state of M_2.

3. Regular Language is Closed under Kleene Star

Let $M_1 = (Q, \Sigma, \delta, q_0, F)$ be the given automata. We can construct a non-deterministic finite automata M_2 such that $L(M_2) = L(M_1)^*$. The construction of M_2 from M_1 is shown in Fig. 3.3.

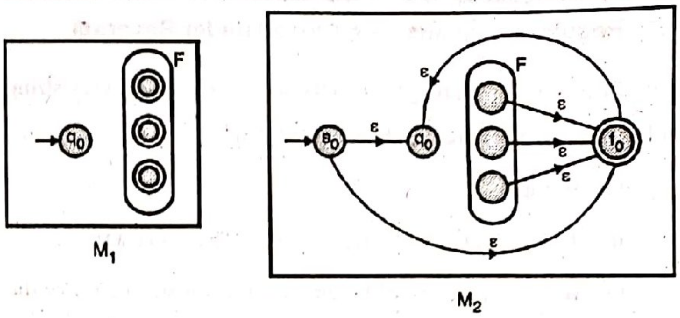


Fig. 3.3 : M_2 is constructed such that $L(M_2) = L(M_1)^*$

M_2 is constructed as given below :

(a) A new start state s_0 is added with an ε-move from s_0 to q_0.

(b) A new final state f_0 is added with ε-moves from every state of F to f_0. An ε-move is added from s_0 to f_0 as ε is a member of $L(M_1)^*$.

$$\text{Machine } M_2 = (Q \cup \{s_0, f_0\}, \Sigma, \delta, s_0, \{f_0\})$$

Machine can accept a string $\in L(M_1)$ and resume back from the start state q_0 through the ε-move from f_0 to q_0. Thus accepting $L(M_1)^*$.

4. Regular Language is Closed under Complementation

Let $M = \{Q, \Sigma, \delta, q_0, F)$ be the given automata. To prove the closure property under complementation, we must show that there is another machine $\overline{M}$ which accepts $L(\overline{M})$ where

$$\overline{L(M)} = L(\overline{M}) = \Sigma^* - L(M)$$

Given machine | Machine after complementation

If M is a deterministic finite automata then $\overline{M}$ can be constructed by interchanging final and non final states of M.

$$\therefore \overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$$

5. Regular Language is Closed under Intersection

If L_1 and L_2 are two regular languages, then

$$L_1 \cap L_2 = ((L_1 \cap L_2)')' = (\overline{L_1} \cup \overline{L_2})'$$

$$= \Sigma^* - [(\Sigma^* - L_1) \cup (\Sigma^* - L_2)]$$

Closeness under intersection follows directly from closeness under union and complementation.

6. Regular Languages are Closed under Difference

Let L_1 and L_2 are two regular languages. The difference $L_1 - L_2$ is the set of strings that are in language L_1 but not in L_2. Construction of a composite automata for $L(M_1) - L(M_2)$ is explained in Chapter 2. Thus regular languages are closed under difference.

7. Regular Languages are Closed under Reversal

Reversal of a language L is obtained by reversing every string in L. Reversal of a language L is represented by L^R.

For example,

if L = {aab, abb, aaa}, then L^R = {baa, bba, aaa}

Let $M_1 = (Q, \Sigma, \delta, q_0, F)$ be the given automata. To prove the closure property under reversal, we must show that there is another machine M_2 which accepts $L(M_1)^R$.

or $L(M_2) = L(M_1)^R$

M_2 can be constructed from M_1 by :

1. By reversing every transition in M_1.
2. Start state of M_1 is made the only final state.
3. A new start state s_0 is added with ε-move to every final state of M_1.

Q. 5 Design a NFA to accept $(a + b)^*aba$ convert it to a reduced DFA. May 2007

Ans. :

$(a + b)^*$ aba

RE to NFA

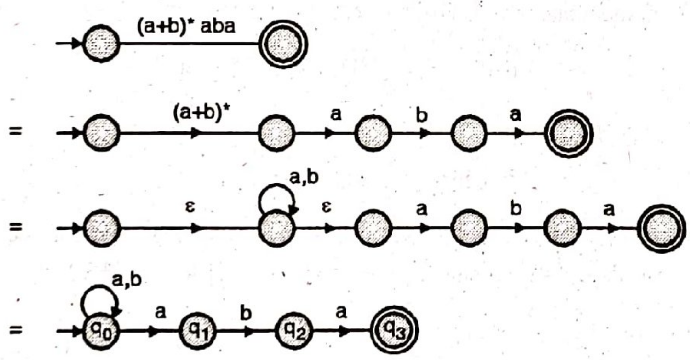


Fig. 3.4 : RE to NFA

NFA to DFA

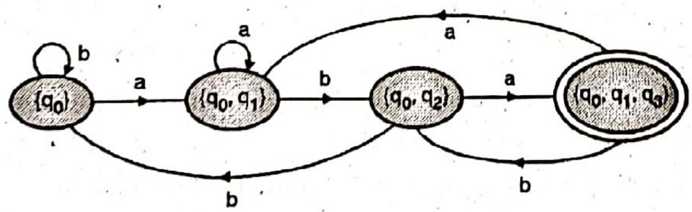


Fig. 3.5 : NFA to DFA

Q. 6 Write RE for the following languages

(I) The set of all string over {0, 1} without length two.

(II) $L = \{a^n b^m \mid (n + m) \text{ is even}\}$

(III) $L = \{\omega \in (a, b)^* \mid (\text{number of a's in } \omega) \bmod 3 = 0\}$

(IV) $L = \{a^n b^m \mid n >= 4, m <= 3\}$

May 2006, Dec. 2007, May 2008

Ans. :

(i) The set of all strings over {0, 1} without length two.

$\varepsilon + (0 + 1) + (0 + 1)(0 + 1)(0 + 1)(0 + 1)^*$

(ii) $L = \{a^n b^m \mid (n + m) \text{ is even}\}$

$((aa)^*ab + bb)(bb)^*$

(iii) $L = \{\omega \in (a, b)^* \mid (\text{number of a's in } \omega) \bmod 3 = 0\}$

$(b + ab^*ab^*a)^*$

(iv) $L = \{a^n b^m \mid n >= 4, m <= 3\}$

$aaaaa^*[\varepsilon + b + bb + bbb]$

Q. 7 Prove $L = \{ (ab)^n a^k \mid n > k, k >= 0 \}$ is not regular. May 2008

Ans. :

Step 1 : Let us assume that L is regular and L is accepted by an FA with n states.

Step 2 : Let us choose a string

$\omega = (ab)^{n+1}a^n$

$|\omega| = 2(n + 1) + n = 3n + 2 \geq n$

Let us write ω as xyz, with

$|y| > 0$

and $|xy| \leq n$

The string xy will contain a maximum of n symbols from $(ab)^n$.

Step 3 : In the string xy^iz with i = 0, at least one 'a' or atleast one 'b' will be erased from $(ab)^{n+1}$ of $(ab)^{n+1}a^n$. This will lead to one of the following situations :

1. Number of a's in $(ab)^n$ is equal to number of a's in a^k of $(ab)^n a^k$.
2. xy^0z will not be of the form $(ab)^n a^k$.

Therefore, $xy^0z \in L$.

Hence, this is proved by contradiction.

Q. 8 Construct a NFA for the RE $(01^* + 1)$ and convert it to DFA. Dec. 2008

Ans. :

$(01^* + 1)$

RE to NFA

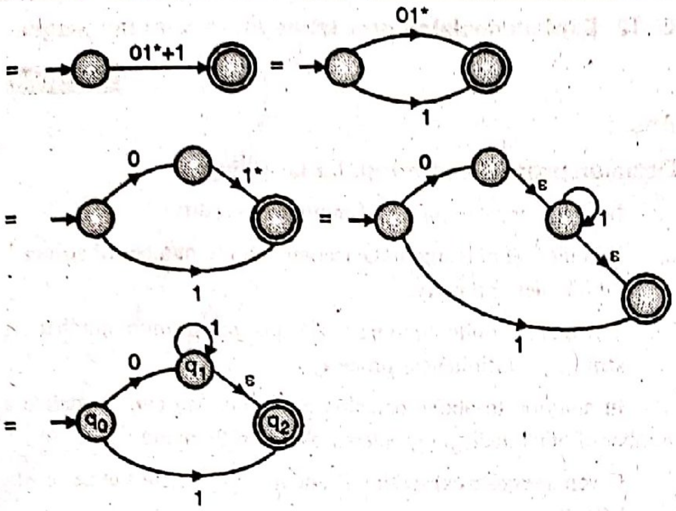


Fig. 3.6(a) : RE to NFA

NFA to DFA

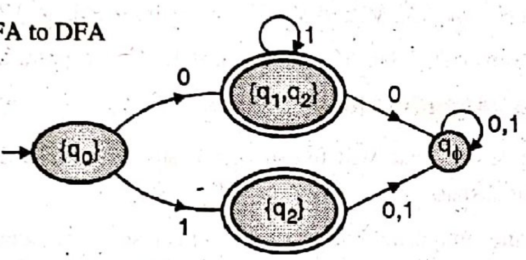


Fig. 3.6(b) : NFA to DFA

Q. 9 Construct an NFA with ε-moves for the RE 10(0 + 01 + 0110)* **May 2009**

Ans. :

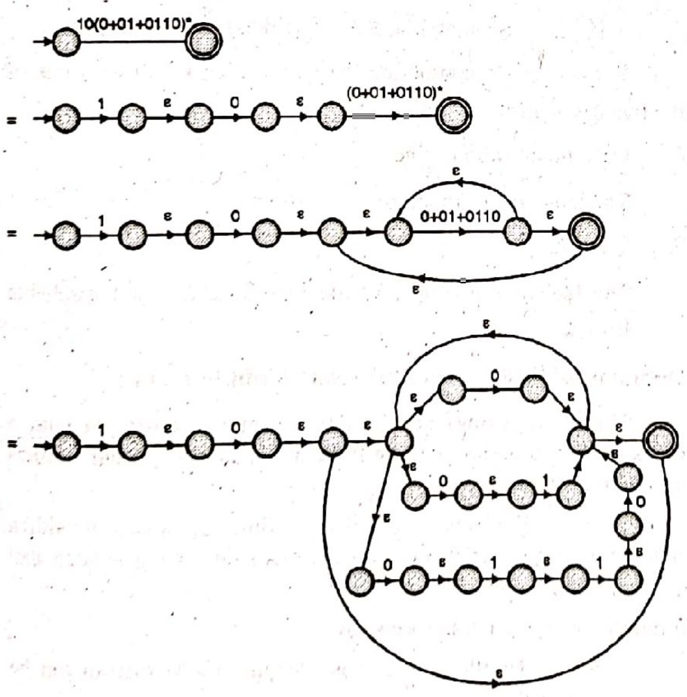


Fig. 3.7

Q. 10 State the pumping lemma for regular language.

Ans. :

Pumping lemma for regular language

Pumping lemma gives a necessary condition for an input string to belong to a regular set.

Pumping lemma does not give sufficient condition for a language to be regular.

Pumping lemma should not be used to establish that a given language is regular.

Pumping lemma should be used to establish that a given language is not regular.

The pumping lemma uses the pigeonhole principle which states that if n pigeons are placed into less than n holes, some holes have to have more than one pigeon in it. Similarly, a string of length $\geq n$ when recognized by a FA with n states will see some states repeating.

Definition of Pumping Lemma

Let L be a regular language and $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automata with n-states. Language L is accepted by m. Let $\omega \in L$ and $|\omega| \geq n$, then ω can be written as xyz, where

(i) $|y| > 0$

(ii) $|xy| \leq n$

(iii) $xy^i z \in L$ for all $i \geq 0$ here y^i denotes that y is repeated or pumped i times.

Interpretation of Pumping Lemma

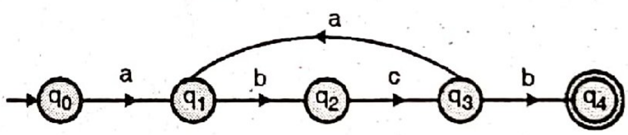


Fig. 3.8 : FA considered for interpretation of pumping lemma

Let us consider the FA of Fig. 3.8

No. of states $= 5$ (q_0 to q_4)

Let us take a string ω with $|\omega| \geq 5$, recognized by the FA.

ω = abcabcb

To recognize the string ω = abcabcb, the machine will transit through various states as shown in Fig. 3.6.2.

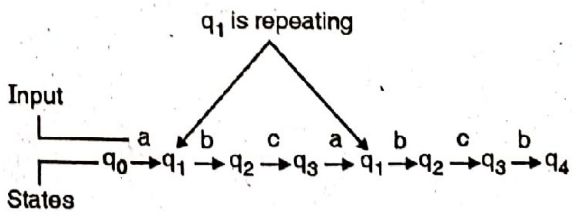


Fig. 3.9 : Transitions of FA on input abcabcb

As the input abcabcb takes the machine through the loop $q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_1$, this loop can repeat any number of times. In terms of abcabcb, we can say that if abcabcb is accepted by FA then every string in a(bca)*bcb will be accepted by the FA of Fig. 3.8. The portion bca is input during the loop.

$q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_4$.

Thus, if abcabcb is accepted by the FA then abcabcb can be written as xyz, with

$x = a$

$y = bca$

$z = bcb$.

Length of abcabcb is $\geq n$

xy^iz for every $i \geq 0$ or $a(bca)^ibcb$ for every $i \geq 0$ will be accepted by the FA of Fig. 3.8.

Q. 11 Construct NFA from (0 + 1)*(00 + 11) and convert into minimized DFA. Dec. 2009

Ans. :

(0 + 1)*(00 + 11)

RE to NFA

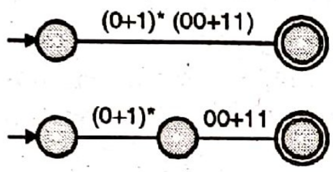


Fig. 3.10 : RE to NFA

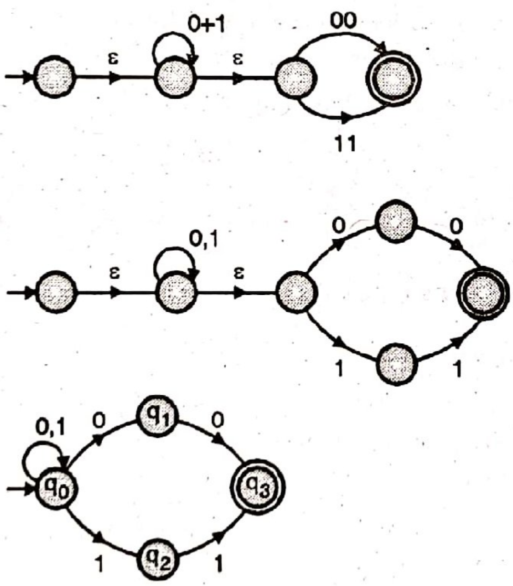


Fig. 3.10(a) : RE to NFA

NFA to DFA

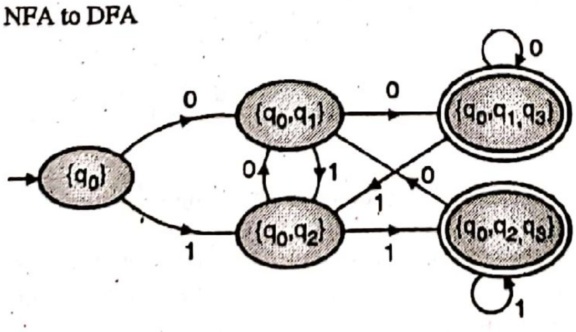


Fig. 3.10(b) : NFA to DFA

Q. 12 Explain decision properties for regular languages. Dec. 2009

Ans. :

Decision properties for regular languages

1. Is a regular set empty ? – Emptiness property.
2. Whether a finite automata accepts a finite number of strings ? – Finiteness property.
3. Whether a finite automata accepts an infinite number of strings ? – Infiniteness property.

In addition to above decision problems, we can formulate a number of other decision problems. Some of them are :

1. Given a regular expression R and a string ω, does ω belong to L(R) ?
2. Given two FAs M_1 and M_2, is $L(M_1) = L(M_2)$?
3. Given two FAs M_1 and M_2, is $L(M_1)$ subset of $L(M_2)$?
4. Given an FA M, is M a minimum state FA accepting L(M) ?

Decision Algorithm for emptiness :

Finite automata will fail to accept any string if it does not have a final state.

Finite automata will fail to accept a string if none of its accepting states is reachable from the initial state.

We can determine the emptiness of language accepted by an FA by calculating Q_k, the set of states that can be reached from q_0 by using strings of length k or less.

$$Q_k = \left\{ \begin{array}{ll} \{q_0\} & \text{if } k = 0 \\ \{Q_{k-1} \cup \{\delta(q, a)\} \mid q \in Q_{k-1} \text{ and } a \in \Sigma\} & \text{if } k > 0 \end{array} \right\}$$

We can go on computing the Q_k for each $k \geq 0$ until one of the two cases arise :

1. Q_k contains a final state.

   The language is not empty.

2. $Q_k = Q_{k-1}$

   The language is empty as the final states are not reachable from q_0.

Decision algorithm for finiteness / infiniteness :

The set of strings accepted by a finite automata M with n states is finite if and only if the finite automata accepts only strings of length less than n.

The set of strings accepted by a finite automata M with n states is infinite if and only if it accepts some string ω such that $n \leq |\omega| < 2n$.

From the pumping lemma we know :

1. If ω with length of ω ≥ n is accepted by M then ω can be written as xyz.
2. For every, i xy^iz will be accepted by M.

We can always design an algorithm to generate all strings over Σ with length between n and 2n.

If any of these strings is accepted by M then L(M) is infinite else L(M) is finite.

Q. 13 Using pumping lemma for regular sets, prove that the language $L = \{\omega\omega^R \mid \omega \in \{0, 1\}^*\}$ is not regular. **May 2010**

Ans. :

Step 1 : Let us assume that L is regular and L is accepted by a FA with n states.

Step 2 : Let us choose a string

$$\omega = \underbrace{a^n b}_{\omega}\ \underbrace{ba^n}_{\omega^R} \longleftarrow \text{from } \omega\omega^R$$

$$|\omega| = 2n + 2 \geq n$$

Let us write w as xyz with

$|y| > 0$ and $|xy| \leq n$

Since $|xy| \leq n$, x must be of the form a^s.

Since $|xy| \leq n$, y must be of the form $a^r \mid r > 0$.

Now,

$$\omega = a^n bba^n = \underbrace{a^s}_{x}\ \underbrace{a^r}_{y}\ \underbrace{a^{n-s-r}bba^n}_{z}$$

Step 3 : Let us check whether $xy^i z$ for i = 2 belongs to L.

$$xy^2z = a^s a^{2r} a^{n-s-r} bba^n = a^{n+r} bba^n$$

Since $r > 0$, $a^{n+r}bba^n$ is not of the form $\omega\omega^R$ as the strings starts with (n + r) a's but ends in (n) a's. Therefore, $xy^2z \notin L$. Hence by contradiction, we can say that the given language is not regular.

Q. 14 Using pumping lemma for regular sets. Prove that the language $L = \{\omega\omega \mid \omega \in \{0, 1\}^*\}$ is not regular. **Dec. 2006, Dec. 2010**

Ans. :

Step 1 : Let us assume that the given language is regular and L is accepted by a FA with a n states.

Step 2 : Let us choose a string

$$\omega = \underbrace{a^n b}_{\omega}\ \underbrace{a^n b}_{\omega} \longleftarrow \text{from } \omega\omega$$

$$|\omega| = 2n + 2 \geq n$$

Let us write ω as xyz with

$|y| > 0$

and $|xy| \leq n$

Since $|xy| \leq n$, x must be of the form a^s.

Since $|xy| \leq n$, y must be of the form $a^r \mid r > 0$.

Now,

$$\omega = a^n ba^n b = \underbrace{a^s}_{x}\ \underbrace{a^r}_{y}\ \underbrace{a^{n-s-r}ba^n b}_{z}$$

Step 3 : Let us check whether xy^iz for i = 2 belongs to L.

$$xy^2z = a^s a^{2r} a^{n-s-r} ba^n b$$
$$= a^{n+r} ba^n b$$

Since $r > 0$, $a^{n+r}ba^nb$ is not of the form $\omega\omega^R$ as the number of a's in the first half is n + r and in the second half is n.

Therefore, $xy^2z \notin L$. Hence by contradiction, the given language is not regular.

Q. 15 Show that the language $L\ \{a^nba^n \mid n > 0\}$ is not regular. **Dec. 2009, Dec. 2011**

Ans. :

Step 1 : Let us assume that L is regular and L is accepted by an FA with n states.

Step 2 : Let us choose a string

$$\omega = a^nba^n$$
$$|\omega| = 2n + 1 \geq n$$

Let us write ω as xyz, with

$|y| > 0$

and $|xy| \leq n$

Since, $|xy| \leq n$, y must be of the form $a^r \mid r > 0$

Since, $|xy| \leq n$, x must be of the form a^s.

Now, a^nb^n can be written as :

$a^s a^r a^{n-s-r} ba^n$

Step 3 : Let us check whether xy^iz for i = 0 belongs to L.

$$xy^0z = a^s (a^r)^0 a^{n-s-r} b a^n$$
$$= a^{n-r} b a^n$$

Since, $r > 0$ the string $a^{n-r}ba^n \notin L$.

Hence by contradiction we can say that the given language is not regular.

Q. 16 Write short note on application areas of R.E. **Dec. 2012**

Ans. :

Application areas of Regular Expression

1. R.E. in Unix

The UNIX regular expression lets us specify a group of characters using a pair of square brackets [ ]. The rules for character classes are :

1. **[ab]** Stand for a + b
2. **[0 – 9]** Stand for a digit from 0 to 9
3. **[A – Z]** Stands for an upper-case letter

4. **[a – z]** Stands for a lower-case letter

5. **[0 – 9A-Za – z]** Stands for a letter or a digit.

   The **grep** utility in UNIX, scans a file for the occurrence of a pattern and displays those lines in which the given pattern is found.

   For example :

   $ grep president emp.txt

   It will list those lines from the file emp.txt which has the pattern "president". The pattern in grep command can be specified using regular expression.

6. ***** matches zero or more occurrences of previous character.

7. **●** matches a single character.

8. **[^ pqr]** Matches a single character which is not a p, q or r.

9. **^ pat** Matches pattern **pat** at the beginning of a line

10. **pat $** Matches pattern at end of line.

Example

(a) The regular expression **[aA] g [ar] [ar] wal** stands for either "Agarwal" or 'agrawal".

(b) **g*** stands for zero or more occurrences of g.

(c) **$grep "A · * thakur" emp.txt** will look for a pattern starting with **A.** and ending with **thakur** in the file emp.txt.

2. Lexical Analysis

Lexical analysis is an important phase of a compiler. The lexical analyser scans the source program and converts it into a steam of tokens. A token is a string of consecutive symbol defining an entity.

For example a C statement **x = y + z** has the following tokens :

x – An identifier

= – Assignment operator

y = An identifier

\+ – Arithmetic operator +

z – An identifier

Keywords, identifiers and operators are common examples of tokens.

The UNIX utility **lex** can be used for writing of a lexical analysis program. Input to **lex** is a set of regular expressions for each type of token and output of **lex** is a C program for lexical analysis.

Q. 17 Design a DFA corresponding to the regular expression. (a + b)* aba (a + b)* **May 2013**

Ans. :

The language associated with the R.E. (a + b)*aba(a + b)* = strings with "aba" as substring.

DFA for strings with aba as substring.

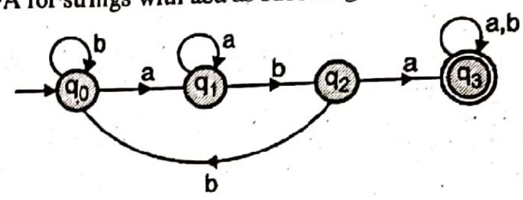


Fig. 3.11

Q. 18 Construct an NFA with epsilon transition for the following RE. (00 + 11)* (10)* **May 2014**

Ans. :

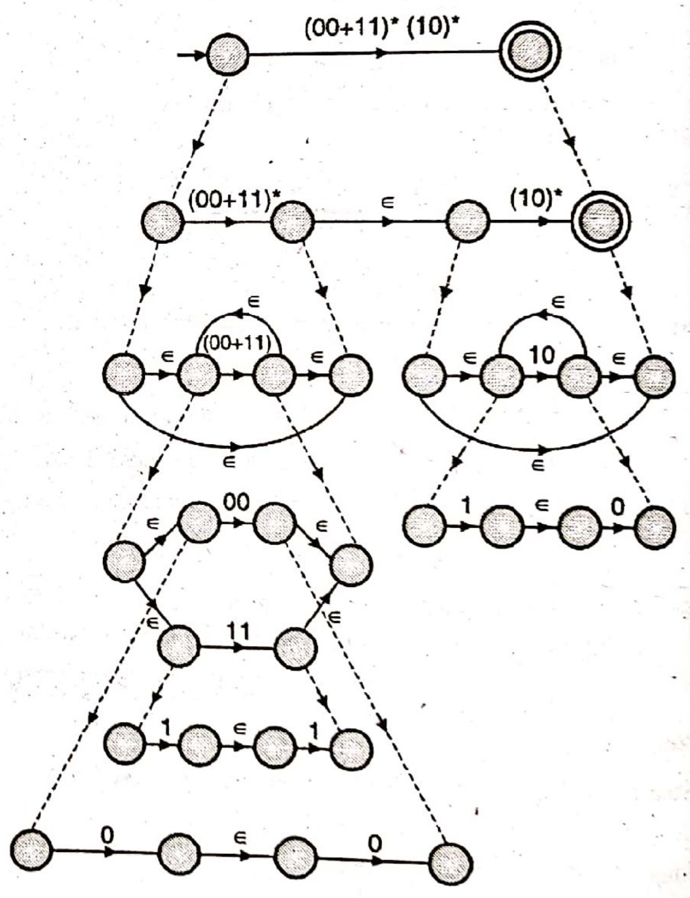


Fig. 3.12

Q. 19 Convert (0 + ∈) (10)* (∈ + 1) into NFA with ∈-moves and hence obtain a DFA. **Dec. 2014**

Ans. :

Step 1 : RE to NFA for (0 + ∈) (10)* (∈ + 1)

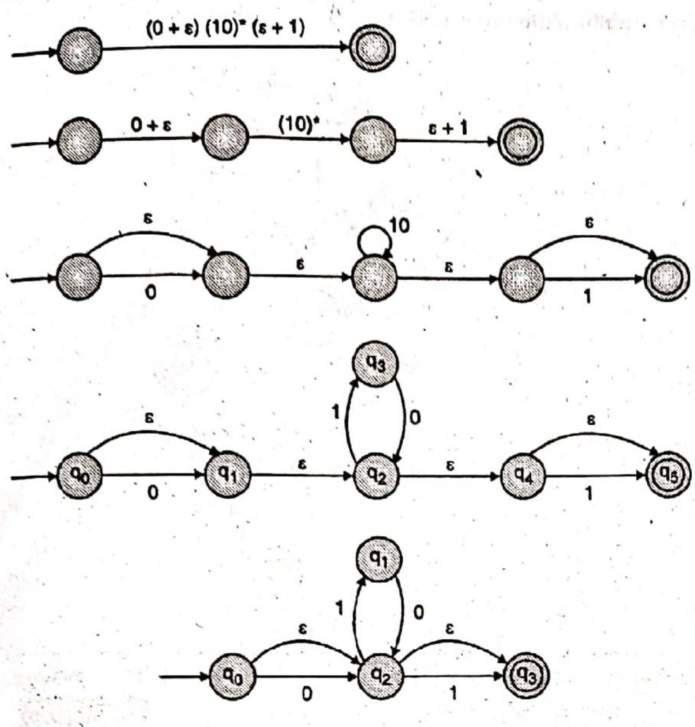


Fig. 3.13

(Note : States have been removed.)

Step 2 : ∈-NFA to DFA

∈-closure of states

$q_0 \rightarrow \{q_0, q_2, q_3\}$, $q_1 \rightarrow \{q_1\}$

$q_2 \rightarrow \{q_2, q_3\}$, $q_3 \rightarrow \{q_3\}$

The DFA using the direct method is given below.

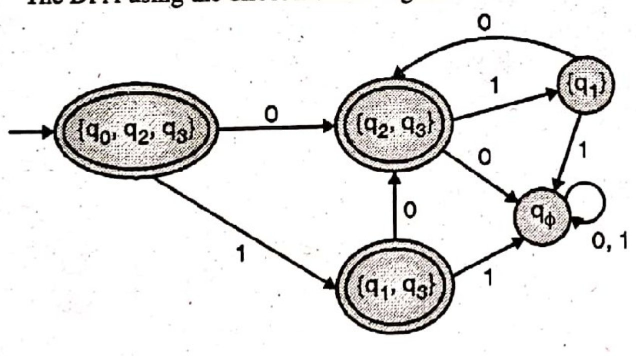


Fig. 3.14

Q. 20 Using pumping lemma for regular sets, prove that the language, $L = \{0^n \mid n$ is a prime$\}$ is not regular.

Dec. 2007, Dec. 2009, Dec. 2015, May 2016

Ans. :

Step 1 : Let us assume that the given language is regular and L is accepted by a FA with n states.

Step 2 : Let us choose a string $\omega = a^p$, where p is a prime and $p > n$.

$$|\omega| = |a^p| = p \geq n$$

Let us write w as xyz with

$$|y| > 0$$

and $|xy| \leq n$

We can assume that $y = a^m$ for $m > 0$.

Step 3 : Length of xy^iz can be written as given below :

$$|xy^iz| = |xyz| + |y^{i-1}| = p + (i-1)\,m$$

as $|y| = |a^m| = m$

Let us check whether P (i − 1) m is a prime for every i.

For $i = p + 1$, $p + (i-1)\,m = P + P_m = P\,(1 + m)$.

P (1 + m) is not a prime as it has two factors p and (1 + m) and

$$|p| > 1,$$

$$|1 + m| > 1$$

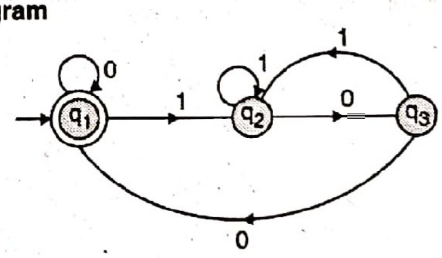
So $xy^{p+1}z \notin L$ Hence by contradiction the given language is not regular.

Q. 21 Draw a state diagram and construct a regular expression corresponding to the following state transition table. **Dec. 2016**

State	0	1
→*q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_1	q_2

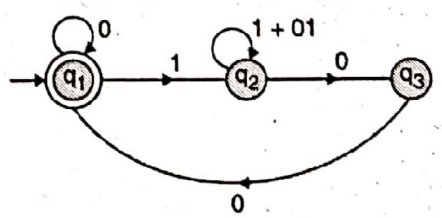
Ans. :

State diagram

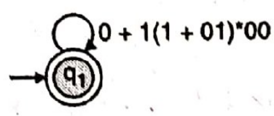


R.E. form state diagram

Step 1 : Removing loop between q_2 and q_3 we get



Step 2 : Removing the main loop, we get



Q. 22 Show that the language $L = \{a^nb^n\}$ is not regular.

Dec. 2006, May 2010, Dec. 2010, Dec. 2012, May 2013, May 2014, Dec. 2016, May 2017, Dec. 2017

Ans. :

Step 1 : Let us assume that L is regular and L is accepted by a FA with n states.

Step 2 : Let us choose a string

$$\omega = a^n b^n$$

$$|\omega| = 2n \geq n$$

Let us write w as xyz, with

$$|y| > 0$$

and $|xy| \leq n$

Since, $|xy| \leq n$, y must be of the form $a^r \mid r > 0$

Since, $|xy| \leq n$, x must be the form a^s.

Now, $a^n b^n$ can be written as

$$\underbrace{a^s}_{x}\ \underbrace{a^r}_{y}\ \underbrace{a^{n-s-r} b^n}_{z}$$

Step 3 : Let us check whether xy^iz for i = 2 belongs to L.

$$xy^2z = a^s(a^r)^2 a^{n-s-r} b^n$$

$$= a^s a^{2r} a^{n-s-r} b^n$$

$$= a^{s+2r+n-s-r} b^n$$

$$= a^{n+r} b^n$$

Since r > 0, number of a's in $a^{n+r} b^n$ is greater than number of b's. Therefore, $xy^2z \notin L$. Hence by contradiction we can say that the given language is not regular.

Q. 23 Construct NFA for given regular expressions :

(i) (a + b)*ab

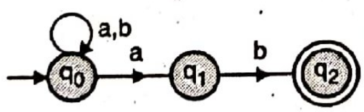
(ii) aa(a + b)*b

(iii) (aba) (a + b)*

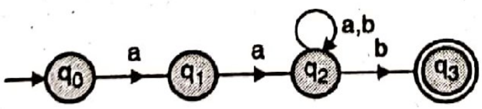
(iv) (ab/ba)*I(aa/bb)*

Ans. :

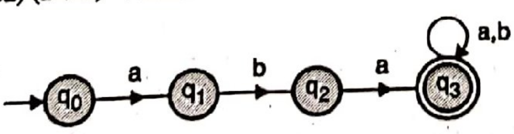
(i) (a + b)*ab : NFA



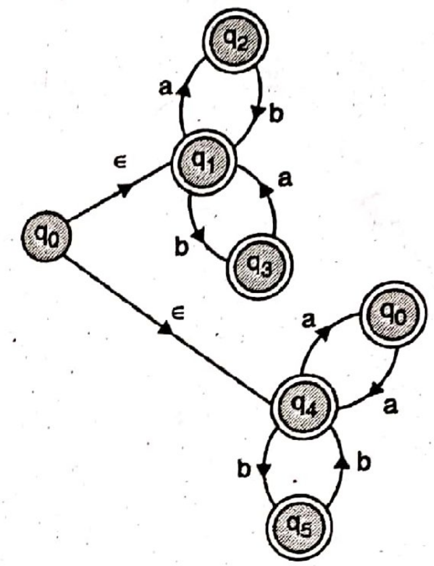
(ii) aa(a + b)*b : NFA



(iii) (aba) (a + b)* : NFA



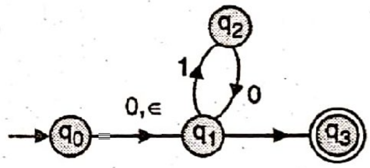
(iv) (ab|ba)*|(aa|bb)* : NFA



Q. 24 Convert (0 + ε) (10)*(ε + 1) into NFA with ε-moves and obtain DFA. Dec. 2017

Ans. :

Step 1 : NFA for the given expression :



Step 2 : ∈-closure of states :

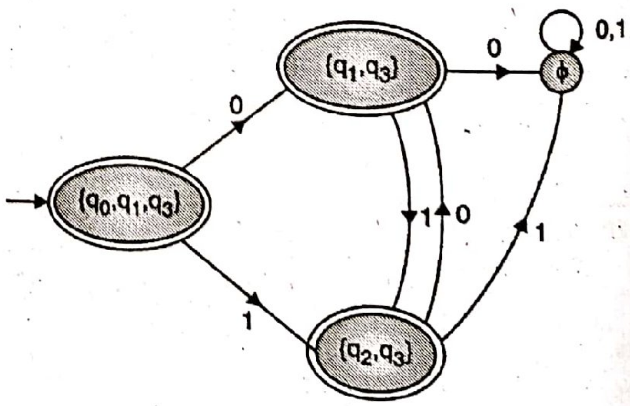
$q_0 \rightarrow \{q_0, q_1, q_3\}$

$q_1 \rightarrow \{q_1, q_3\}$

$q_2 \rightarrow \{q_2\}$

$q_3 \rightarrow \{q_3\}$

Step 3 : DFA using direct method :

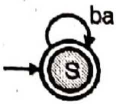


Chapter 4 : Regular Grammar (RG)

Q. 1 Construct right linear grammar and left linear grammar for the language (ba)*. Dec. 2006

Ans. :

Transition system for (ba)* is given by :



We can write left linear grammar and the right linear grammar form the transition systems.

Right linear grammar :

$S \rightarrow baS \mid \varepsilon$

Left linear grammar :

$S \rightarrow Sba \mid \varepsilon$

Q. 2 Final the equivalent DFA accepting the regular language defined by the right linear grammar given as :

$S \rightarrow aA \mid bB$, $A \rightarrow aA \mid bc \mid a$ $B \rightarrow aB \mid b$ $C \rightarrow bB$ May 2009

Ans. :

A new final state F is being introduced to handle productions like,

$A \rightarrow a$, $B \rightarrow b$

Step 1 : Adding transitions corresponding to every production, we get the FA shown in Fig. 4.1(a).

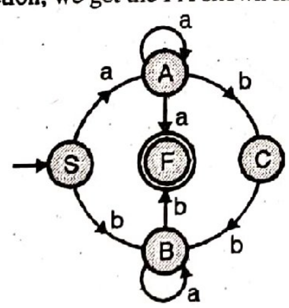


Fig. 4.1(a)

Step 2 : Drawing an equivalent DFA, we get :

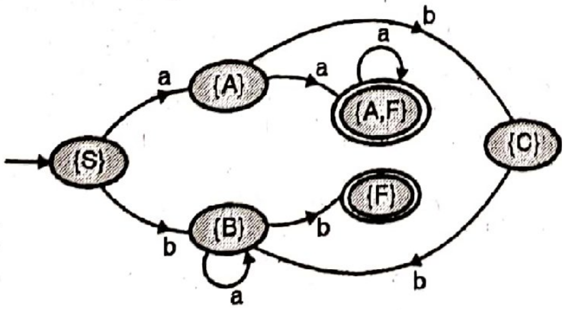


Fig. 4.1(b)

Step 3 : A dead state is added to handle ϕ–transition. The resulting DFA is shown in Fig. 4.1(c).

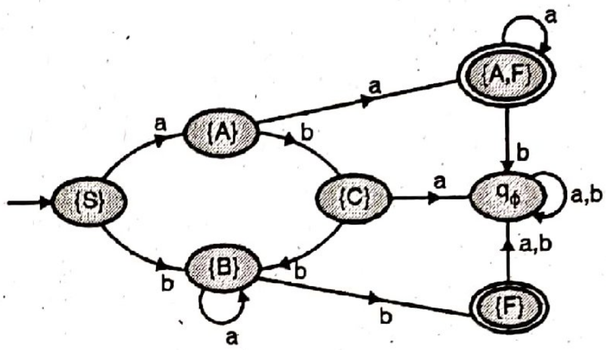


Fig. 4.1(c)

Q. 3 Construct left linear and right linear grammar for the regular expression.

((01 + 10)*11)*00)* May 2009

Ans. :

The given expression can be represented using a transition system as shown below :

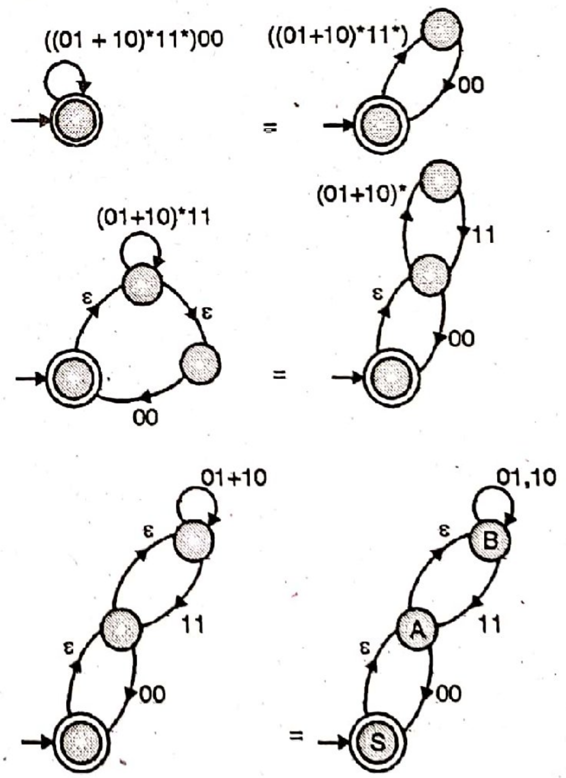


Fig. 4.2(a)

Removing ε – transitions, we get :

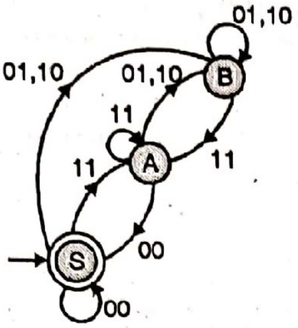


Fig. 4.2(b)

Writing of right linear grammar we get,

S → 00S | 11A | 01B | 10B | ε

A → 11A | 01B | 10B | 00S

B → 01B | 10B | 11A

For writing of left linear grammar, we interchange the start state and the final state and change direction of all transitions. The resulting transition system is given by :

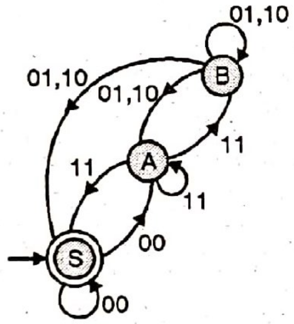


Fig. 4.2(c)

Writing of left linear grammar we get,

S → S00 | A00 | ε

A → A11 | B11 | S11

B → B01 | 10B | S01 | S10 | A01 | A10

Q. 4 Convert the following right-linear grammar to an equivalent DFA.

S → bB

B → bC

B → aB

C → a

B → b

Ans. :

Re-writing the production we get

S → bB

B → bC | b

B → aB

C → a

Step 1 : Adding transitions corresponding to every production, we get

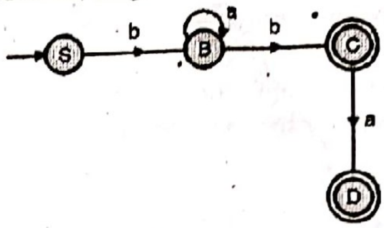


Fig. 4.3(a)

Step 2 : Adding a state E to handle ϕ-transitions, we get the final DFA.

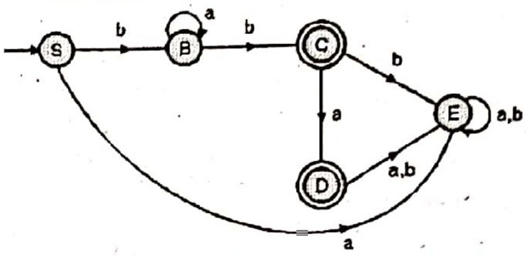


Fig. 4.3(b) : Final DFA

Q. 5 Convert following RG to DFA

S → 0A | 1B, A → 0C | 1A | 0,

B → 1B | 1A | 1, C → 0 | 0A.

Ans. :

A new final state F is being introduced to handle productions like, A → 0, B → 1, C → 0.

Step 1 : Adding transitions corresponding to every production, we get the FA shown in Fig. 4.4(a).

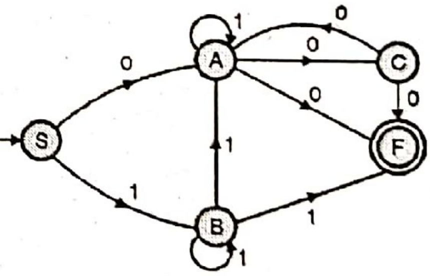


Fig. 4.4(a)

Step 2 : Drawing an equivalent DFA, we get

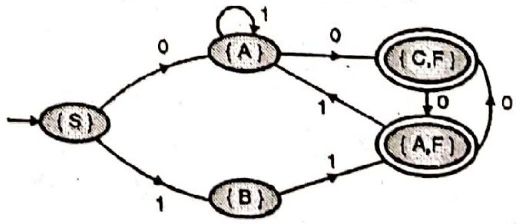


Fig. 4.4(b)

Step 3 : States {S}, {A}, {B}, {C,F}, and {A, F} are renamed as q_0, q_1, q_2, q_3, q_4 and a dead state q_ϕ is introduced to handle ϕ – transitions. The resulting DFA is shown in Fig. 4.4(c) :

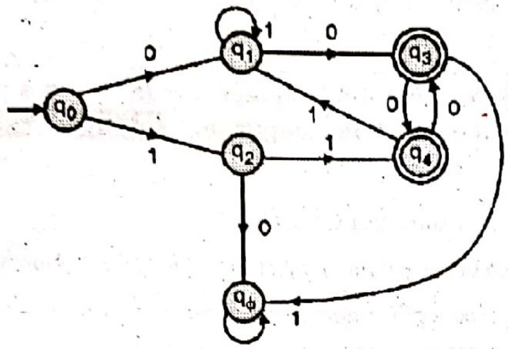


Fig. 4.4(c) : Final DFA

Q. 6 Write an equivalent left linear grammar from the given right linear grammar.

S → 0A | 1B

A → 0C | 1A | 0

B → 1B | 1A | 1

C → 0 | 0A

Ans. :

Step 1 : Transition system for the given right linear grammar is as shown in Fig. 4.5(a).

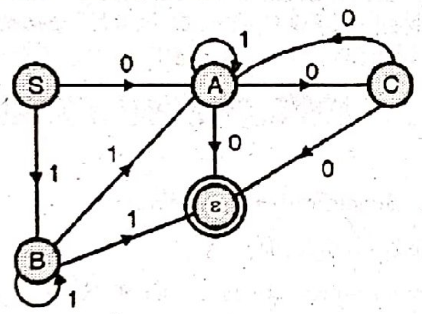


Fig. 4.5(a) : Transition graph

Step 2 : Interchanging the start state with the final state and reversing direction of transitions, we get

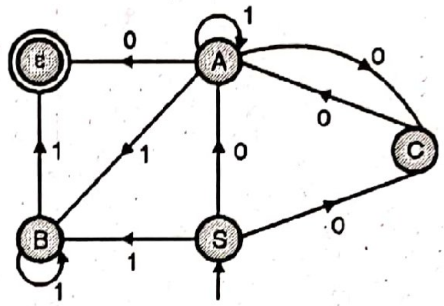


Fig. 4.5(b)

Step 3 : Writing of left linear grammar from the transition system, we get :

S → C0 | A0 | B1

A → A1 | C0 | B1 | 0

B → B1 | 1

C → A0.

Chapter 5 : Context Free Grammars (CFG)

Q. 1 Write an unambiguous CFG for arithmetic expressions with operators : +, *, /, ^, unary minus and operand a, b, c, d, e, f. Also, if should be possible to generate brackets with your grammar. Derive (a + b) ^ d / e + (– f) from your grammar. **Dec. 2005**

Ans. :

An unambiguous grammar is given below.

E → E + T | T [+ has lowest priority with L → R associativity]

T → T * F | T / F | F [* and / has higher priority over + with L → R associativity]

F → F ^ G | G [^ has higher priority over * and / with L → R associativity]

G → – H | H [unary – has the highest priority]

H → a | b | c | d | e | f | (E) [to handle brackets and identifiers]

Derivation tree for (a + b) ^ d / e + (– f)

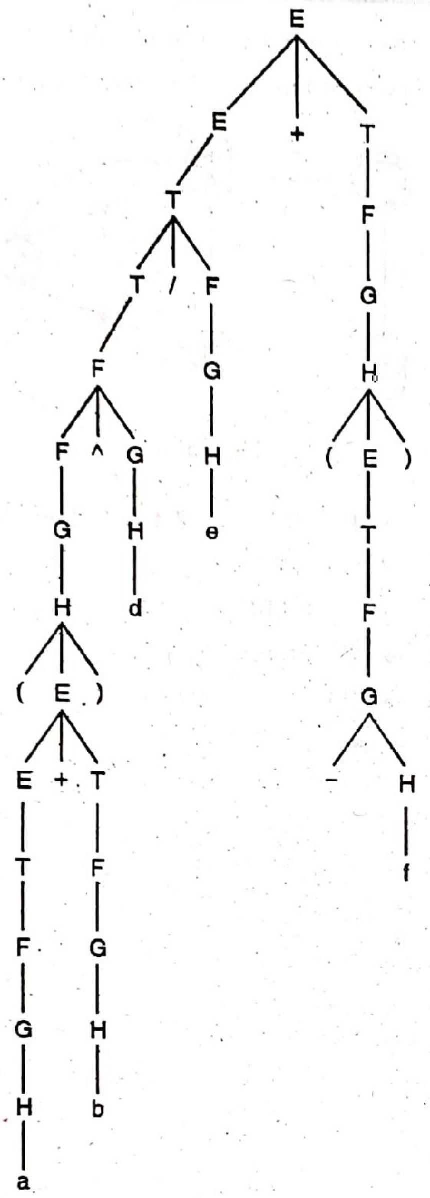


Fig. 5.1 : Derivation tree for (a + b) ^ d / e + (– f)

Q. 2 Convert the following CFG to GNF :

S → aSa | bSb | c **Dec. 2005**

Ans. :

The grammar can be brought to GNF through simple substitutions $C_a \rightarrow a$ and $C_b \rightarrow b$.

$S \rightarrow aSC_a \mid bSC_b \mid C$

$C_a \rightarrow a$

$C_b \rightarrow b$

Q. 3 Write short note on GNF.

Ans. :

Greibach Normal Form (GNF)

A context free grammar G = (V, T, P, S) is said to be in GNF if every production is of the form :

$A \rightarrow a\alpha,$

Where, $a \in T$ is a terminal and α is a string of zero or more variables.

The language L (G) should be without ε.

Right hand side of each production should start with a terminal followed by a string of non-terminals of length zero or more.

Q. 4 Prove that the language $L = \{a^p \mid p \text{ is a prime}\}$ is not context free language. **May 2006, May 2012**

Ans. :

1. Let us assume that L is a CFL.
2. Let n be the natural number for L, as per the pumping lemma.
3. Let p be a prime number greater than n. Then $z = a^p \in L$. We can write z = uvxyz.
4. By pumping lemma $uv^0xy^0z = uxz \in L$. Therefore,

   | uxz | is a prime number.

   Let us assume that | uxz | = q.

   Now, let us consider a string uv^qxy^qz,

   The length of uv^qxy^qz is given by :

$|uv^q xy^q z| = q + q(|v| + |y|)$, which is not a prime with q is a factor.

Thus, $uv^qxy^qz \notin L$. This is a contradiction.

Therefore, L is not a context free language.

Q. 5 Given a CFG G, find G' in CNF generating L (G) - ε
S → ASB | ε A → AaS | a B → SbS | A | bb

May 2006, May 2009, May 2010, Dec. 2011

Ans. :

Step 1 : Simplification of grammar

Symbol S is nullable.

After removing ε-productions, the set of productions is given by

$S \rightarrow ASB \mid AB$

$A \rightarrow AaS \mid Aa \mid a$

$B \rightarrow SbS \mid Sb \mid bS \mid b \mid A \mid bb$

Unit production $B \rightarrow A$ is removed, the resulting set of productions is given by

$S \rightarrow ASB \mid AB$

$A \rightarrow AaS \mid Aa \mid a$

$B \rightarrow SbS \mid Sb \mid bS \mid b \mid AaS \mid Aa \mid a \mid bb$

Step 2 : Every symbol in α, in productions of the form $A \rightarrow \alpha$ where $|\alpha| \geq 2$ should be a variable.

This can be done by adding two productions :

$C_a \rightarrow a$

and $C_b \rightarrow b$

The set of productions after the above changes is

$S \rightarrow ASB \mid AB$

$A \rightarrow AC_aS \mid AC_a \mid a$

$B \rightarrow SC_bS \mid SC_b \mid C_bS \mid b \mid AC_aS \mid AC_a \mid a \mid C_bC_b$

$C_a \rightarrow a$

$C_b \rightarrow b$

Step 3 : Finding an equivalent CNF

Original production	Equivalent productions in CNF
$S \rightarrow ASB$	$S \rightarrow AC_1$ $C_1 \rightarrow SB$
$S \rightarrow AB$	$S \rightarrow AB$
$A \rightarrow AC_aS$	$A \rightarrow AC_2$ $C_2 \rightarrow C_aS$
$A \rightarrow AC_a$	$A \rightarrow AC_a$
$A \rightarrow a$	$A \rightarrow a$
$B \rightarrow SC_bS$	$B \rightarrow SC_3$ $C_3 \rightarrow C_bS$
$B \rightarrow SC_b \mid C_bS \mid b$	$S \rightarrow SC_b \mid C_bS \mid b$
$B \rightarrow AC_aS$	$B \rightarrow AC_2$
$B \rightarrow AC_a \mid a \mid C_bC_b$	$B \rightarrow AC_a \mid a \mid C_bC_b$
$C_a \rightarrow a$	$C_a \rightarrow a$
$C_b \rightarrow b$	$C_b \rightarrow b$

Q. 6 Convert the following grammar into GNF

$S \rightarrow XY1 \mid 0 \quad X \rightarrow 00X \mid Y \quad Y \rightarrow 1X1$

May 2006, May 2012

Ans. :

Simplification of grammar

The unit production $x \rightarrow y$ is removed, the equivalent set of productions is given by :

$S \rightarrow XY1 \mid 0$

$X \rightarrow 00X \mid 1X1$

$Y \rightarrow 1X1$

The symbol X is non-generating.

The set of productions after elimination of X is given by :

$S \rightarrow 0$, it is in GNF

Q. 7 Find CFG for generating

(i) String containing alternate sequence of 0's and 1's, $\Sigma = \{0, 1\}$

(ii) The string containing no consecutive 'b's but 'a's can be consecutive.

(iii) The set of all string over alphabet {a, b} with exactly twice as many a's as b's.

(iv) Language having number of a's greater than number of b's.

Dec. 2006, May 2009, Dec. 2009

Ans. :

(i) String containing alternate sequence of 0's and 1's, $\Sigma = \{0, 1\}$

Since, any binary number will satisfy the condition of alternate sequence of 0's and 1's, the language $L = (0 + 1)^*$

The set of productions are :

$S \rightarrow 0S \mid 1S \mid \varepsilon$

$\therefore$ CFG G = $(\{S\}, \{0, 1\}, \{S \rightarrow 0S \mid 1S \mid \in\}, S)$

(ii) The string containing no consecutive b's but a's can be consecutive.

The set of productions for the given language L are :

P = {

$S \rightarrow aS \mid bX \mid b \mid \varepsilon$

$X \rightarrow aS \mid a$

}

These production can easily be written from the FA for the above language. The FA is shown in Fig. Ex. 5.2.33.

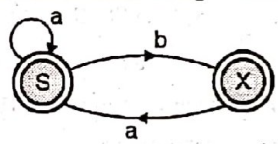


Fig. 5.2

Set of variables V = {S, X}

Set of terminals T = {a, b}

Start symbol = S

(iii) The set of all strings over alphabet {a, b} with exactly twice as many a's as b's.

The CFG G = (V, T, P, S)

Where V = {S}

T = {a, b}

P = $\{S \rightarrow aSaSbS \mid aSbSaS \mid bSaSaS \mid \varepsilon\}$

S = Start symbol

(iv) Language having number of a's greater than number of b's.

The set of productions for the grammar are given by :

P = {

$S \rightarrow SaS \mid aSS \mid SSa \mid a \mid aX \mid Xa$

$X \rightarrow aB \mid bA$

$A \rightarrow aX \mid bAA \mid a$

$B \rightarrow bX \mid aBB \mid b$

}

The variable X generates a string having equal number of a's and b's. Group of excess a's over b's are generated by S-productions.

Where

Set of variables $V = \{S, X, A, B\}$

Set of terminals $T = \{a, b\}$

Start symbol $= S$

Q. 8 Convert the given grammar to GNF.
S → SSlaSblab **Dec. 2006**

Ans. :

Step 1 : Other than the first symbol on the RHS of every production, every symbol must be a variable.

We can make the substitution X for b.

The resulting set of productions after the above substitution is :

$S \rightarrow SS | aSX | aX$

$X \rightarrow b$

Step 2 : Removing left recursion from s-production, we get :

$S \rightarrow aSXS_1 | aXS_1 | aSX | aX$

$S_1 \rightarrow SS_1 | S$

$X \rightarrow b$

Step 3 : S_1-productions are not in GNF. They can be brought to GNF by substituting S.

$S \rightarrow aSXS_1 | aXS_1 | aSX | aX$

$S_1 \rightarrow aSXS_1S_1 | aXS_1S_1 | aSXS_1 | aXS_1 | aSXS_1 | aXS_1 | aSX | aX$

$X \rightarrow b$

Q. 9 Prove that $L = \{0^i 1^i 2^j 3^j \mid i >= 1 \text{ and } j >= 1\}$ is not context free. **Dec. 2007**

Ans. :

1. Let us assume that L is CFL
2. Let us pick up a word $\omega = 0^n 1^n 2^n 3^n$, where the constant n is given as per the pumping lemma.
3. ω is rewritten as uvxyz where $|vxy| \leq n$ and $v \cdot y \neq \varepsilon$ i.e. both v and y are not null.
4. From pumping lemma, if uvxyz ∈ L then uv^ixy^iz is in L(G) for each i = 0,1,2,...

There are two case :

Case I : vy contains three symbols. These three symbols could be 0,1,2 or 1,2,3.

The exact ordering of 0,1,2,3 will be broken in uv^2xy^2z and hence $uv^2xy^2z \notin L(G)$

Case II : If vy does not contain three symbols then uv^2xy^2z will have either unequal number of 0's and 2's or unequal number of 1's and 3's. Hence, $uv^2xy^2z \notin L(G)$.

Thus, proved by contradiction.

Q. 10 Prove that $L = \{a^i b^i c^i \mid i \geq 1\}$ is not a CFL. **May 2008**

Ans. :

1. Let us assume that L is CFL.
2. Let us pick up a word $w = a^n b^n c^n$ where the constant n is given as per the pumping lemma.
3. w is rewritten as uvxyz.

   Where $|vxy| \leq n$ and $v \cdot y \neq \varepsilon$ i.e., both v and y are not null.
4. From pumping lemma, if uvxyz ∈ L then uv^ixy^iz is in L (G) for each i = 0, 1, 2, ...

There are two cases :

Case I : vy contains all three symbols a, b and c.

If vy contains all three symbols a, b and c then either v or y contains two symbols. The exact ordering of a, b and c will be broken in uv^2xy^2z and hence $uv^2xy^2z \notin L(G)$

Case II : If vy does not contain three symbols a, b and c then uv^2xy^2z will have unequal number of a's, b's and c's and hence $uv^2xy^2z \notin L(G)$.

Hence, it is proved by contradiction.

Q. 11 Convert the following grammar to CNF S → AACD A → aAb | ε C → aC | a A → aDa | bDb | ε **May 2008**

Ans. :

First of all, the grammar must be simplified.

Step 1 : Removing null productions.

Nullable set = {A}

Null productions are removed with the resulting set of production as :

$S \rightarrow AACD | ACD | CD$

$A \rightarrow aAb | ab$

$C \rightarrow aC | a$

$A \rightarrow aDa | bDb$

Step 2 : Removing non-generating symbol

Symbol S and D are non-generating.

Since, the starting symbol itself is non-generating, it is an **invalid grammar**.

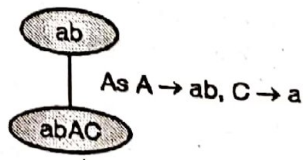


Fig. 5.3

Q. 12 Given a CFG G, find G' in CNF generating L (G) - ε

S → ASB | ε

A → AaS | a

B → SbS | A | bb

May 2006, May 2009, May 2010, Dec. 2011

Ans. :

Step 1 : Simplification of grammar

Symbol S is nullable.

After removing ε-productions, the set of productions is given by

S → ASB | AB

A → AaS | Aa | a

B → SbS | Sb | bS | b | A | bb

Unit production B → A is removed, the resulting set of productions is given by

S → ASB | AB

A → AaS | Aa | a

B → SbS | Sb | bS | b | AaS | Aa | a | bb

Step 2 : Every symbol in α, in productions of the form A → α where |α| ≥ 2 should be a variable.

This can be done by adding two productions :

C_a → a

and C_b → b

The set of productions after the above changes is

S → ASB | AB

A → AC_aS | AC_a | a

B → SC_bS | SC_b | C_bS | b | AC_aS | AC_a | a | C_bC_b

C_a → a

C_b → b

Step 3 : Finding an equivalent CNF

Original production	Equivalent productions in CNF
S → ASB	S → AC_1 C_1 → SB
S → AB	S → AB
A → AC_aS	A → AC_2 C_2 → C_aS
A → AC_a	A → AC_a
A → a	A → a
B → SC_bS	B → SC_3 C_3 → C_bS
B → SC_b | C_bS | b	S → SC_b | C_bS | b
B → AC_aS	B → AC_2
B → AC_a | a | C_bC_b	B → AC_a | a | C_bC_b
C_a → a	C_a → a
C_b → b	C_b → b

Q. 13 Let G = (V, T, P, S) be the CFG having following set of productions. Derive the string "aabbaa" using leftmost derivation and rightmost derivation.

S → aAS | a, A → SbA | SS | ba **May 2009**

Ans. :

(i) Leftmost derivation :

Leftmost derivation of aabbaa is being shown with the help of the parse tree.

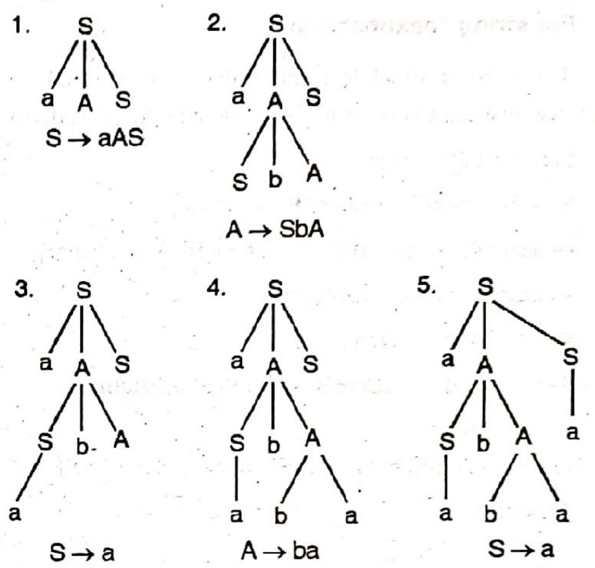


Fig. 5.4(a)

S → aAS → aSbAS → aabAS → aabbas → aabbaa

(ii) Rightmost derivation :

Rightmost derivation of aabbaa is being shown with the help of the parse tree.

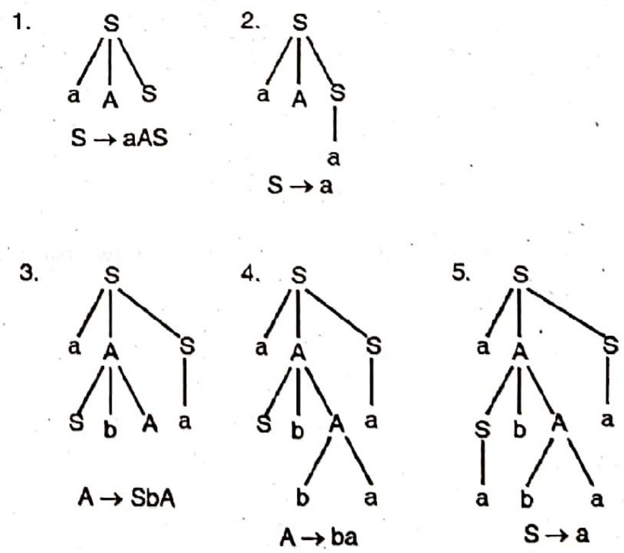


Fig. 5.4(b)

S → aAS → aAa → aSbAa → aSbbaa → aabbaa

Q. 14 Let G be the grammar S → aB | bA A → a | aS | bAA B → b | bS | aBBFind

(i) Left most derivation

(ii) Right most derivation

(iii) Parse Tree

(iv) Is the grammar unambiguous ?

For given strings (A) aaabbabbba (B) bbaaabbaba (C) 00110101 Dec. 2009, Dec. 2012, May 2013

Ans. :

(A) For string "aaabbabbba"

It will be worthwhile to draw the parse tree and from the parse tree, one can easily write left most and right most derivation.

(i) Left most derivation :

S → aB → aaBB → aaaBBB → aaabBB

→ aaabbB → aaabbaBB → aaabbabB → aaabbabbS

→ aaabbabbbA → aaabbabbba

(ii) Right most derivation :

S→aB→aaBB→aaBaBB→aaBaBbS→aaBaBbbA

→aaBaBbba

→aaBabbba→aaaBBabbba→aaaBbabbba→aaabbabbba

(iii) Parse tree :

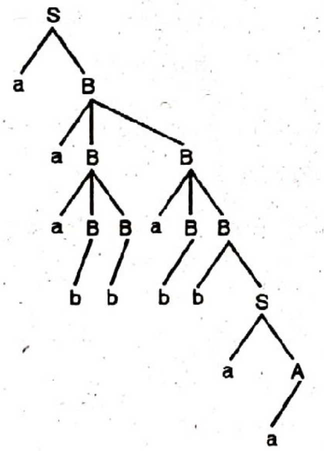
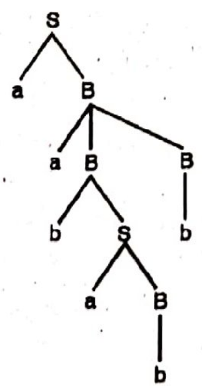
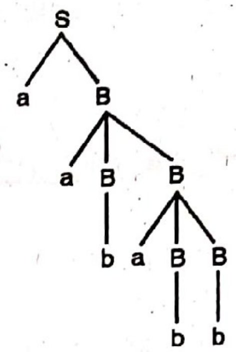


Fig. 5.5

(iv) The grammar is ambiguous as we can draw two parse trees for aababb :



(a)



(b)

Fig. 5.5

(B) For string "bbaaabbaba"

(i) Leftmost derivation

S → bA → bbAA → bbaA → bbaaS

→ bbaaaB → bbaaabs → bbaaabbA

→ bbaaabbas → bbaaabbabA → bbaaabbaba

(ii) Rightmost derivation

S → bA → bbAA → bbAaS → bbAaaB

→ bbAaabS → bbAaabbA → bbAaabbaS

→ bbAaabbabA → bbAaabbaba → bbaaabbaba

(iii) Parse tree for bbaaabbaba

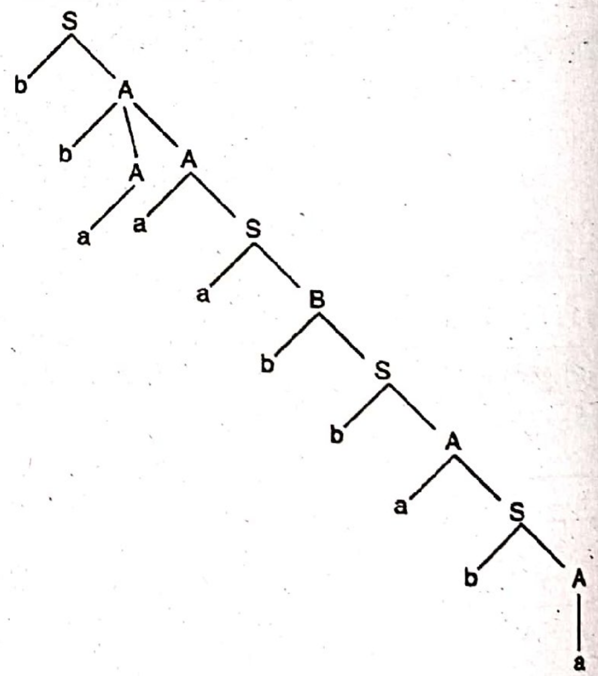


Fig. 5.5(c)

(C) For the string 00110101

(i) Leftmost derivation

S → 0BB → 00BB → 001B → 0011S

→ 00110 B → 001101S → 0011010B

→ 00110101

(ii) Rightmost derivation

$$S \to 0B \to 00BB \to 00B1S \to 00B10B$$
$$\to 00B101S \to 00B1010B \to 00B10101$$
$$\to 001110101$$

(iii) Parse tree

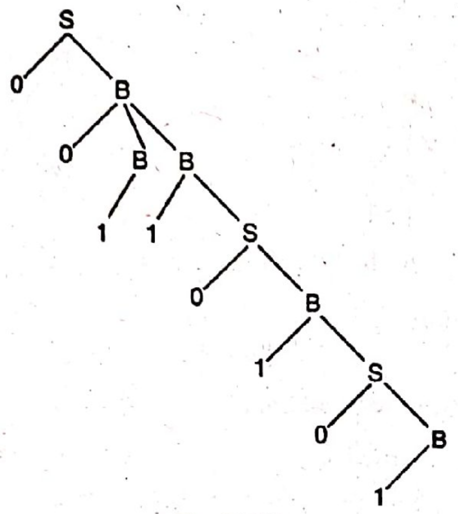


Fig. 5.5(d)

Q. 15 Obtain a grammar to generate the language $L = \{0^n 1^{2n} \mid n \geq 0\}$. **May 2010**

Ans. :

Productions for the required language are as follows.

$$P = \{S \to 0S11 \mid \varepsilon\}$$

CFG for the above language is ({S}, {0, 1}, P, S)

Q. 16 Reduce the following grammar to GNF. $S \to AB$, $A \to BSB \mid BB \mid b$, $B \to aAb \mid a$ **May 2011**

Ans. :

Step 1 : Making every symbol other than the first symbol (in derived string α in $A \to \alpha$) as a variable :

Variables C_b is substituted for b with resulting set of productions give as :

$$S \to AB$$
$$A \to BSB \mid BB \mid b$$
$$B \to aAC_b \mid a, \; C_b \to b$$

Step 2 : The variables S, A, B and C_b are renamed as A_1, A_2, A_3 and A_4 respectively. The resulting set of productions is given below.

$$A_1 \to A_2A_3$$
$$A_2 \to A_3A_1A_3 \mid A_3A_3 \mid b$$
$$A_3 \to aA_1A_4 \mid a$$
$$A_4 \to b$$

Step 3 : Convert to CFG

Given production		Equivalent Production in GNF
$A_4 \to b$	$\to$	$A_4 \to b$
$A_3 \to aA_1A_4 \mid a$	$\to$	$A_3 \to aA_1A_4 \mid a$
$A_2 \to A_3A_1A_3$ ⇓	Substituting A_3	$A_2 \to aA_1A_4A_1A_3 \mid aA_1A_3$
$A_2 \to A_3A_3$ ⇓	Substituting A_3	$A_2 \to aA_1A_4A_3 \mid aA_3$
$A_2 \to b$	$\to$	$A_2 \to b$
$A_1 \to A_2A_3$ ⇓	Substituting A_2	$A_1 \to aA_1A_4A_1A_3A_3 \mid aA_1A_3A_3 \mid aA_1A_4A_1A_3A_3 \mid aA_3A_3 \mid bA_3$

$\therefore$ The final set of productions is :

$$A_1 \to aA_1A_4A_1A_3A_3 \mid aA_1A_3A_3 \mid aA_1A_4A_3A_3 \mid aA_3A_3 \mid bA_3$$
$$A_2 \to aA_1A_4A_1A_3 \mid aA_1A_3 \mid aA_1A_4A_3 \mid aA_3 \mid b$$
$$A_3 \to aA_1A_4 \mid a$$
$$A_4 \to b.$$

Q. 17 Reduce the following grammars to GNF $B \to aAb \mid a$ $S \to AA \mid 1$ $A \to SS \mid 1$ **May 2011**

Ans. :

Step 1 : Renaming of variables by substituting A_1 for S and A_2 for A.

$$A_1 \to A_2A_2 \mid 1$$
$$A_2 \to A_1A_1 \mid 1$$

Step 2 : Every production of the form $A_i \to A_j\alpha$ with $i > j$ must be modified to make $i \leq j$.

A_2 – production, $A_2 \to A_1A_1$ should be modified. We must substitute $A_2A_2 \mid 1$ for the first A_1.

$$[A_2 \to A_1A_1] \Rightarrow \begin{bmatrix} A_2 \to A_2A_2A_1 \\ A_2 \to 1A_1 \end{bmatrix}$$

The resulting set of productions is :

$$A_1 \to A_2A_2 \mid 1$$
$$A_2 \to A_2A_2A_1 \mid 1A_1 \mid 1$$

Step 3 : Removing left recursion :

The A_2 – production contains left recursion. Left recursion can be removed through

$$A_2 \to 1A_1B_2 \mid 1B_2$$

$B_2 \rightarrow A_2 A_1 B_2 \mid A_2 A_1$

The resulting set of productions is :

$A_1 \rightarrow A_2 A_2 \mid 1$

$A_2 \rightarrow 1 A_1 B_2 \mid 1 B_2 \mid 1 A_1 \mid 1$

$B_2 \rightarrow A_2 A_1 B_2 \mid A_2 A_1$

Step 4 : A_2 – productions are in GNF.

A_1 and B_2 productions can be converted to GNF with the help of A_2 – productions.

$A_2 \rightarrow 1 A_1 B_2 \mid 1 B_2 \mid 1 A_1 \mid 1$

$A_1 \rightarrow 1 A_1 B_2 A_2 \mid 1 B_2 A_2 \mid 1 A_1 A_2 \mid 1 A_2 \mid 1$

$B_2 \rightarrow 1 A_1 B_2 A_1 B_2 \mid 1 B_2 A_1 B_2 \mid 1 A_1 A_1 B_2$
$\mid 1 A_1 B_2 \mid 1 A_1 B_2 A_1 \mid 1 B_2 A_1 \mid 1 A_1 A_1 \mid 1 A_1$

Q. 18 Let G be the grammar S → aB | bAA → a | aS | bAA B → b | bS | aBB Find

(i) Left most derivation

(ii) Right most derivation

(iii) Parse Tree

(iv) Is the grammar unambiguous ?

For given strings (A) aaabbabbba (B) bbaaabbaba (C) 00110101 **Dec. 2009, Dec. 2012, May 2013**

Ans. :

(A) For string "aaabbabbba"

It will be worthwhile to draw the parse tree and from the parse tree, one can easily write left most and right most derivation.

(i) Left most derivation :

S →aB → aaBB → aaaBBB → aaabBB

→ aaabbB → aaabbaBB → aaabbabB → aaabbabbS

→ aaabbabbbA → aaabbabbba

(ii) Right most derivation :

S→aB→aaBB→aaBaBB→aaBaBbS→aaBaBbbA

→aaBaBbba

→aaBabbba→aaaBBabbba→aaaBbabbba→aaabbabbba

(iii) Parse tree :

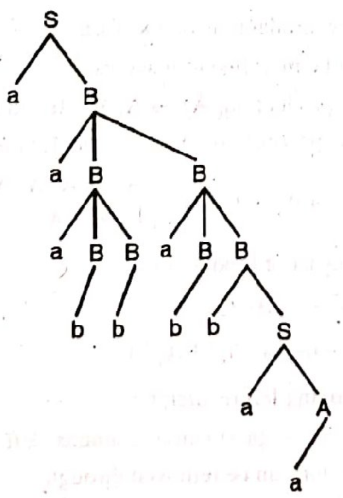
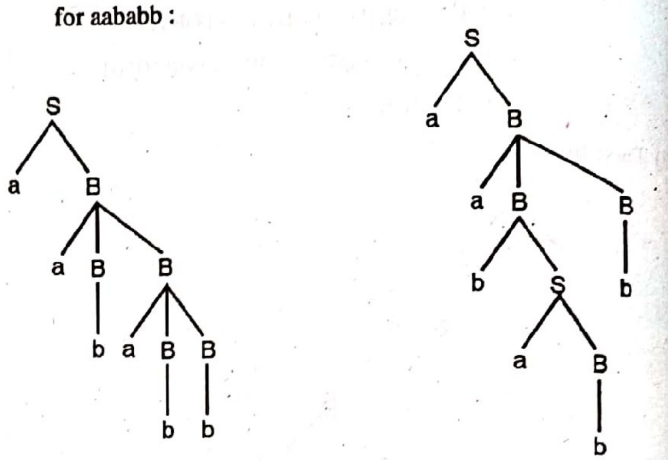


Fig. 5.6

(iv) The grammar is ambiguous as we can draw two parse trees for aababb :



(a) (b)

Fig. 5.6

(B) For string "bbaaabbaba"

(i) Leftmost derivation

S → bA → bbAA → bbaA → bbaaS

→ bbaaaB → bbaaabs → bbaaabbA

→ bbaaabbas → bbaaabbabA → bbaaabbaba

(ii) Rightmost derivation

S → bA → bbAA → bbAaS → bbAaaB

→ bbAaabS → bbAaabbA → bbAaabbaS

→ bbAaabbabA → bbAaabbaba → bbaaabbaba

(iii) Parse tree for bbaaabbaba

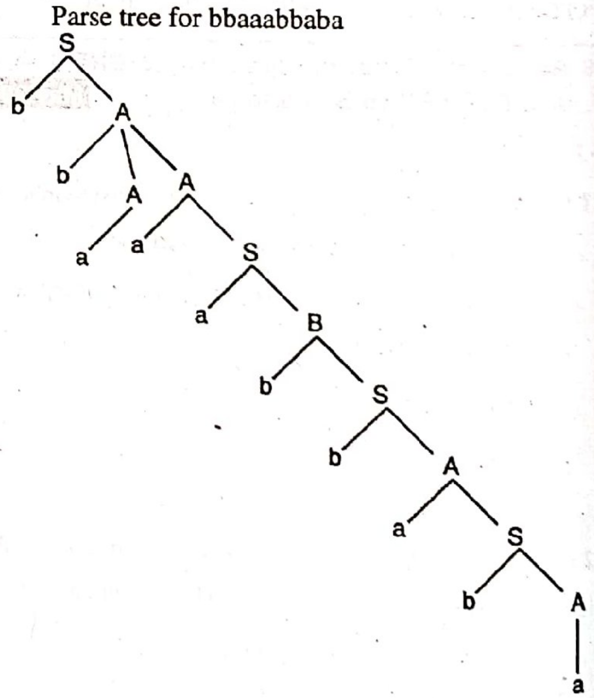


Fig. 5.6(c)

(C) For the string 00110101

(i) Leftmost derivation

S → 0BB → 00BB → 001B → 0011S

$\rightarrow 00110 \quad B \rightarrow 001101S \rightarrow 0011010B$

$\rightarrow 00110101$

(ii) Rightmost derivation

$S \rightarrow 0B \rightarrow 00BB \rightarrow 00B1S \rightarrow 00B10B$

$\rightarrow 00B101S \rightarrow 00B1010B \rightarrow 00B10101$

$\rightarrow 001110101$

(iii) Parse tree

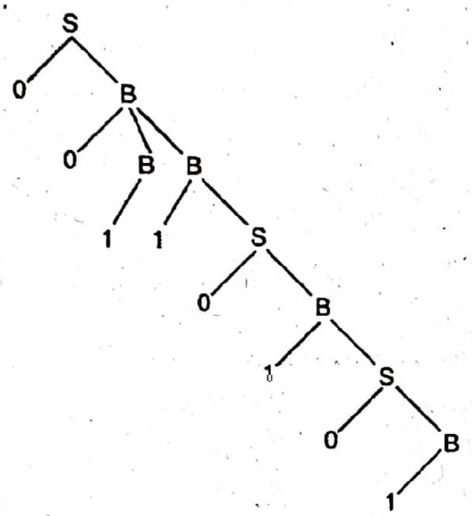


Fig. 5.6(d)

Q. 19 Consider the following grammar :
S → iCtS | iCtSeS | a C → b For the String 'ibtibtaea' find the following : (i) Leftmost derivation (ii) Rightmost derivation (iii)Parse Tree (iv) Check if the above grammar is Ambiguous

May 2014

Ans. :

(i) Leftmost derivation :	**(ii) Rightmost derivation :**
$S \rightarrow iCtS$ [using $S \rightarrow iCtS$]	$S \rightarrow iCtS$ [using $S \rightarrow iCtS$]
$\rightarrow ibtS$ [using $C \rightarrow b$]	$\rightarrow iCtiCtSeS$ [using $S \rightarrow iCtSeS$]
$\rightarrow ibtiCtSeS$ [using $S \rightarrow iCtSeS$]	$\rightarrow iCtiCtSea$ [using $S \rightarrow a$]
$\rightarrow ibtibtSeS$ [using $C \rightarrow b$]	$\rightarrow iCteCtaea$ [using $S \rightarrow a$]
$\rightarrow ibtibtaeS$ [using $S \rightarrow a$]	$\rightarrow iCtebtaea$ [using $C \rightarrow b$]
$\rightarrow ibtibtaea$ [using $S \rightarrow a$]	$\rightarrow ibtebtaea$ [using $C \rightarrow b$]

(iii) Parse Tree :

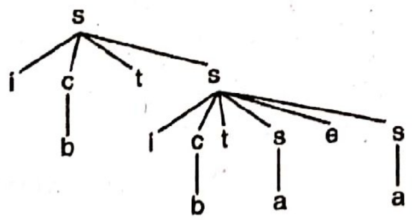


Fig. 5.7

Q. 20 Convert the following Grammar to CNF form :
S → ABA A → aA | bA | ∈ B → bB | aA | ∈

May 2014

Ans. :

1. The non-terminals {S, A, B} are nullable. Null productions are removed. The resulting grammar is :

   $S \rightarrow ABA \mid BA \mid AB \mid AA \mid A \mid B$

   $A \rightarrow aA \mid bA \mid a \mid b$

   $B \rightarrow bB \mid aA \mid b \mid a$

2. Removing unit productions, we get

   $S \rightarrow ABA \mid BA \mid AB \mid AA \mid aA \mid bA \mid a \mid b \mid bB \mid aA$

   $A \rightarrow aA \mid bA \mid a \mid b$

   $B \rightarrow bB \mid aA \mid b \mid a$

3. Every symbol in α, in production of the form $A \rightarrow \alpha$ where $|\alpha| \geq 2$ should be a variable.

   This can be done by adding two productions.

   $C_a \rightarrow a$

   $C_b \rightarrow b$

   The set of productions after the above changes is :

   $S \rightarrow ABA \mid BA \mid AB \mid AA \mid C_a A \mid C_b A \mid a \mid b \mid C_b B \mid C_a A$

   $A \rightarrow C_a A \mid C_b A \mid a \mid b$

   $B \rightarrow C_b B \mid C_a A \mid b \mid a$

   $C_a \rightarrow a, C_b \rightarrow b$

4. Finding an equivalent CNF.

Original production	Equivalent productions in CNF
$S \rightarrow ABA$	$S \rightarrow A C_1, C_1 \rightarrow BA$
$S \rightarrow BA \mid AB \mid AA \mid C_a A \mid C_b A \mid a \mid b \mid C_b B \mid C_a A$	$S \rightarrow BA \mid AB \mid AA \mid C_a A \mid C_b A \mid a \mid b \mid C_b B \mid C_a A$
$A \rightarrow C_a A \mid C_b A \mid a \mid b$	$A \rightarrow C_a A \mid C_b A \mid a \mid b$
$B \rightarrow C_b B \mid C_a A \mid b \mid a$	$B \rightarrow C_b B \mid C_a A \mid b \mid a$
$C_a \rightarrow a$	$C_a \rightarrow a$
$C_b \rightarrow b$	$C_b \rightarrow b$

Q. 21 Obtain leftmost derivation, rightmost derivation and derivation tree for the string "cccbaccba". The grammar is S → SS a | SSb | c

Dec. 2014

Ans. :

Derivation tree :

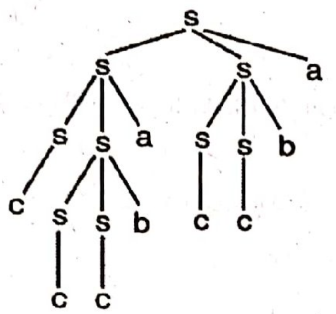


Fig. 5.8

Left most derivation	Right most derivation
S → SSa	S → SSa
→ SSaSa	→ SSSba
⇒ cSaSa	→ SScba
→ cSSbaSa	→ Sccba
→ ccSbaSa	→ SSaccba
→ cccbaSa	→ SSSbaccba
→ cccbaSSba	→ SScbaccba
→ cccbacSba	→ Sccbaccba
→ cccbaccba	→ cccbaccba

Q. 22 Convert following grammar to CNF and GNF.

S → ASB | a | bb

A → aSA | a

B → SbS | bb

Ans. :

S → ASB | a | bb

A → aSA | a

B → SbS | bb

Converting to CNF :

Re-writing the grammar, we get,

$S \rightarrow ASB \mid a \mid V_1V_1$

$A \rightarrow V_2SA \mid a$

$B \rightarrow SV_1S \mid V_1V_1$

$V_1 \rightarrow b$

$V_2 \rightarrow a$

Now, re-writing each production in its equivalent CNF form, we get,

Productions	CNF forms
S → ASB	$S \rightarrow AV_3$, $V_3 \rightarrow SB$
S → a	S → a
$S \rightarrow V_1 V_1$	$S \rightarrow V_1 V_1$
$A \rightarrow V_2 SA \mid a$	$S \rightarrow V_2 V_4$, $V_4 \rightarrow SA$ A → a
$B \rightarrow S V_1S \mid V_1 V_1$	$B \rightarrow S V_5$, $V_5 \rightarrow V_1 S$ $B \rightarrow V_1 V_1$
$V_1 \rightarrow b$	$V_1 \rightarrow b$
$V_2 \rightarrow a$	$V_2 \rightarrow a$

Converting to GNF :

Step 1 : Substituting symbols, we get,

$S \rightarrow ASB \mid a \mid b X_1$

$A \rightarrow a S A \mid a$

$B \rightarrow S X_2 S \mid b X_1$

$X_1 \rightarrow b$

$X_2 \rightarrow a$

Step 2 : Re-writing production in GNF :

	Productions	CNF forms
(1)	$X_1 \rightarrow b$	$X_1 \rightarrow b$
(2)	$X_2 \rightarrow a$	$X_2 \rightarrow a$
(3)	$A \rightarrow aSA \mid a$	$A \rightarrow a S A \mid a$
(4)	$S \rightarrow ASB \mid a \mid b X_1$	$S \rightarrow aSASB \mid aSB$ [substituting A] $S \rightarrow a \mid b X_1$
(5)	$B \rightarrow SX_2 S \mid bX$	$S \rightarrow aSASBX_2S \mid aSBX_2S \mid aX_2S \mid b X_1 X_2 S$ [substituting for S] S → bX

Q. 23 Consider the following grammar G = (V, T P, S), V = (S, X), T = {0, 1} and productions P are

S → 0 |0X1| 01S1

X → 0XX1 | 1S

S is start symbol. Show that above grammar is ambiguous. **Dec. 2015**

Ans. :

A grammar is said to be ambiguous grammar if the language generated by the grammar contains some strings that has 2 parse trees.

Ex. : Let us consider the given grammar

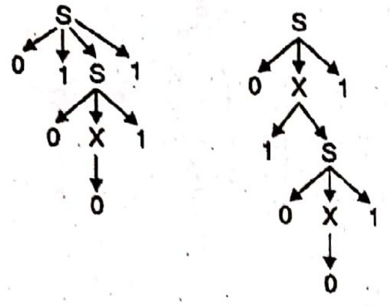
S → 0 | 0X1 | 01S1

X → 0XX1 | 1S

where, S is the start symbol.

A string 010011 is generated by the given grammar.

The grammar generates the string 010011 in 2 different ways. The 2 deviations are shown in Fig. 1(a)-Q. 61 and Fig. 1(b)-Q. 61. As the same string has 2 different parse trees. The given grammar is ambiguous grammar.



(a) (b)

Fig. 5.9

Q. 24 Consider the following grammar G = (V, T, P, S), V = {S, X}, T = {a, b} and productions P are

S → aSb | aX

X → Xa | Sa | a

Convert this grammar in Greibach Normal Form (GNF). **May 2016**

Ans. :

Given set of productions

$S \rightarrow aSb \mid aX$

$X \rightarrow Xa \mid Sa \mid a$

Substituting C_a for a, C_b for b, A_1 for S and A_2 for X.

$A_1 \rightarrow a A_1 C_b \mid a A_2$

$A_2 \rightarrow A_2 C_a \mid A_1 C_a \mid a$

$C_a \rightarrow a$

$C_b \rightarrow b$

Removing left recursion form A_2 production, we get

$C_a \rightarrow a, \quad C_b \rightarrow b$

$A_1 \rightarrow a A_1 C_b \mid a A_2$

$A_2 \rightarrow A_1 C_a A_3 \mid a A_3 \mid A_1 C_a \mid a$

$A_3 \rightarrow C_a A_3 \mid A_2$

Re-writing productions in GNF from

$A_1 \rightarrow a A_1 C_b \mid a A_2$

$A_2 \rightarrow a A_1 C_b C_a A_3 \mid a A_2 C_a A_3 \mid a A_3 \mid a A_1 C_b C_a \mid a A_2 C_a \mid a$

$A_3 \rightarrow a A_3 \mid a A_1 C_b C_a A_3 \mid a A_2 C_a A_3 \mid a A_3 \mid a A_1 C_b C_a \mid a A_2 C_a \mid a$

$C_a \rightarrow a$

$C_b \rightarrow b$

Q. 25 Construct a grammar in GNF which is equivalent to the grammar S → AA | a, A → SS | b.

May 2008, Dec. 2011, Dec. 2016

Ans. :

Step 1 : Grammar is already in a simple form without :

1. ε-productions.
2. Unit productions.
3. Useless symbol.

We can proceed for renaming of variables, Variables S and A are renamed as A_1 and A_2 respectively. The set of productions after renaming becomes :

$A_1 \rightarrow A_2A_2$

$A_1 \rightarrow a$

$A_2 \rightarrow A_1A_1$

$A_2 \rightarrow b$

Productions after renaming

Step 2 : Every production of the form $A_i \rightarrow A_j\alpha$ with $i > j$ must be modified to make $i \leq j$.

A_2 – production $A_2 \rightarrow A_1 A_1$ should be modified.

⇓

We must substitute $A_2A_2 \mid a$ for the first A_1. We should not touch the second A_1 of A_1A_1.

$$[A_2 \rightarrow A_1A_1] \Rightarrow \begin{bmatrix} A_2 \rightarrow A_2A_2A_1 \\ A_2 \rightarrow a A_1 \end{bmatrix}$$

The resulting set of productions is :

$A_1 \rightarrow A_2A_2 \mid a$

$A_2 \rightarrow A_2A_2A_1 \mid aA_1 \mid b$

Step 3 : Removing left recursion :

The A_2 – productions $A_2 \rightarrow A_2A_2A_1 \mid aA_1 \mid b$ contains left recursion. Left recursion from A_2–production can be removed through introduction of B_2-production.

$A_2 \rightarrow aA_1B_2 \mid bB_2$

$B_2 \rightarrow A_2A_1B_2 \mid A_2A_1$

The resulting set of productions is :

$A_1 \rightarrow A_2A_2 \mid a$

$A_2 \rightarrow aA_1B_2 \mid aB_2 \mid aA_1 \mid b$

$B_2 \rightarrow A_2A_1B_2 \mid A_2A_1$

Step 4 : A_2 – productions are in GNF.

A_1 and B_2 productions can be converted to GNF with the help of A_2–productions.

$A_2 \rightarrow aA_1B_2 \mid bB_2 \mid aA_1 \mid b$ ... in GNF

$A_1 \rightarrow A_2 A_2$

⇓ Substitute $aA_1B_2 \mid bB_2 \mid aA_1 \mid b$ for first A_2

$A_1 \rightarrow aA_1B_2A_2 \mid bB_2A_2 \mid aA_1A_2 \mid bA_2$

$A_1 \rightarrow a$ ... in GNF

Now, for B_2 – Production

$B_2 \rightarrow A_2A_1B_2$

⇓ Substitute $aA_1B_2 \mid bB_2 \mid aA_1 \mid b$ for the first A_2

$B_2 \rightarrow aA_1B_2A_1B_2 \mid bB_2A_1B_2 \mid aA_1A_1B_2 \mid bA_1B_2$

$B_2 \rightarrow A_2A_1$

⇓ Substitute $a A_1 B_2 \mid b B_2 \mid a A_1 \mid b$ for the first A_2

$B_2 \rightarrow aA_1B_2A_1 \mid bB_2A_1 \mid aA_1A_1 \mid bA_1$

The final set of productions is :

$A_2 \rightarrow aA_1B_2 \mid bB_2 \mid aA_1 \mid b$

$A_1 \rightarrow aA_1B_2A_2 \mid bB_2A_2 \mid aA_1A_2 \mid bA_2 \mid a$

A set of productions P

$B_2 \rightarrow aA_1B_2A_1B_2 \mid bB_2A_1B_2 \mid aA_1A_1B_2 \mid bA_1B_2 \mid$
$aA_1B_2A_1 \mid bB_2A_1 \mid aA_1A_1 \mid bA_1$

where,Set of variables V = (A_1, A_2, B_2)

Set of terminals T = (a, b)

Start symbol = A_1

Set of productions P = Given above.

Q. 26 Consider the following grammar :

S → ICtSIICtSeSIa

C → b

For the string 'ibtibtaea' find the following :

(i) Leftmost derivation

(ii) Rightmost derivation

(iii) Parse tree

(iv) Check if above grammar is ambiguous.

Dec. 2017

Ans. :

(I) Leftmost derivation

$S \Rightarrow iCtSeS \xRightarrow{C \rightarrow b} ibtSeS \xRightarrow{S \rightarrow iCtS}$

$ibtiCtSeS \xRightarrow{C \rightarrow b} ibtibtSeS \xRightarrow{S \rightarrow a} ibtibtaeS$

$\Rightarrow ibtibtaea$

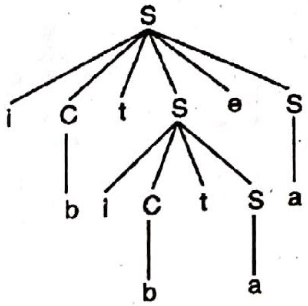
(ii) Rightmost derivation

$S \Rightarrow iCtSeS \xRightarrow{S \rightarrow a} iCtSea \xRightarrow{S \rightarrow iCtS}$

$iCtiCtSea \xRightarrow{S \rightarrow a} iCtiCtaea$

$\xRightarrow{C \rightarrow b} iCtibtaea \xRightarrow{C \rightarrow b} ibtibtaea$

(iii) Parse tree



(iv) It is an ambiguous grammar due to laughing if problem.

Q. 27 Reduce following grammar to GNF.

S → AB

A → BSBIBBIb

B → alaAb

(i) S → 01SI01

S → 10SI10

S → 00I∈

Ans. :

Removing ∈-production, we get,

S → 01SI01I10SI10I00

It can be converted into GNF in an easy way by introducing two production

X → 1 and Y → 0

∴ Productions in GNF are

S → 0XSI0XI1YSI1YI0Y

X → 1

Y → 0

Chapter 6 : Pushdown Automata (PDA)

Q. 1 Distinguish between NPDA and DPDA. **Dec. 2005**

Ans. :

Distinguish between NPDA and DPDA

A NPDA provides non-determinism to PDA.

In a DPDA there is only one move in every situation. Where as, in case of NPDA there could be multiple moves under a situation. DPDA is less powerful than NPDA.

Every context free language can not be recognized by a DPDA but it can be recognized by NPDA. The class of language a DPDA can accept lies in between a regular language and CFL. A palindrome can be accepted by NPDA but it can not be accepted by a DPDA

Q. 2 Design a PDA to accept $(bdb)^n c^n$.

Ans. :

To solve this problem, we can take a stack symbol x. For every 'bdb', one x will be pushed on top of the stack. After reading $(bdb)^n$, the stack should contain n number of x's. These x's will be matched with c's. The transitions for the PDA accepting through an empty stack are given in Fig. 6.1.

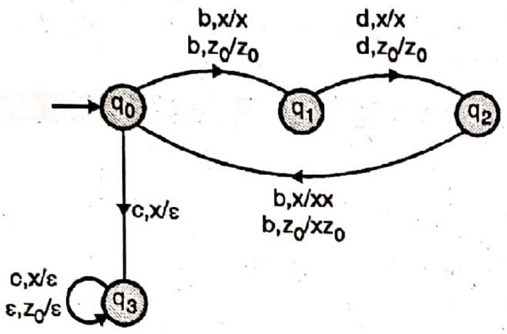


Fig. 6.1

A cycle through $q_0 \to q_1 \to q_2 \to q_0$ traces a group of bdb.

The PDA $M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, \phi\}$

Where,

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{b, d, c\}$, $\Gamma = \{x, z_0\}$

q_0 is the initial state, z_0 is initial stack symbol.

The transition function δ is given by,

$\delta(q_0, b, z_0) = (q_1, z_0)$

$\delta(q_0, b, x) = (q_1, x)$

$\delta(q_1, d, z_0) = (q_2, z_0)$

$\delta(q_1, d, x) = (q_2, x)$

$\delta(q_2, b, z_0) = (q_0, xz_0)$

$\delta(q_2, b, x) = (q_0, xx)$

$\delta(q_0, c, x) = (q_3, \varepsilon)$

$\delta(q_3, c, x) = (q_3, \varepsilon)$

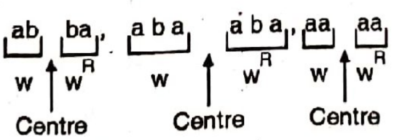
$\delta(q_3, \varepsilon, z_0) = (q_3, \varepsilon)$ Accept through empty stack.

Q. 3 Design a PDA for detection of even palindrome over {a, b}.

Dec. 2005, May 2006, May 2007, May 2016

Ans. :

An even palindrome will be of the form ww^R



If the length of w is n then a palindrome of even length is :

First n characters are equal to the last n characters in the reverse order.

The character immediately before the middle position will be identical to the character immediately after the middle position.

Algorithm :

There is no way of finding the middle position by a PDA; therefore the middle position is fixed non-deterministically.

1. First n characters are pushed onto the stack. n is non-deterministic.
2. The n characters on the stack are matched with the last n characters of the input string.
3. n is decided non-deterministically. Every character out of first n characters, whose previous character is same as itself should be considered for two cases :

(a) It is first character of the second half.

- Pop the current stack symbol using the transitions :

$\delta(q_0, a, a) \Rightarrow (q_1, \varepsilon)$

$\delta(q_0, b, b) \Rightarrow (q_1, \varepsilon)$

Must be identical

(b) It belongs to first half.

- Push the current input

$\delta(q_0, a, \varepsilon) \Rightarrow (q_0, a)$

$\delta(q_0, b, \varepsilon) \Rightarrow (q_0, b)$

4. n is decided non-deterministically. Every character out of first n characters, whose previous character is not same as itself should be pushed onto the stack.

- Push the current symbol using the transitions :

$\delta(q_0, a, b) \Rightarrow (q_0, ab)$

$\delta(q_0, b, a) \Rightarrow (q_0, ba)$

The transition table for the PDA is given below :

$\delta(q_0, a, z_0) \Rightarrow \{(q_0, az_0)\}$

$\delta(q_0, b, z_0) \Rightarrow \{(q_0, bz_0)\}$

$\delta(q_0, a, a) \Rightarrow \{(q_0, aa)\ (q_1, \varepsilon)\}$

$\delta(q_0, a, b) \Rightarrow \{(q_0, ab)\}$

$\delta(q_0, b, a) \Rightarrow \{(q_0, ba)\}$

$\delta(q_0, b, b) \Rightarrow \{(q_0, bb), (q_1, \varepsilon)\}$

$\delta(q_1, a, a) \Rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, b, b) \Rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) \Rightarrow \{(q_1, \varepsilon)\}$ [Accept through an empty stack]

Where,

the set of states $Q = \{q_0, q_1\}$

the set of input symbols $\Sigma = \{a, b\}$

the set of stack symbols $\Gamma = \{a, b, z_0\}$

Starting state $= q_0$

Initial stack symbol $= z_0$

Q. 4 Construct a PDA equivalent to the following CFG.
$S \rightarrow 0BB$
$\rightarrow 0S \mid 1S \mid 0$
Test if 010^4 is in the language

May 2006, May 2011, May 2012

Ans. :

The equivalent PDA, M is given by

$$M = (\{q\}, \{0, 1\}, \{0, 1, S, B\}, \delta, q, S, \phi),$$

where δ is given by

$\delta(q, \varepsilon, S) \Rightarrow \{(q, 0BB)\}$
$\delta(q, \varepsilon, B) \Rightarrow \{(q, 0S), (q, 1S), (q, 0)\}$ — For each production in the given grammar

$\delta(q, 0, 0) \Rightarrow \{(q, \varepsilon)\}$
$\delta(q, 1, 1) \Rightarrow \{(q, \varepsilon)\}$ — For each terminal

Acceptance of 010^4 by M :

$\delta(q, 010000, S)$ $\quad \delta(q, \varepsilon, S) = (q, 0BB)$

$\longrightarrow (q, 010000, 0BB)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\longrightarrow (q, 10000, BB)$

$\delta(q, \varepsilon, B) = (q, 1S)$

$\longrightarrow (q, 10000, 1SB)$

$\delta(q, 1, 1) = (q, \varepsilon)$

$\longrightarrow (q, 0000, SB)$

$\delta(q, \varepsilon, S) = (q, 0BB)$

$\longrightarrow (q, 0000, 0BBB)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\longrightarrow (q, 000, BBB)$

$\delta(q, \varepsilon, B) = (q, 0)$

$\longrightarrow (q, 000, 0BB)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\longrightarrow (q, 00, BB)$

$\delta(q, \varepsilon, B) = (q, 0)$

$\longrightarrow (q, 00, 0B)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\longrightarrow (q, 0, B)$

$\delta(q, \varepsilon, B) = (q, 0)$

$\longrightarrow (q, 0, 0)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\longrightarrow (q, \varepsilon, \varepsilon)$

Thus the string 010^4 is accepted by M using an empty stack.

$\therefore \; 010^4 \in L$

Q. 5 Construct a PDA accepting $\{a^nb^ma^n \mid m, n \geq 1\}$ by null store.

Dec. 2006, Dec. 2010, May 2012, May 2013

Ans. :

Algorithm :

1. The sequence of a's should be pushed onto the stack in state q_0

$$\delta(q_0, a, z_0) = (q_0, az_0)$$
$$\delta(q_0, a, a) = (q_0, aa)$$

2. On first b, the machine moves to q_1 and remains there for b's. b's will have no effect on the stack.

3. For every 'a', an 'a' is erased from the stack.

The PDA accepting through empty stack is given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, z_0\}, \delta, q_0, z_0, \phi)$$

Where the transition function δ is :

1. $\delta(q_0, a, z_0) = (q_0, az_0)$ [First 'a' is pushed]
2. $\delta(q_0, a, a) = (q_0, aa)$ [Subsequent a's are pushed]
3. $\delta(q_0, b, a) = (q_1, a)$ [Input symbols b's are skipped]
4. $\delta(q_1, b, a) = (q_1, a)$
5. $\delta(q_1, a, a) = (q_2, \varepsilon)$ [An a is erased on first a of last a's]
6. $\delta(q_2, a, a) = (q_2, \varepsilon)$ [An a is erased on subsequent a's of last a's]
7. $\delta(q_2, \varepsilon, z_0) = (q_2, \varepsilon)$ [Accepting through empty stack]

Q. 6 Design a PDA to accept $(ab)^n(cd)^n$. May 2007

Ans. :

To solve this problem, we can take a stack symbol x. For every 'ab', one x will be pushed on top of the stack. After reading $(ab)^n$, the stack should contain n number of x's. These x's will be matched with $(cd)^n$. For every 'cd' one x will be popped.

The transitions for the PDA accepting through an empty stack are given in Fig. 6.2.

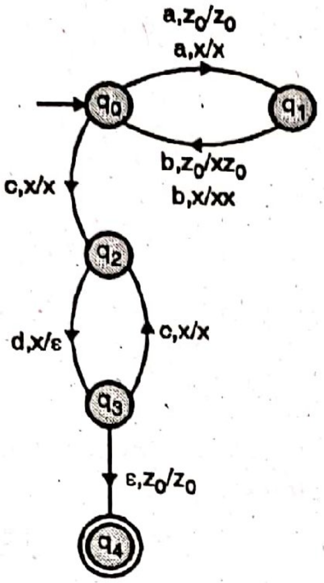


Fig. 6.2

PDA accepts through the final state q_4.

The PDA M = $\{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

Where,

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b, c, d\}$

$\Gamma = \{x, z_0\}$

The transition function δ is given by,

$\delta(q_0, a, z_0) = (q_1, z_0)$

$\delta(q_0, a, x) = (q_1, x)$

$\delta(q_1, b, z_0) = (q_0, x\, z_0)$

$\delta(q_1, b, x) = (q_0, xx)$

$\delta(q_0, c, x) = (q_2, x)$

$\delta(q_2, d, x) = (q_3, \varepsilon)$

$\delta(q_3, c, x) = (q_2, x)$

$\delta(q_2, \varepsilon, z_0) = (q_4, z_0)$

q_0 is initial state,

z_0 is initial stack symbol.

Set of final states $F = \{q_4\}$

Q.7 Design a PDA for detection of odd palindrome over {a, b}. **Dec. 2007**

Ans. :

An odd palindrome will be of the form :

1. waw^R

ab a ba, aba a aba, aa a aa
(w a w^R, w a w^R, w a w^R)

2. wbw^R

ab b ba, aba b aba, aa b aa
(w b w^R, w b w^R, w b w^R)

If the length of w is n then a palindrome of odd length is :

First n characters are equal to the last n characters in reverse order with middle character as 'a' or 'b'.

Algorithm :

There is no way of finding the middle position of a string by a PDA, therefore the middle position is fixed non-deterministically.

1. First n characters are pushed onto the stack, where n is non-deterministic.
2. The n characters on the stack are matched with the last n characters of the input string.
3. n is decided non-deterministically. Every character out of first n characters should be considered for two cases :
   (a) It is not the middle character – push the current character using the transition :

   $\delta(q_0, a, \varepsilon) \Rightarrow (q_0, a)$

   $\delta(q_0, b, \varepsilon) \Rightarrow (q_0, b)$

   (b) It is a middle character – go for matching of second half with the first half.

   $\delta(q_0, a, \varepsilon) \Rightarrow (q_1, \varepsilon)$

   $\delta(q_0, b, \varepsilon) \Rightarrow (q_1, \varepsilon)$

The status of the stack and the state of the machine is shown in the Fig. 6.3. Input applied is ababa.

Left child → current input is taken as the middle character

Right child → current input is not a middle character.

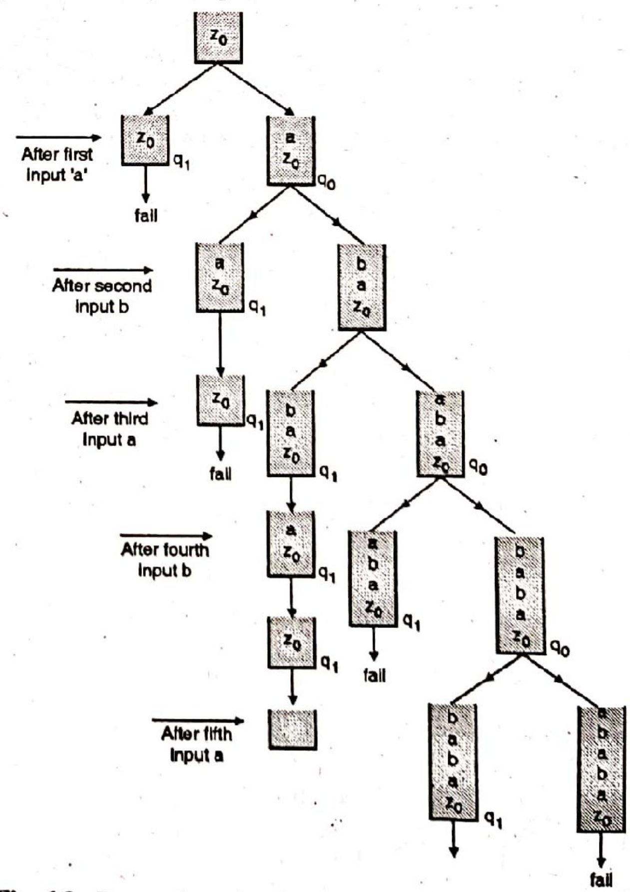


Fig. 6.3 : Processing of string by the PDA. String is taken as "ababa"

The transition table for the PDA is given below,

$\delta(q_0, a, \varepsilon) \Rightarrow \{(q_1, \varepsilon), (q_0, a)\}$

ε - Indicates that irrespective of the current stack symbol, perform the transition.

$\delta(q_0, b, \varepsilon) \Rightarrow \{(q_1, \varepsilon), (q_0, b)\}$

$\delta(q_1, a, a) \Rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, b, b) \Rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) \Rightarrow \{(q_1, \varepsilon)\}$ [Accept through an empty stack]

Where, The set of states $Q = \{q_0, q_1\}$

The set input alphabet $\Sigma = \{a, b\}$

The set of stack symbols $\Gamma = \{a, b, z_0\}$

Starting state $= q_0$

Initial stack symbol $= z_0$

Q. 8 Give the CFG generating the language accepted by the following PDA : $M = (\{q_0, q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$ when δ is given below : $\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$ $\delta(q_0, 1, x) = \{(q_0, xx)\}$ $\delta(q_0, 0, x) = \{(q_1, x)\}$ $\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$ $\delta(q_1, 1, x) = \{(q_1, \varepsilon)\}$ $\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

Dec. 2007

Ans. :

Step 1 : Add productions for the start symbol

$S \rightarrow [q_0 \, {}^{z_0} \, q_0]$

$S \rightarrow [q_0 \, {}^{z_0} \, q_1]$

Step 2 : Add productions for $\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$[q_0 \, {}^{z_0} \, q_0] \rightarrow 1 \, [q_0 \, {}^{x} \, q_0] \, [q_0 \, {}^{z_0} \, q_0]$

$[q_0 \, {}^{z_0} \, q_0] \rightarrow 1 \, [q_0 \, {}^{x} \, q_1] \, [q_1 \, {}^{z_0} \, q_0]$

$[q_0 \, {}^{z_0} \, q_1] \rightarrow 1 \, [q_0 \, {}^{x} \, q_0] \, [q_0 \, {}^{z_0} \, q_1]$

$[q_0 \, {}^{z_0} \, q_1] \rightarrow 1 \, [q_0 \, {}^{x} \, q_1] \, [q_1 \, {}^{z_0} \, q_1]$

Step 3 : Add productions for $\delta(q_0, 1, x) \Rightarrow \{(q_0, xx)\}$

$[q_0 \, {}^{x} \, q_0] \rightarrow 1 \, [q_0 \, {}^{x} \, q_0] \, [q_0 \, {}^{x} \, q_0]$

$[q_0 \, {}^{x} \, q_0] \rightarrow 1 \, [q_0 \, {}^{x} \, q_1] \, [q_1 \, {}^{x} \, q_0]$

$[q_0 \, {}^{x} \, q_1] \rightarrow 1 \, [q_0 \, {}^{x} \, q_0] \, [q_0 \, {}^{x} \, q_1]$

$[q_0 \, {}^{x} \, q_1] \rightarrow 1 \, [q_0 \, {}^{x} \, q_1] \, [q_1 \, {}^{x} \, q_1]$

Step 4 : Add productions for $\delta(q_0, 0, x) \Rightarrow \{(q_1, x)\}$

$[q_0 \, {}^{x} \, q_0] \rightarrow 0 \, [q_1 \, {}^{x} \, q_0]$

$[q_0 \, {}^{x} \, q_1] \rightarrow 0 \, [q_1 \, {}^{x} \, q_1]$

Step 5 : Add productions for $\delta(q_0, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

$[q_0 \, {}^{z_0} \, q_1] \rightarrow \varepsilon$

Step 6 : Add production for $\delta(q_1, 1, x) \Rightarrow \{(q_1, \varepsilon)\}$

$[q_1 \, {}^{x} \, q_1] \rightarrow 1$

Step 7 : Add productions for $\delta(q_1, 0, z_0) \Rightarrow \{(q_0, z_0)\}$

$[q_1 \, {}^{z_0} \, q_0] \Rightarrow 0 \, [q_0 \, {}^{z_0} \, q_0]$

$[q_1 \, {}^{z_0} \, q_1] \Rightarrow 0 \, [q_0 \, {}^{z_0} \, q_1]$

Q. 9 Design a PDA for accepting a language $L = \{ WcW^T \mid W \in \{a, b\}^* \}$

May 2008, May 2010, May 2011

Ans. :

W^T stands for reverse of W. A string of the form WcW^T is an odd length palindrome with the middle character as c.

Algorithm :

If the length of the string is 2n + 1, then the first n symbols should be matched with the last n symbols in the reverse order. A stack can be used to reverse the first n input symbols.

Status of the stack and state of the machine is shown in Fig. 6.4. Input applied is abbcbba.

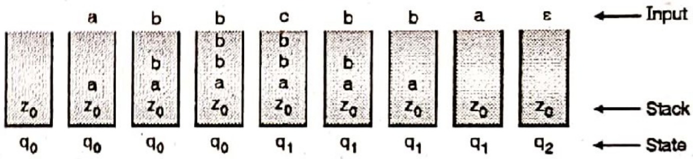


Fig. 6.4 : A PDA on input abbcbba

The PDA accepting through final state is given by

$M = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_2\})$

Where the transition function δ is given below :

1. $\delta(q_0, a, \varepsilon) = (q_0, a)$ — First n symbols are pushed onto the stack
2. $\delta(q_0, b, \varepsilon) = (q_0, b)$
3. $\delta(q_0, c, \varepsilon) = (q_1, \varepsilon)$ — [State changes on c]
4. $\delta(q_1, a, a) = (q_1, \varepsilon)$ — Last n symbols are matched with first n symbols in reverse order
5. $\delta(q_1, b, b) = (q_1, \varepsilon)$
6. $\delta(q_1, \varepsilon, z_0) = (q_2, z_0)$ — [Accepted through final state]

A transition of the form $\delta(q_0, a, \varepsilon) = (q_0, a)$ implies that always push a, irrespective of stack symbol.

Q. 10 Convert the following expression grammar to PDA $I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$ $E \rightarrow I \mid E * E \mid E * E \mid (E)$

Dec. 2008

Ans. :

The equivalent PDA, M is given by,

$M = (\{q\}, \{0, 1, a, b, *, +, (,)\}, \{0, 1, a, b, *, +, (,), I, E\}, \delta, q, E, \phi)$

where, δ is given by,

$$\delta(q, \varepsilon, E) = \{(q, I), (q, E * E), (q, E + E), (q, (E))\}$$
$$\delta(q, \varepsilon, I) = \{(q, a), (q, b), (q, Ib), (q, Ia), (q, I0), (q, I1)\}$$
$$\delta(q, 0, 0) = \{(q, \varepsilon)\}$$
$$\delta(q, 1, 1) = \{(q, \varepsilon)\}$$
$$\delta(q, a, a) = \{(q, \varepsilon)\}$$
$$\delta(q, b, b) = \{(q, \varepsilon)\}$$
$$\delta(q, +, +) = \{(q, \varepsilon)\}$$
$$\delta(q, *, *) = \{(q, \varepsilon)\}$$
$$\delta(q, (, () = \{(q, \varepsilon)\}$$
$$\delta(q,),)) = \{(q, \varepsilon)\}$$

Q. 11 Design a PDA for CFL that checks the well formedness of parenthesis i.e. the language L of all "balanced" string of two types of parenthesis say "( )" and "[ ]". Trace the sequence of moves made corresponding to input string (([ ])[ ]).

May 2009, May 2014, Dec. 2017

Ans. :

The transition function of the PDA is given below :

1. $\delta(q_0, (, z_0) = (q_0, (z_0)$
2. $\delta(q_0, (, () = (q_0, (()$
3. $\delta(q_0, (, [) = (q_0, ([)$

   ] Push the opening bracket '('

4. $\delta(q_0, [, z_0) = (q_0, [z_0)$
5. $\delta(q_0, [, () = (q_0, [()$

   ] Push the opening bracket '['

6. $\delta(q_0, [, [) = (q_0, [[)$
7. $\delta(q_0,),)) = (q_0, \varepsilon)$

   ] POP an opening bracket for a closing bracket.

8. $\delta(q_0,],]) = (q_0, \varepsilon)$
9. $\delta(q_0, \varepsilon, z_0) = (q_f, z_0)$

   ] Accept through a final state.

Simulation of PDA for the input string (([])[])

$(q_0, (([])[]), z_0) \xrightarrow{\text{Rule 1}} (q_0, ([])[]), (z_0)$

$\xrightarrow{\text{Rule 2}} (q_0, [])[]), ((z_0)$

$\xrightarrow{\text{Rule 5}} (q_0,])[]), [((z_0)$

$\xrightarrow{\text{Rule 8}} (q_0,)[]), ((z_0)$

$\xrightarrow{\text{Rule 7}} (q_0, []), (z_0)$

$\xrightarrow{\text{Rule 5}} (q_0,]), [(z_0)$

$\xrightarrow{\text{Rule 8}} (q_0,), (z_0)$

$\xrightarrow{\text{Rule 7}} (q_0, \varepsilon, z_0)$

$\xrightarrow{\text{Rule 9}} (q_f, \varepsilon, z_0)$

Q. 12 Consider the PDA with the following moves : $\delta(q_0, a, z_0) = \{(q_0, az_0)\}$ $\delta(q_0, a, a) = \{(q_0, aa)\}$ $\delta(q_0, b, a) = \{(q_1, \varepsilon)\}$ $\delta(q_1, b, a) = \{(q_1, \varepsilon)\}$ $\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$ Obtain CFG equivalent to PDA.

May 2009

Ans. :

Step 1 : Add productions for the start symbol.

$$S \rightarrow [q_0\ ^{z_0}\ q_0]$$
$$S \rightarrow [q_0\ ^{z_0}\ q_1]$$

Step 2 : Add productions for $(q_0, a, a) = \{(q_0, aa)\}$

$$[q_0\ ^{a}\ q_0] \rightarrow a\, [q_0\ ^{a}\ q_0]\, [q_0\ ^{a}\ q_0]$$
$$[q_0\ ^{a}\ q_0] \rightarrow a\, [q_0\ ^{a}\ q_1]\, [q_1\ ^{a}\ q_0]$$
$$[q_0\ ^{a}\ q_1] \rightarrow a\, [q_0\ ^{a}\ q_0]\, [q_0\ ^{a}\ q_1]$$
$$[q_0\ ^{a}\ q_1] \rightarrow a\, [q_0\ ^{a}\ q_1]\, [q_1\ ^{a}\ q_1]$$

Step 3 : Add productions for $\delta(q_0, b, a) = \{(q_1, \varepsilon)\}$

$$[q_0\ ^{a}\ q_1] \rightarrow b$$

Step 4 : Add productions for $\delta(q_1, b, a) = \{(q_1, \varepsilon)\}$

$$[q_1\ ^{a}\ q_1] \rightarrow b$$

Step 5 : Add productions for $\delta(q_1, \varepsilon, z_0) \rightarrow \{(q_1, \varepsilon)\}$

$$[q_1\ ^{z_0}\ q_1] \rightarrow \varepsilon$$

Q. 13 Write short note on DPDA. **Dec. 2009**

Ans. :

DPDA

In a DPDA there is only one move in every situation. A DPDA is less powerful than NPDA.

Every context free language cannot be accepted by a DPDA. For example, a string of the form ww^R can not be processed by a DPDA.

The class of a language a DPDA can accept lies in between a regular language and CFL.

A DPDA is defined as :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F), \text{ where}$$

$\delta(q, a, x)$ has one move for any $q \in Q$, $X \in \Gamma$ and $a \in \Sigma$.

Q. 14 Design a PDA for detection of palindromes over {a, b}. **Dec. 2012**

Ans. :

A palindrome will be of the form :

1. ww^R – even palindrome
2. waw^R
3. wbw^R – odd palindrome

If the length of w is n then a palindrome is :

First n characters are equal to the last n characters in the reverse order with the middle character as :

(1) a [For odd palindrome]

(2) b [For odd palindrome]

(3) ε [For even palindrome]

The transition table for the PDA is given below :

$\delta(q_0, a, z_0) \Rightarrow \{(q_1, z_0), (q_0, az_0)\}$

$\delta(q_0, b, z_0) \Rightarrow \{(q_1, z_0), (q_0, bz_0)\}$

$\delta(q_0, a, a) \Rightarrow \{(q_0, aa)\ (q_1, a), (q_1, \varepsilon)\}$

$\delta(q_0, a, b) \Rightarrow \{(q_0, ab), (q_1, b)\}$

$\delta(q_0, b, a) \Rightarrow \{(q_0, ba), (q_1, a)\}$

$\delta(q_0, b, b) \Rightarrow \{(q_0, bb), (q_1, b), (q_1, \varepsilon)\}$

$\delta(q_1, a, a) \Rightarrow \{(q_1, \varepsilon)\}$

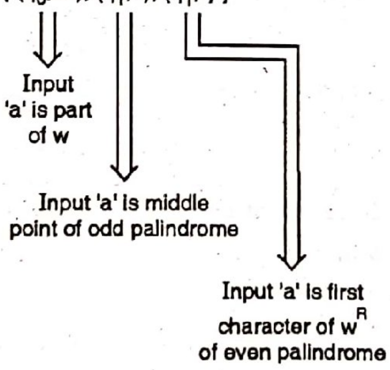
$\delta(q_1, b, b) \Rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) \Rightarrow \{(q_1, \varepsilon)\}$

[Accept through an empty stack].

Details of important transitions :

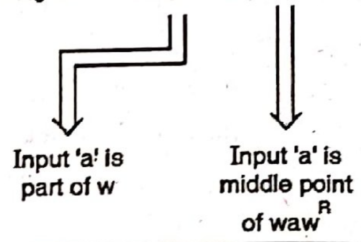
The transaction, $\delta(q_0,a,a) \Rightarrow \{ (q_0,aa), (q_1,a), (q_1,\varepsilon) \}$



The transition rule for $\delta(q_0, a, a)$, must consider the three cases :

1. Input 'a' is part of w of the palindrome.
2. Input 'a' is middle character of waw^R
3. Input 'a' is the first character of w^R.

The transaction, $\delta(q_0,a,b) \Rightarrow \{ (q_0, ab), (q_1, b) \}$



Q. 15 Write application of PDA. **Dec. 2012**

Ans. :

Applications of PDA

PDA is a machine for CFL.

A string belonging to a CFL can be recognized by a PDA.

PDA is extensively used for parsing.

PDA is an abstract machine; it can also used for giving proofs of lemma on CFL.

Q. 16 Design a PDA to accept language $\{a^{n-1} b^{2n+1} \mid n \geq 1\}$ **Dec. 2014**

Ans. :

For every 'a' in the input, 2 b's are pushed onto the stack.

For the first 'b' in the input, 2 b's are pushed onto the stack.

For every 'b' in the input, 1 'b' is popped out from the stack.

Finally the stack should become empty.

Transitions

$\delta(q_0, a, z_0) = (q_0, bb\ z_0)$

$\delta(q_0, a, b) = (q_0, bbb)$

$\delta(q_0, b, z_0) = (q_1, bb\ z_0)$

$\delta(q_0, b, b) = (q_1, bbb)$

$\delta(q_1, b, b) = (q_1, \in)$

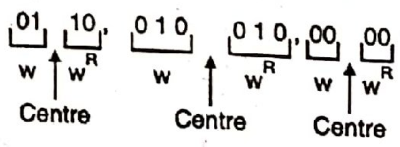
$\delta(q_1, \in, z_0) = (q_1, \in)$

[Accept using empty stack]

Q. 17 Design PDA to check even palindrome over $\Sigma = \{0, 1\}$

Ans. :

An even palindrome will be of the form ww^R



If the length of w is n then a palindrome of even length is :

First n characters are equal to the last n characters in the reverse order.

The character immediately before the middle position will be identical to the character immediately after the middle position.

Algorithm :

There is no way of finding the middle position by a PDA; therefore the middle position is fixed non-deterministically.

1. First n characters are pushed onto the stack. n is non-deterministic.
2. The n characters on the stack are matched with the last n characters of the input string.
3. n is decided non-deterministically. Every character out of first n characters, whose previous character is same as itself should be considered for two cases :
   (a) It is first character of the second half.
   - Pop the current stack symbol using the transitions :
   $\delta(q_0, 0, 0) \Rightarrow (q_1, \varepsilon)$
   $\delta(q_0, 1, 1) \Rightarrow (q_1, \varepsilon)$
   Must be identical
   (b) It belongs to first half.
   - Push the current input
   $\delta(q_0, 0, \varepsilon) \Rightarrow (q_0, 0)$
   $\delta(q_0, 1, \varepsilon) \Rightarrow (q_0, 1)$
4. n is decided non-deterministically. Every character out of first n characters, whose previous character is not same as itself should be pushed onto the stack.
   - Push the current symbol using the transitions :
   $\delta(q_0, 0, 1) \Rightarrow (q_0, 01)$
   $\delta(q_0, 1, 0) \Rightarrow (q_0, 10)$

The transition table for the PDA is given below :

$\delta(q_0, 0, z_0) \Rightarrow \{(q_0, 0z_0)\}$
$\delta(q_0, 1, z_0) \Rightarrow \{(q_0, 1z_0)\}$
$\delta(q_0, 0, 0) \Rightarrow \{(q_0, 00)\ (q_1, \varepsilon)\}$
$\delta(q_0, 0, 1) \Rightarrow \{(q_0, 01)\}$
$\delta(q_0, 1, 0) \Rightarrow \{(q_0, 10)\}$
$\delta(q_0, 1, 1) \Rightarrow \{(q_0, 11), (q_1, \varepsilon)\}$
$\delta(q_1, 0, 0) \Rightarrow \{(q_1, \varepsilon)\}$
$\delta(q_1, 1, 1) \Rightarrow \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, z_0) \Rightarrow \{(q_1, \varepsilon)\}$

[Accept through an empty stack]

Where,

the set of states $Q = \{q_0, q_1\}$
the set of input symbols $\Sigma = \{0, 1\}$
the set of stack symbols $\Gamma = \{0, 1, z_0\}$
Starting state $= q_0$
Initial stack symbol $= z_0$

Q. 18 Design DPDA to accept language $L = \{x \in \{a, b\}^* \mid N_a(x) > N_b(x)\}$, $N_a(x) > N_b(x)$ means number of a's are greater than number of b's in string x.

Dec. 2015

Ans. :

The PDA is being designed to accept the string using final state. The stack is being used to store excess of a's over b's or excess of b's over a's out of input seen so far.

Transitions

1.	$\delta(q_0, a, z_0) = (q_0, a\, z_0)$	[Extra 'a' is pushed]
2.	$\delta(q_0, b, z_0) = (q_0, b\, z_0)$	[Extra 'b' is pushed]
3.	$\delta(q_0, a, a) = (q_0, aa)$	[Excess a's are pushed]
4.	$\delta(q_0, a, b) = (q_0, \in)$	[Excess b's decreased by 1]
5.	$\delta(q_0, b, b) = (q_0, bb)$	[Excess b's are pushed]
6.	$\delta(q_0, b, a) = (q_0, \in)$	[Excess a's decreased by 1]
7.	$\delta(q_0, \in, a) = (q_1, \in)$	[Input ends with excess a's on the stack]

The PDA is given by :

$M = (\{q_0, q_1\}\ \{a, b\}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_1\})$

Q. 19 Construct PDA accepting the language $L = \{a^{2n} b^n \mid n > 0\}$.

May 2016

Ans. :

Algorithm :

1. For every pair of leading a's, one X is inserted in the stack.
2. X's on the stack are matched with trailing b's.

The PDA is given by

$M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}\ \{X, Z_0\}, \delta, q_0, Z_0, \phi)$

where the transition function δ is

1. $\delta(q_0, a, Z_0) = (q_1, Z_0)$
2. $\delta(q_1, a, Z_0) = (q_2, X Z_0)$
3. $\delta(q_2, a, X) = (q_1, X)$
4. $\delta(q_1, a, X) = (q_2, XX)$
5. $\delta(q_2, b, X) = (q_3, \in)$
6. $\delta(q_3, b, X) = (q_3, \in)$
7. $\delta(q_3, \in, Z_0) = (q_3, \in)$

Accept through empty stack.

Q. 20 Design a PDA corresponding to the grammar :

$S \rightarrow aSA \mid \in$
$A \rightarrow bB$
$B \rightarrow b$

Dec. 2016

Ans. :

The equivalent PDA, M is given by :

$M = (\{q\}, \{a, b\}, \{a, b, S, A, B\}, \delta, q, S, \phi)$

where δ is given by :

$\delta(q, \in, S) \Rightarrow \{(q, aSA), (q, 6)\}$

$\delta(q, \in, A) \Rightarrow \{(q, bB)\}$

$\delta(q, \in, B) = \{(q, b)\}$

$\delta(q, a, a) = \{(q, \in)\}$

$\delta(q, b, b) = \{(q, \in)\}$

Q. 21 Design a PDA to accept language $\{a^{n-1} b^{2n+1} \mid n >= 1\}$ Dec. 2017

Ans. :

1. $\delta(q_0, a, Z_0) \Rightarrow (q_1, aaZ_0)$
2. $\delta(q_1, a, a) \Rightarrow (q_1, aa)$
3. $\delta(q_1, b, a) \Rightarrow (q_2, a)$
4. $\delta(q_2, b, a) \Rightarrow (q_1, \in)$
5. $\delta(q_2, \in, Z_0) \Rightarrow (q_2, \in)$

Accept through empty stack.

Chapter 7 : Turing Machine (TM)

Q. 1 Write short note on : Universal TM.

Dec. 2005, May 2007, Dec. 2007, May 2008, Dec. 2008, May 2009, May 2010, Dec. 2011, May 2012, Dec. 2012, Dec. 2015

Ans. :

Universal TM

A general-purpose computer can be programmed to solve different types of problems. A TM can also behave like a general-purpose computer. A general purpose computer solves a problem as given below :

1. A program is written in a high level language and its machine-code is obtained with the help of a complier.
2. Machine code is loaded in main memory.
3. Input to the program can also be loaded in memory.
4. Program stored in memory is executed line by line. Execution involves reading a line of code pointed by IP (instruction pointer), decoding the code and executing it.

We can follow a similar approach for a TM. Such, a TM is known as **Universal Turing Machine.** Universal Turing Machine (UTM) can solve all sorts of solvable problems.

A Turing machine M is designed to solve a particular problem p, can be specified as :

1. The initial state q_0 of the TM M.
2. The transition function δ of M can be specified as given :

If the current state of M is q_i and the symbol under the head is a_i then the machine moves to state q_j while changing a_i to a_j. The move of tape head may be :

1. To-left,
2. To-Right or
3. Neutral

Such a move of TM can be represented by tuple

$\{(q_i, a_i, q_j, a_j, m_t) : q_i, q_j \in Q ; a_i, a_j \in \Gamma ; m_t \in$ {To- left, To-Right, Neutral}}

UTM should be able to simulate every turing machine. Simulation of a Turing will involve :

1. Encoding behaviour of a particular TM as a program.
2. Execution of the above program by UTM.

A move of the form $(q_i, a_i, q_j, a_j, m_t)$ can be represented as 10^{i+1} $10^i\ 10^{j+1}\ 10^j\ 10^K$,

Where K = 1, if move is to the left

K = 2, if move is to the right

K = 3, if move is 'no-move'

State q_0 is represented by 0,

State q_1 is represented by 00,

State q_n is represented by 0^{n+1}.

First symbol can be represented by 0,

Second symbol can be represented by 00 and so on.

Two elements of a tuple representing a move are separated by 1.

Two moves are separated by 11.

Execution by UTM :

We can assume the UTM as a 3-tape turing machine.

1. Input is written on the first tape.
2. Moves of the TM in encoded form is written on the second tape.
3. The current state of TM is written on the third tape.

The control unit of UTM by counting number of 0's between 1's can find out the current symbol under the head. It can find the current state from the tape 3. Now, it can locate the appropriate move based on current input and the current state from the tape 2. Now, the control unit can extract the following information from the tape 2 :

1. Next state
2. Next symbol to be written
3. Move of the head.

Based on this information, the control unit can take the appropriate action.

Q. 2 Design a TM which recognizes palindromes over alphabet {a,b}

May 2006, May 2009, May 2014, Dec. 2017

Ans. :

A palindrome can have one of the following forms :

1. $\omega\omega^R$
2. $\omega a\omega^R$
3. $\omega b\omega^R$

Where ω is a string over {a,b} with $|\omega| \geq 0$

Algorithm :

1. Algorithm requires n cycles, where $|\omega| = n$.
2. In each cycle, first character is matched with the last character and both are erased.

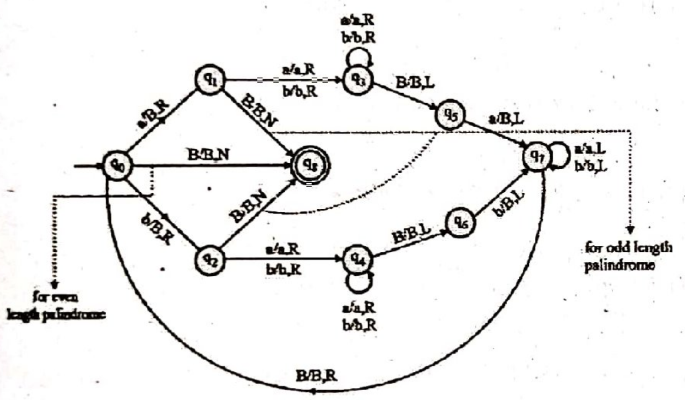


Fig. 7.1(a) : Transition diagram

If the leftmost character is 'a' the machine takes a path through $q_0 \to q_1 \to q_3 \to q_5 \to q_7$, looking for last character as 'a'.

If the leftmost character is 'b', the machine takes a path through

$q_0 \to q_2 \to q_4 \to q_6 \to q_7$, looking for last character as 'b'.

The Turing machine M is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, B\}$

The transition function δ is given in Fig. 7.1(a)

q_0 = initial state

B = blank symbol

F = $\{q_8\}$, halting state

Working of TM for input abbabba is shown in Fig. 7.1(a) :

```
BabbabbaB |- BBbbabbaB |- BBbbabbaB |- BbbabbaB
 ↑             ↑             ↑            ↑
 q0            q1            q3           q3
|- BbbabbaB |- BbbabbaB |- BbbabbaB |- BbbabbaB
       ↑            ↑            ↑            ↑
       q3           q3           q3           q3
|- BbbabbaB |- BbbabbBB |- BbbabbB |- BbbabbB
         ↑          ↑          ↑         ↑
         q5         q7         q7        q7
|- BbbabbB |- BbbabbB |- BbbabbB |- BbbabbB
     ↑         ↑         ↑           ↑
     q7        q7        q7          q0
|- BBbabbB |- BbabbB |- BbabbB |- BbabbB
     ↑          ↑          ↑          ↑
     q2         q4         q4         q4
|- BbabbB |- BbabbB |- BbabBB |- BbabB
        ↑        ↑        ↑        ↑
        q4       q6       q7       q7
|- BbabB |- BbabB |- BbabB |- BBabB
    ↑       ↑         ↑         ↑
    q7      q7        q0        q2
|- BabB |- BabB |- BabB |- BaBB |- BaB
     ↑        ↑      ↑      ↑      ↑
     q4       q4     q6     q7     q7
|- BaB |- BBB |- BBB
    ↑       ↑      ↑
    q0      q1     q8
```

Fig. 7.1(a)

Q. 3 Design a TM to compute multiplication of two unary numbers. **May 2007**

Ans. :

Multiplication algorithm is being explained with the help of an example.

3 × 5 will require three cycles.

Initial	B000 # 00000 # B (3) (5) (Product will be stored here)	
Cycle 1	BX00 # 00000 # 00000B 1×5 = 5	1×5 = 5
Cycle 2	BXX0 # 00000 # 000000000B 2×5 = 10	2×5 = 10
Cycle 3	BXXX # 00000 # 00000 00000 00000 B 3×5 = 15	3×5 = 15

To calculate 3 × 5, three times, 5 zero's are appended.

Unary representation of 3 is 000.

Unary representation of 5 is 00000.

3, 5 and the result, are separated by #.

Inside each major cycles (three cycles for 3), there will be a number of minor cycles (5 minor cycles for 5) to append 0's one at a time.

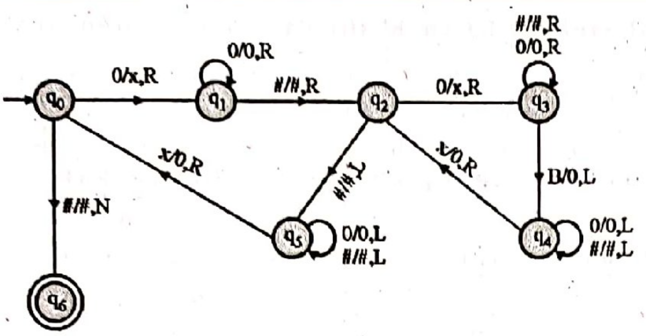


Fig. 7.2 : Transition diagram for TM

Let us assume that the two numbers to be multiplied are x_1 and x_2.

x_1 is represented by ω_1, where ω_1 is a string of 0's.

x_2 is represented by ω_2, where ω_2 is a string of 0's.

$x_1 * x_2$ is represented by ω_3, where ω_3 is a string 0's.

separates ω_1 and ω_2, ω_2 and ω_3.

In the TM shown in Fig. Ex. 7.3.6, there are two cycles.

The cycle $q_0 \to q_1 \to q_2 \to q_5 \to q_0$ appends ω_2 to ω_3 for every zero in ω_1, with the help of cycle $q_2 \to q_3 \to q_4 \to q_2$

Working of TM for 2×2 is shown in Fig. 7.2(a) :

B00#00#B ⊢Bx0#00#B ⊢Bx0#00#B ⊢Bx0#00#B
↑q_0 ↑q_1 ↑q_1 ↑q_2

⊢Bx0#x0#B ⊢Bx0#x0#B
↑q_3 ↑q_3

⊢Bx0#x0#B ⊢Bx0#x0#0B
↑q_3 ↑q_4

⊢Bx0#x0#0B ⊢Bx0#x0#0B
↑q_4 ↑q_4

⊢Bx0#00#0B ⊢Bx0#0x#0B
↑q_2 ↑q_3

⊢Bx0#0x#0B ⊢Bx0#0x#0B
↑q_3 ↑q_3

⊢Bx0#0x#00B ⊢Bx0#0x#00B
↑q_4 ↑q_4

⊢Bx0#0x#00B ⊢Bx0#00#00B
↑q_4 ↑q_2

⊢Bx0#00#00B ⊢Bx0#00#00B ⊢Bx0#00#00B
↑q_5 ↑q_5 ↑q_5

⊢Bx0#00#00B ⊢Bx0#00#00B ⊢B00#00#00B
↑q_5 ↑q_5 ↑q_0

⊢B0x#00#00B ⊢B0x#00#00B
↑q_1 ↑q_2

Fig. 7.2Contd...

⊢B0x#x0#00B⊢B0x#x0#00B
↑q_3 ↑q_3

⊢B0x#x0#00B⊢B0x#x0#00B
↑q_3 ↑q_3

⊢B0x#x0#00B⊢B0x#x0#000B
↑q_3 ↑q_4

⊢B0x#x0#000B⊢B0x#x0#000B
↑q_4 ↑q_4

⊢B0x#x0#000B⊢B0x#x0#000B
↑q_4 ↑q_4

⊢B0x#00#000B⊢B0x#0x#000B
↑q_2 ↑q_3

⊢B0x#0x#000B⊢B0x#0x#000B
↑q_3 ↑q_3

⊢B0x#0x#000B⊢B0x#0x#000B
↑q_3 ↑q_3

⊢B0x#0x#0000B⊢B0x#0x#0000B
↑q_4 ↑q_4

⊢B0x#0x#0000B⊢B0x#0x#0000B
↑q_4 ↑q_4

⊢B0x#0x#0000B⊢B0x#00#0000B
↑q_4 ↑q_4

⊢B0x#00#0000B⊢B0x#00#0000B⊢B0x#00#0000B
↑q_5 ↑q_5 ↑q_5

⊢B0x#00#0000B⊢B00#00#0000B⊢B00#00#0000B
↑q_5 ↑q_0 ↑q_6 (0000 : result)

Fig. 7.2(a)

Q. 4 Design a TM to find the value of $\log_2(n)$, where n is any binary number. **Dec. 2007**

Ans. :

$\log_2(n)$ of any number n lying between 2^n and 2^{n+1} is given by n.

i.e. if $2^n \le n < 2^{n+1}$, then $\log_2(n) = n$

Let us consider the case of a number

$$n = 36$$

$$2^5 \le 36 < 2^6$$

Therefore, $\log_2(36) = 5$

36 can be written as 100100.

Any number n satisfying the condition $2^5 \le n < 2^6$ can be written as 1XXXXX (where X stands for either 1 or 0). $\log_2$ (1XXXXX) can be calculated by erasing the most significant bit 1 and renaming other bits as '0'. Unary representation of 5 is 00000.

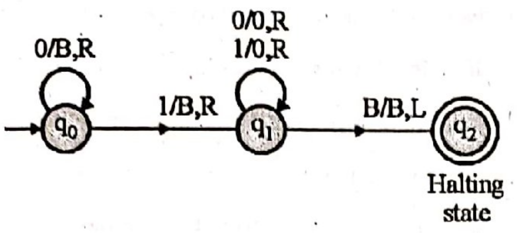


Fig. 7.3(a) : Transition diagram

	0	1	B
$\rightarrow q_0$	(q_0,B,R)	(q_1,B,R)	–
q_1	$(q_1,0,R)$	$(q_1,0,R)$	(q_2,B,L)
q_2^*	q_2	q_2	q_2 ← Halting state

Fig. 7.3(b) : Transition table

Working of TM for $(36)_{10}$ is shown in Fig. 7.3(c) :

$(36)_{10}$ = $(0100100)_2$

B0100100B (↑q_0) ⊢ B100100B (↑q_0) ⊢ B00100B (↑q_1)

⊢ B00100B (↑q_1) ⊢ B00100B (↑q_1) ⊢ B00000B (↑q_1)

⊢ B00000B (↑q_1) ⊢ B00000B (↑q_1) ⊢ B00000B (↑q_2)

Fig. 7.3(c)

Q. 5 Design a Turing machine to compute n!.

Dec. 2008

Ans. :

It is assumed that n is represented in unary system.

Factorial of n can be calculated through repeated application of :

1. Multiplication
2. Copy

Operations.

Algorithm is being explained with the help of example.

Algorithm for $\lfloor 3$.

Initial configuration: 0#000#BB... (000 = n)

Cycle 1 :

1. Multiplication: 0#000#000B... (000 after second # = Product)

2. Copy n − 1, i.e. 2: 0#000#000#00 (n, n×1, n−1)

Cycle 2 :

1. Multiplication: 0#000#000#00#000000 (n, n×1, n−1, n×(n−1))

2. Copy n − 2, i.e. 1: 0#000#000#00#000000#0 (n, n×1, n−1, n×(n−1), n−2)

Cycle 3 :

1. 0#000#000#00#000000#0#000000# (n, 1×n, n−1, n×(n−1), n−2, n×(n−1)(n−2))

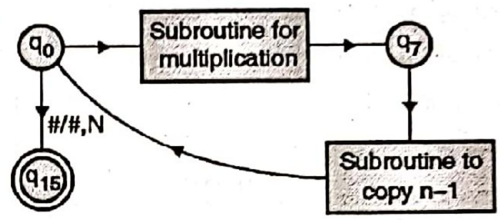


Fig. 7.4(a)

Subroutine for multiplication :

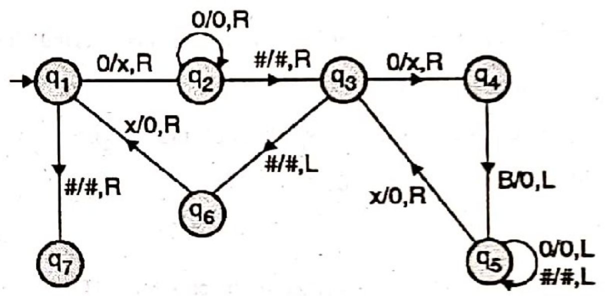


Fig. 7.4(b)

Subroutine to copy n − 1 :

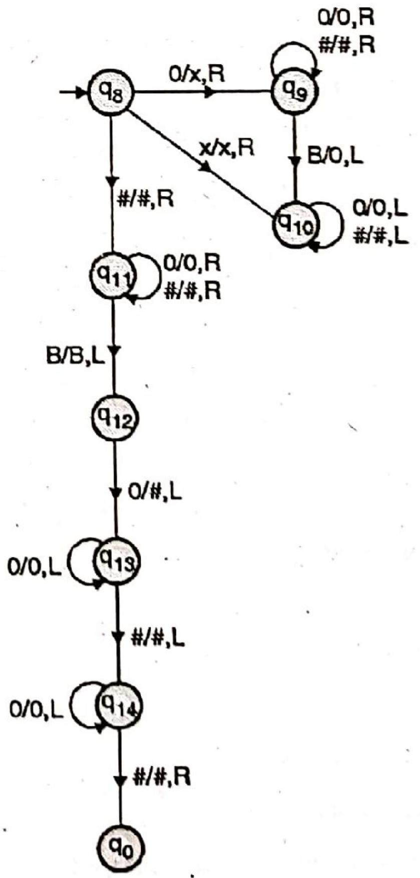


Fig. 7.4(c)

Q. 6 Write note on 'Multiple Turing machine'.

Dec. 2007

Ans. :

Multiple Turing machine

1. A Turing Machine with Multiple Heads

A turing machine with single tape can have multiple heads. Let us consider a turing machine with two heads H_1 and H_2. Each head is capable of performing read/write /move operation independently.

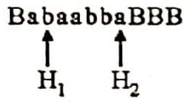


Fig. 7.5 : A Turing machine with two heads

The transition behavior of 2-head one tape Turing machine can be defined as given below :

δ (State, Symbol under H_1, Symbol under H_2) = (New state, (S_1, M_1), (S_2, M_2))

Where,

S_1 is the symbol to be written in the cell under H_1.

M_1 is the movement (L, R, N) of H_1.

S_2 is the symbol to be written in the cell under H_2.

M_2 is the movement (L, R, N) of H_2.

2. Multi-Tape Turing Machine

Multi-Tape turing machine has multiple tuples with each tape having its own independent head. Let us consider the case of a two tape turing machine. It is shown in Fig. 7.6.

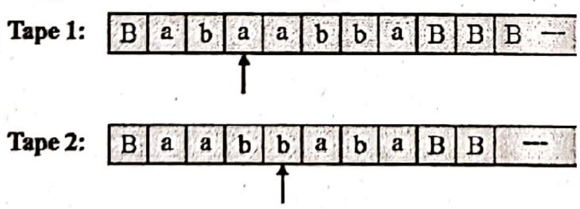


Fig. 7.6 : A two-tape turing machine

The transition behavior of a two-tape Turing machine can be defined as :

$\delta(q_1, a_1, a_2) = (q_2, (S_1, M_1), (S_2, M_2))$

Where,

q_1 is the current state,

q_2 is the next state,

a_1 is the symbol under the head on tape 1,

a_2 is the symbol under the head on tape 2,

S_1 is the symbol written in the current cell on tape 1,

S_2 is the symbol written in the current cell on tape 2,

M_1 is the movement (L, R, N) of head on tape 1,

M_2 is the movement (L, R, N) of head on tape 2.

Q. 7 Design a TM which recognizes words of the form $a^n b^n c^n \mid n \geq 1$.

May 2006. May 2008, Dec. 2011. Dec. 2016

Ans. :

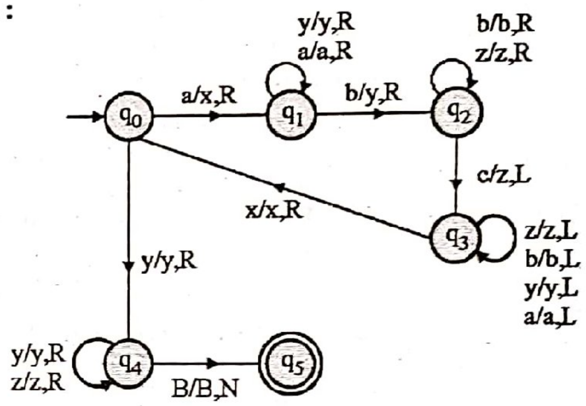


Fig. 7.7(a) : Transition diagram

	a	b	c	x	y	z	B
→q_0	(q_1,x,R)	–	–	–	(q_4,y,R)	–	–
q_1	(q_1,a,R)	(q_2,y,R)	–	–	(q_1,y,R)	–	–
q_2	–	(q_2,b,R)	(q_3,z,R)	–	–	$(q_2,z\ R)$	–
q_3	(q_3,a,L)	(q_3,b,L)	–	(q_0,x,R)	(q_3,y,L)	(q_3,z,L)	–
q_4	–	–	–	–	(q_4,y,R)	(q_4,z,R)	(q_5,B,N)
q_5^*	q_5	q_5	q_5	q_5	q_5	q_5	q_5

Hating state

Fig. 7.7(b) : Transition table

The Turing machine M is given by :

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b, c, x, y, z, B\}$

δ = The transition is given Fig. 7.7(a, b)

q_0 = Initial state

B = Blank symbol

F = $\{q_5\}$, Halting state

Algorithm :

For a string $a^n b^n c^n$, the TM will need n cycles. In each cycle :

1. Leftmost a is written as x
2. Leftmost b is written as y
3. Leftmost c is written as z

At the end of n cycles, the tape should contain only x's, y's and z's.

Working of the TM for input $a^3b^3c^3$ is shown in Fig. 7.7(c) :

```
 BaaabbbcccB ⊢ BxaabbbcccB ⊢ BxaabbbcccB
  ↑               ↑               ↑
  q0              q1              q1
⊢BxaabbbcccB ⊢ BxaaybbcccB ⊢ BxaaybbcccB
     ↑               ↑               ↑
     q1              q2              q2
⊢BxaaybbcccB ⊢ BxaaybbzccB ⊢ BxaaybbzccB
       ↑              ↑               ↑
       q2             q3              q3
⊢BxaaybbzccB ⊢ BxaaybbzccB ⊢ BxaaybbzccB
    ↑               ↑               ↑
    q3              q3              q3
⊢BxaaybbzccB ⊢ BxaaybbzccB ⊢ BxxaybbzccB
  ↑               ↑               ↑
  q3              q0              q1
⊢BxxaybbzccB ⊢ BxxaybbzccB ⊢ BxxayybzccB
     ↑               ↑               ↑
     q1              q1              q2
⊢BxxayybzccB ⊢ BxxayybzccB ⊢ BxxayybzzcB
       ↑               ↑              ↑
       q2              q2             q3
⊢BxxayybzzcB ⊢ BxxayybzzcB ⊢ BxxayybzzcB
      ↑              ↑              ↑
      q3             q3             q3
⊢BxxayybzzcB ⊢ BxxayybzzcB ⊢ BxxayybzzcB
   ↑              ↑              ↑
   q3             q3             q0
⊢BxxxyybzzcB ⊢ BxxxyybzzcB ⊢ BxxxyybzzcB
     ↑              ↑               ↑
     q1             q1              q1
⊢BxxxyyyzzcB ⊢ BxxxyyyzzcB ⊢ BxxxyyyzzcB
       ↑               ↑               ↑
       q2              q2              q2
⊢BxxxyyyzzzB ⊢ BxxxyyyzzzB ⊢ BxxxyyyzzzB
       ↑              ↑               ↑
       q3             q3              q3
⊢BxxxyyyzzzB ⊢ BxxxyyyzzzB ⊢ BxxxyyyzzzB
      ↑              ↑              ↑
      q3             q3             q3
```

Fig. 7.7(c)

Q. 8 Design a turing machine to check whether a string over {a,b} contains equal number of a's and b's. Dec. 2009, May 2008, Dec. 2015

Ans. :

Algorithm :

1. Locate first a or first b.
2. If it is 'a' then locate 'b' rewrite them as x.
3. If it is 'b' then locate 'a' rewrite them as x.
4. Repeat steps from 1 to 3 till every a or b is re-written as x.

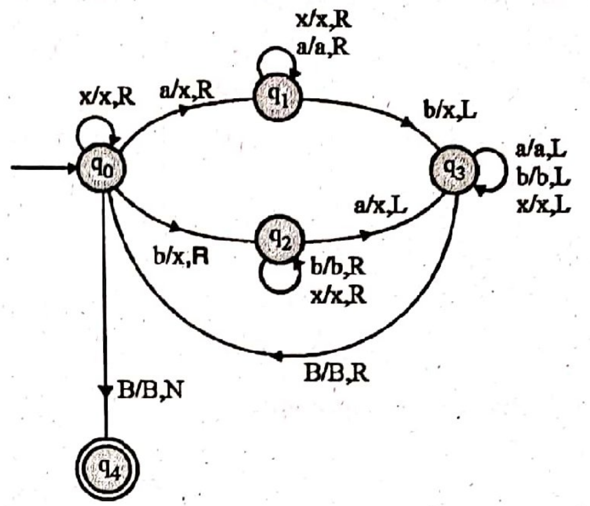


Fig. 7.8(a) : State transition diagram

	a	b	X	B	
→q_0	(q_1,X,R)	(q_2,X,R)	(q_0,X,R)	(q_4,B,N)	
q_1	(q_1,a,R)	(q_3,X,L)	(q_1,X,R)	–	
q_2	(q_3,X,L)	(q_2,b,R)	(q_2,X,R)	–	
q_3	(q_3,a,L)	(q_3,b,L)	(q_3,X,L)	(q_0,B,R)	
q_4^*	q_4	q_4	q_4	q_4	← Halting state

Fig. 7.8(b) : Transition table

The turing machine M is given by :

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, X, B\}$

q_0 = Initial state

B = Blank symbol

$F = \{q_4\}$

Working of machine for an input abba is shown in Fig. 7.8(c)

```
⊢BabbaB ⊢ BxbbaB ⊢ BxxbaB ⊢ BxxbaB
   ↑         ↑         ↑         ↑
   q0        q1        q3        q3
⊢BxxbaB ⊢ BxxbaB ⊢ BxxbaB ⊢ BxxxaB
   ↑           ↑         ↑           ↑
   q0          q0        q0          q2
```

Fig. 7.8(c) Contd....

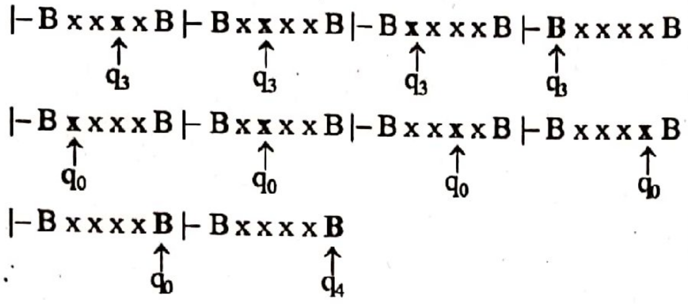


Fig. 7.8(c)

Q. 9 What is Turing machine ? **Dec. 2008**

Ans. :

Turing machine : Formal Definition of Turing Machine

A Turing machine M is a 7-tuple given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where

1. Q is finite set of states
2. Σ is finite set of input alphabet not containing B.
3. Γ is a finite set of tape symbols. Tape symbols include B.
4. $q_0 \in Q$ is the initial symbol.
5. $B \in \Gamma$ is a special symbol representing an empty cell.
6. $F \subseteq Q$ is the set of final states, final states are also known as halting states.
7. The transition function δ is a function from

   $Q \times \Gamma$ to $Q \times \Gamma \times (L, R, N)$

   A transition in turing machine is written as,

   $\delta(q_0, a) = (q_1, b, R)$, which implies, when in state q_0 and scanning symbol a, the machine will enter state q_1, it will rewrite a as b and move to the right cell.

   A transition $\delta(q_0, a) = (q_1, a, R)$, implies that the machine will enter state q_1, it will not change the symbol being scanned and move to the right cell.

   Movement of Read / Write head is given L, R or N

   L $\rightarrow$ Move to left cell

   R $\rightarrow$ Move to right cell

   N $\rightarrow$ Remain in the current cell (No movement)

Q. 10 Design a TM to compute proper subtraction of two unary numbers. The proper subtraction function f is defined as follows :

$$f(m, n) = \begin{cases} m - n & \text{if } m > n \\ 0 & \text{otherwise} \end{cases}$$

May 2009, Dec. 2009

Ans. :

The working of the TM is being explained with subtraction of 3 from 5.

In unary system, 5 is represented as 00000.

In unary system, 3 is represented as 000.

In unary system, 0 is represented by a blank tape.

Subtraction will require several cycle. In each cycle :

1. Leftmost 0 is erased
2. Rightmost 0 is erased.

Situation of tape after each cycle is shown below :

Initial	B	0	0	0	0	0	#	0	0	0	B	--
After 1st cycle	B	B	0	0	0	0	#	0	0	B	B	--
After 2nd cycle	B	B	B	0	0	0	#	0	B	B	B	--
After 3rd cycle	B	B	B	B	0	0	#			B	B	---

Transition diagram and transition table are given in Fig. 7.9(a) and (b).

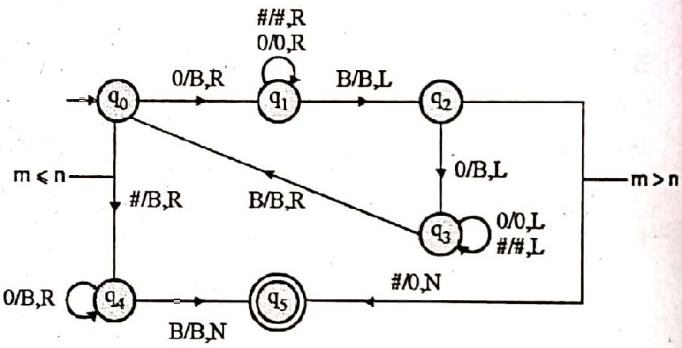


Fig. 7.9(a) : Transition diagram

	0	#	B	
$\rightarrow q_0$	(q_1, B, R)	(q_4, B, R)	–	
q_1	$(q_1, 0, R)$	$(q_1, \#, R)$	(q_2, B, L)	
q_2	(q_3, B, L)	$(q_5, 0, N)$	–	
q_3	$(q_3, 0, L)$	$(q_3, \#, L)$	(q_0, B, R)	
q_4	(q_4, B, R)	–	(q_5, B, N)	
q_5^*	q_5	q_5	q_5	$\leftarrow$ Halting state

Fig. 7.9(b) : Transition table

The Turing machine M is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where,

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{0, 1, \#\}$

$\Gamma = (0, 1, \#, B)$

The transition function δ is given in Fig. 7.9(a) and (b)

q_0 = initial state,

B = blank symbol

F = $\{q_5\}$, Halting state

The working of TM is being simulated for 5-3 is shown in Fig. Ex. 7.3(c) :

```
B00000#000B ⊢ BB0000#000B ⊢ BB0000#000B
q0             q1             q1
⊢ BB 0000#000B ⊢ BB0000#000B ⊢ BB0000#000B
q1               q1             q1
⊢ BB 0000#000B ⊢ BB0000#000B ⊢ BB0000#000B
q1               q1             q1
⊢ BB 0000#000B ⊢ B0000#000B ⊢ B0000#00BB
q1               q2            q3
⊢ B0000#00B ⊢ B0000#00B ⊢ B0000#00B ⊢ B0000#00B
q3            q3          q3          q3
⊢ B0000#00B ⊢ B0000#00B ⊢ B0000#00B ⊢ B0000#00B
q3            q3          q3          q0
⊢ BB000#00B ⊢ BB000#00B ⊢ BB000#00B ⊢ B000#00B
q1            q1          q1          q1
⊢ B000#00B ⊢ B000#00B ⊢ B000#00B ⊢ B000#00B
q1           q1         q1         q2
⊢ B000#0BB ⊢ B000#0B ⊢ B000#0B ⊢ B000#0B
q3           q3        q3        q3
⊢ B000#0B ⊢ B000#0B ⊢ B000#0B ⊢ BB00#0B
q3          q3        q0        q1
⊢ B00#0B ⊢ B00#0B ⊢ B00#0B ⊢ B00#0B ⊢ B00#0B
q1         q1       q1       q1       q2
⊢ B00#BB ⊢ B00#B ⊢ B00#B ⊢ B00#B
q3         q3      q3      q3
⊢ B00#B ⊢ BB0#B ⊢ BB0#B
q0        q1      q1
⊢ B0#B ⊢ B0#B ⊢ B00B
q1       q2     q5
```

Fig. 7.9(c)

Q. 11 Write short note on Variants of TM.

Dec. 2006, Dec. 2008, Dec. 2009, Dec. 2010, May 2014, May 2015, May 2017

Ans. :

1. Two-way Infinite Turing Machine

In a standard turing machine number of positions for leftmost blanks is fixed and they are included in instantaneous description, where the right-hand blanks are not included.

In the two way infinite Turing machine, there is an infinite sequence of blanks on each side of the input string. In an instantaneous description, these blanks are never shown.

2. A Turing Machine with Multiple Heads

A turing machine with single tape can have multiple heads. Let us consider a turing machine with two heads H_1 and H_2. Each head is capable of performing read/write /move operation independently.

```
BabaabbaBBB
 ↑  ↑
 H1 H2
```

Fig. 7.10 : A Turing machine with two heads

The transition behavior of 2-head one tape Turing machine can be defined as given below :

δ (State, Symbol under H_1, Symbol under H_2) = (New state, (S_1, M_1), (S_2, M_2))

Where,

S_1 is the symbol to be written in the cell under H_1.

M_1 is the movement (L, R, N) of H_1.

S_2 is the symbol to be written in the cell under H_2.

M_2 is the movement (L, R, N) of H_2.

3. Multi-Tape Turing Machine

Multi-Tape turing machine has multiple tuples with each tape having its own independent head. Let us consider the case of a two tape turing machine. It is shown in Fig. 7.11.

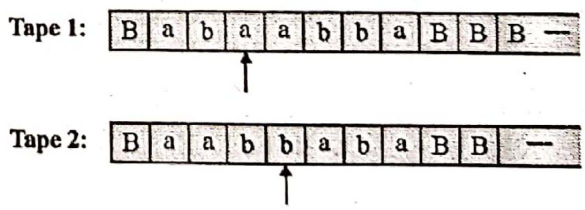


Fig. 7.11 : A two-tape turing machine

The transition behavior of a two-tape Turing machine can be defined as :

$$\delta(q_1, a_1, a_2) = (q_2, (S_1, M_1), (S_2, M_2))$$

Where,

Scanned by CamScanner

q_1 is the current state,

q_2 is the next state,

a_1 is the symbol under the head on tape 1,

a_2 is the symbol under the head on tape 2,

S_1 is the symbol written in the current cell on tape 1,

S_2 is the symbol written in the current cell on tape 2,

M_1 is the movement (L, R, N) of head on tape 1,

M_2 is the movement (L, R, N) of head on tape 2.

4. Non-deterministic Turing Machine

Non-deterministic is a powerful feature. A non-deterministic TM machine might have, on certain combinations of state and symbol under the head, more than one possible choice of behaviour.

Non-deterministic does not make a TM more powerful.

For every non-deterministic TM, there is an equivalent deterministic TM.

It is easy to design a non-deterministic TM for certain class of problems.

A string is said to be accepted by a NDTM, if there is at least one sequence of moves that takes the machine to final state.

An example of non-deterministic move for a TM is shown in Fig. 7.12.

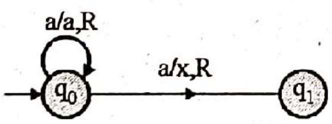


Fig. 7.12 : A sample move for NDTM

The transition behaviour for state q_0 for TM of Fig. 7.12 can be written as

$$\delta(q_0, a) = \{(q_0, a, R)(q_1, x, R)\}$$

Q. 12 Design a turing machine to replace string 110 by 101 in binary input string. **May 2010**

Ans. :

The turing machine will look for every occurrence of the string 110.

State q_2 is for previous two symbols as 11.

Next symbol as 0 in state q_2, will initiate the replacement process to replace 110 by 101.

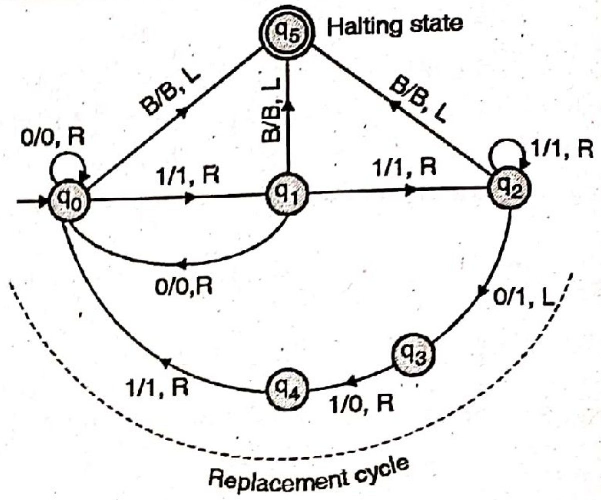


Fig. 7.13

The turing machine M is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, B\}$

δ = Transition function is shown using the transition diagram

B = Blank symbol for the tape

F = $\{q_5\}$, halting state

Working of the machine for input 0101101 is shown in Fig. 7.13(a) :

```
0101101B ⊢ 0101101 ⊢ 0101101B ⊢ 0101101
↑           ↑            ↑              ↑
q0          q0           q1             q0

⊢0101101B ⊢ 0101101B
   ↑            ↑
   q1           q2

⊢0101111B ⊢ 0101011B
   ↑            ↑
   q3           q4

⊢010111B ⊢ 010111B ⊢ 010111B
    ↑          ↑          ↑
    q0         q1         q5 (halt)
```

Fig. 7.13(a)

Q. 13 Design Turing machine as generator to add two binary numbers and hence simulate for "110 + 10". **Dec. 2014**

Ans. :

This problem can be solved using a 3-tape Turing machine.

First machine T1 stores the first binary number. Second machine T2 stores the second binary number. Third machine T3 stores the result.

The Turing machine will have 3 states :

q_0 – previous carry as 0

q_1 – previous carry as 1

q_2 – Halting state

(0, 0, L) (0, 0, L) (B, 0, L)	(1, 1, L) (0, 0, L) (B, 0, L)
(1, 1, L) (0, 0, L) (B, 1, L)	(1, 1, L) (B, B, L) (B, 0, L)
(0, 0, L) (1, 1, L) (B, 1, L)	(0, 0, L) (1, 1, L) (B, 0, L)
(B, B, L) (0, 0, L) (B, 0, L)	(B, B, L) (1, 1, L) (B, 0, L)
(0, 0, L) (B, B, L) (B, 0, L)	(1, 1, L) (1, 1, L) (B, 1, L)
(B, B, L) (1, 1, L) (B, 1, L)	
(1, 1, L) (B, B, L) (B, 1, L)	

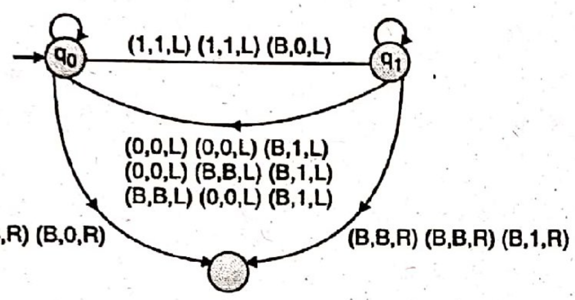


Fig. 7.14

Simulation for 110 + 10

	B	B	1	0	
	B	1	1	0	
	B	B	B	B	

↑

State q_0

	B	B	1	0	
	B	1	1	0	
	B	B	B	0	

↑

State q_0

	B	B	1	0	
	B	1	1	0	
	B	B	0	0	

↑

State q_1

	B	B	1	0	
	B	1	1	0	
	B	0	0	0	

↑

State q_1

B	B	B	1	0	
B	B	1	1	0	
B	1	0	0	0	

↑

q_2 (Halt)

Q. 14 Design a Turing machine as acceptor for the language $\{a^n b^m \mid n, m \geq 0 \text{ and } m \geq n\}$. **Dec. 2014**

Ans. :

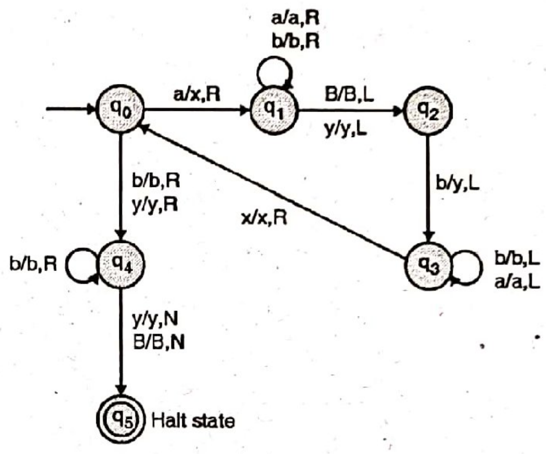


Fig. 7.15

Q. 15 Construct turning machine that accepts the string over $\Sigma = \{0, 1\}$ and converts every occurrence of 111 to 101. **May 2015**

Ans. :

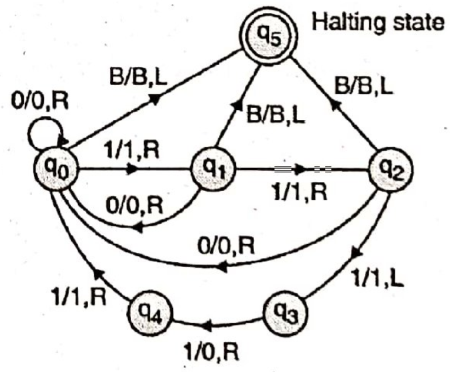


Fig. 7.16

The turing machine M is given by :

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, B\}$

δ = Transition function is shown using the transition diagram

B = Blank symbol for the tape

F = $\{q_5\}$, halting state

Q. 16 Construct a TM for checking well for medness of parentheses. **May 2012, May 2015, May 2017**

Ans. :

In each cycle, the left-most ')' is written as X, then the head moves left to locate the nearer '(' and it is changed to X.

The cycles of computation are shown below.

Input string is assumed to be (()())().

Cycle No.	Tape
Initial	B (()())() B
1.	B (XX())() B
2.	B (XXXX)() B
3.	B XXXXXX() B
4.	B XXXXXXXX B

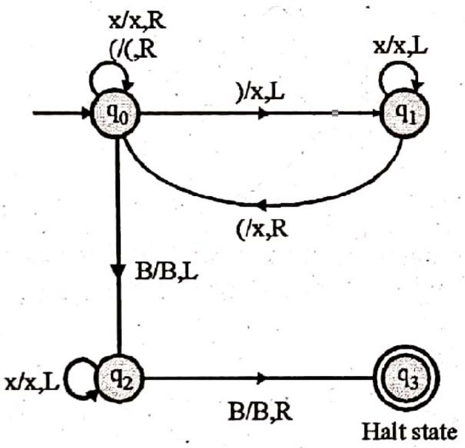


Fig. 7.17(a) : State transition diagram

	(	)	x	B
q_0	$(q_0,(,R)$	(q_1,x,L)	(q_0,x,R)	(q_2,B,L)
q_1	(q_0,x,R)	–	(q_1,x,L)	–
q_2	–	–	(q_2,x,L)	(q_3,B,R)
q_3^*	q_3	q_3	q_3	q_3

⇓

Halting state

Fig. 7.17(b) : State transition table

The Turing machine M is given by :

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

where, $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{(,)\}$

$\Gamma = \{(,), x, B\}$

δ is given in Fig. 7.17(a) or 7.17(b)

q_0 = Initial state

B = Blank symbol

F = $\{q_3\}$, halting state

Making of the machine for input (()())() is given in Fig. 7.17(c) :

B(()())()B ⊢ B(()())()B ⊢ B(()())()
↑q_0 ↑q_0 ↑q_0

⊢ B((x())()B ⊢ B(xx())()B ⊢ B(xx())()B
↑q_1 ↑q_0 ↑q_0

⊢ B(xx())()B ⊢ B(xx(x)()B ⊢ B(xxxx)()B
↑q_0 ↑q_1 ↑q_0

⊢ B(xxxx)()B ⊢ B(xxxxx()B ⊢ B(xxxxx()B
↑q_0 ↑q_1 ↑q_1

⊢ B(xxxxx()B ⊢ B(xxxxx()B ⊢ B(xxxxx()B
↑q_1 ↑q_1 ↑q_1

⊢ Bxxxxxx()B ⊢ Bxxxxxx()B ⊢ Bxxxxxx()B
↑q_0 ↑q_0 ↑q_0

⊢ Bxxxxxx()B ⊢ Bxxxxxx()B ⊢ Bxxxxxx()B
↑q_0 ↑q_0 ↑q_0

⊢ Bxxxxxx()B ⊢ Bxxxxxx(xB
↑q_0 ↑q_1

⊢ BxxxxxxxxB ⊢ BxxxxxxxxB
↑q_0 ↑q_0

⊢ BxxxxxxxxB ⊢ BxxxxxxxxB ⊢ BxxxxxxxxB
↑q_2 ↑q_2 ↑q_2

⊢ BxxxxxxxxB ⊢ BxxxxxxxxB ⊢ BxxxxxxxxB
↑q_2 ↑q_2 ↑q_2

⊢ BxxxxxxxxB ⊢ BxxxxxxxxB ⊢ BxxxxxxxxB
↑q_2 ↑q_2 ↑q_2

⊢ BxxxxxxxxB
↑q_3

Fig. 7.17(c)

Q. 17 Design a turing machine to check whether a string over {a,b} contains equal number of a's and b's. **Dec. 2009, May 2008, Dec. 2015**

Ans. :

Algorithm :

1. Locate first a or first b.
2. If it is 'a' then locate 'b' rewrite them as x.
3. If it is 'b' then locate 'a' rewrite them as x.
4. Repeat steps from 1 to 3 till every a or b is re-written as x.

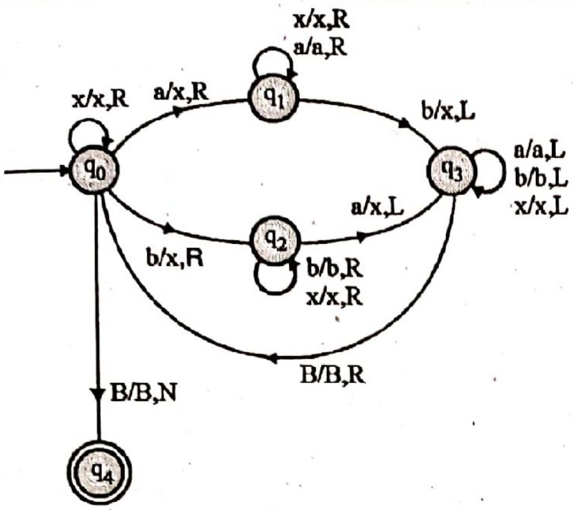


Fig. 7.18(a) : State transition diagram

	a	b	X	B
$\rightarrow q_0$	(q_1,X,R)	(q_2,X,R)	(q_0,X,R)	(q_4,B,N)
q_1	(q_1,a,R)	(q_3,X,L)	(q_1,X,R)	–
q_2	(q_3,X,L)	(q_2,b,R)	(q_2,X,R)	–
q_3	(q_3,a,L)	(q_3,b,L)	(q_3,X,L)	(q_0,B,R)
q_4^*	q_4	q_4	q_4	q_4 ← Halting state

Fig. 7.18(b) : Transition table

The turing machine M is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, X, B\}$

q_0 = Initial state

B = Blank symbol

$F = \{q_4\}$

Working of machine for an input abba is shown in Fig. 7.18(c) :

⊢ B **a** b b a B ⊢ B x **b** b a B ⊢ B **x** x b a B ⊢ **B** x x b a B
↑q_0 ↑q_1 ↑q_3 ↑q_3

⊢ B **x** x b a B ⊢ B x **x** b a B ⊢ B x x **b** a B ⊢ B x x x **a** B
↑q_0 ↑q_0 ↑q_0 ↑q_2

Fig. 7.18(c) Contd....

⊢ B x x **x** x B ⊢ B x **x** x x B ⊢ B **x** x x x B ⊢ **B** x x x x B
↑q_3 ↑q_3 ↑q_3 ↑q_3

⊢ B **x** x x x B ⊢ B x **x** x x B ⊢ B x x **x** x B ⊢ B x x x **x** B
↑q_0 ↑q_0 ↑q_0 ↑q_0

⊢ B x x x x **B** ⊢ B x x x x **B**
↑q_0 ↑q_4

Fig. 7.18(c)

Q. 18 Design a Turing machine as an acceptor for the language $\{a^n b^m \mid n, m \geq 0 \text{ and } m \geq n\}$ **May 2016**

Ans. :

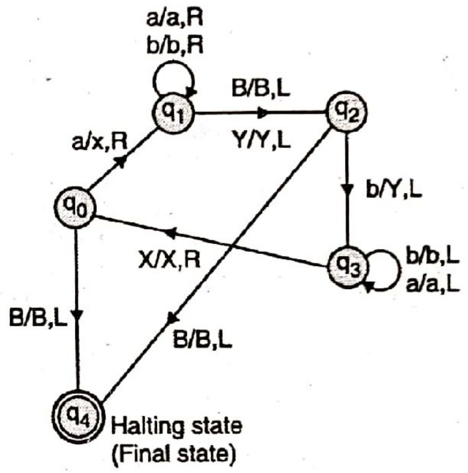


Fig. 7.19

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Where, $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, X, Y, B\}$

q_0 = initial state

B = Blank symbol

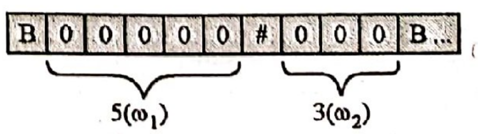
$F = \{q_4\}$

Q. 19 Design a TM to add two unary numbers. **May 2011, Dec. 2016**

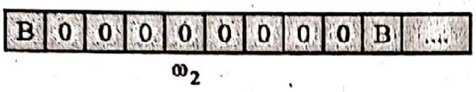
Ans. :

Addition of two unary numbers can be performed through append operation. To add two numbers 5 (say ω_1) and 3 (say ω_2) will require following steps :

1. Initial configuration of tape :



2. ω_1 is appended to ω_2.



While every '0' from ω_1 is getting appended to ω_2, '0' from ω_1 is erased. ω_2 contains 8 0's, which is sum of 5 and 3.

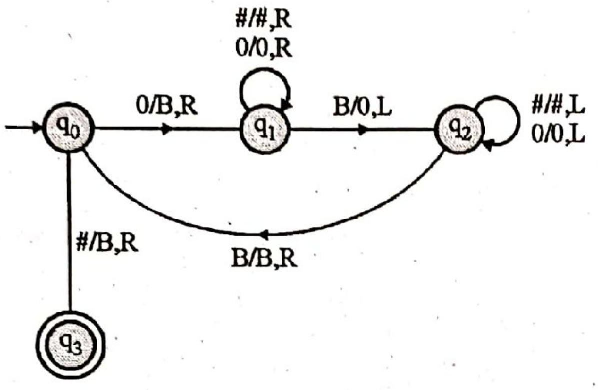


Fig. 7.20(a) : Transition diagram

	0	#	B	
$\rightarrow q_0$	(q_1,B,R)	(q_3,B,R)	–	
q_1	$(q_1,0,R)$	$(q_1,\#,R)$	$(q_2,0,L)$	
q_2	$(q_2,0,L)$	$(q_2,\#,L)$	(q_0,B,R)	
q_3^*	q_3	q_3	q_3	← Halting state

Fig. 7.20(b) : Transition table

The turing machine M is given by :

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Where $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, \#\}$

$\Gamma = \{0, \#, B\}$

δ = Transition function is given in Fig. Ex. 7.3.10 (a), (b)

q_0 = initial state

B = blank symbol

F = $\{q_3\}$, halting state.

Q. 20 Write short note on : Church-Turing Thesis.

May 2017

Ans. :

Church-Turing Thesis

The Turing machine is a general model of computation. Any algorithmic procedure can be solved by G computer can also be solved by a TM. Problems computed by a computer or a TM are also known as partial recursive functions. Some enhancements to TM made the Church-Turing thesis acceptable. These enhancements are :

1. Multi-tape
2. Multi-head
3. Infinite tapes
4. Non-determinism.

Since the introduction of TM, no one has suggested an algorithm than can be solved by a computer but cannot be solved by a TM.

Chapter 8 : Undecidability

Q. 1 Write short note on : Recursive and Recursively Enumerable Languages.

Dec. 2005, Dec. 2009, Dec. 2010, May 2014, Dec. 2014, May 2015, Dec. 2015, May 2016, Dec. 2016, Dec. 2017

Ans. :

Recursive and Recursively Enumerable Languages

There is a difference between recursively enumerable (Turing Acceptable) and recursive (Turing Decidable) language.

Following statements are equivalent :

1. The language L is **Turing acceptable.**
2. The language L is **recursively enumerable.**

Following statements are equivalent

1. The language L is **Turing decidable.**
2. The language L is **recursive.**
3. There is an algorithm for recognizing L.

Every Turing decidable language is Turing acceptable.

Every Turing acceptable language need not be Turing decidable.

Turing Acceptable Language

A language $L \subseteq \Sigma^*$ is said to be a Turing Acceptable language if there is a Turing machine M which halts on every $\omega \in L$ with an answer 'YES'. However, if $\omega \notin L$, then M may not halt.

Turing Decidable Language

A language $L \subseteq \Sigma^*$ is said to be turing being decidable if there is a turing machine M which always halts on every $\omega \in \Sigma^*$. If $\omega \in L$ then M halts, with answer 'YES', and if $\omega \notin L$ then M halts, with answer 'NO'.

A set of solutions for any problem defines a language.

A problem P is said to be decidable /solvable if the language $L \subseteq \Sigma^*$ representing the problem (set of solutions) is turing decidable.

If P is solvable / decidable then there is an algorithm for recognizing L, representing the problem. It may be noted that an algorithm terminates on all inputs.

Following statements are equivalent :

1. The language L is Turing decidable.
2. The language L is recursive.
3. There is an algorithm for recognizing L.

Every turing decidable language is turing acceptable.

Every turing acceptable language need not be turing decidable.

A language $L \subseteq \Sigma^*$ many not be turing acceptable and hence not turing decidable. Thus we cannot design a turing machine / algorithm which halts for every $\omega \in L$.

Q. 2 Two recursive languages L_1 and L_2 is recursive : $L_1 \cup L_2$

Ans. :

$L_1 \cup L_2$ is recursive

Let the turing machine M_1 decides L_1 and M_2 decides L_2.

If a word $\omega \in L_1$ then M_1 returns "Y" else it returns "N". Similarly, if a word $\omega \in L_2$ then M_2 returns "Y" else it returns "N".

Let us construct a turing machine M_3 as shown in Fig. 8.1.

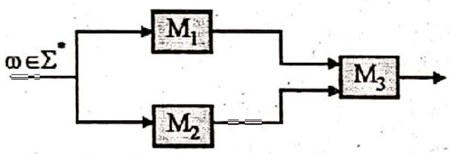


Fig. 8.1 : A turing machine for $L_1 \cup L_2$

Output of machine M_1 is written on the tape of M_3.

Output of machine M_2 is written on the tape of M_3.

The machine M_3 returns "Y" as output, if at least one of the outputs of M_1, or of M_2 is "Y".

It should be clear that M_3 decides $L_1 \cup L_2$. As both L_1 and L_2 are turing decidable, after a finite time both M_1 and M_2 will halt with answer "Y" or "N". The machine M_3 is activated after M_1 and M_2 are halted. The machine M_3 halts with answer "Y" if $\omega \in L_1$ or $\omega \in L_2$, else M_3 halts with output "N".

Thus $L_1 \cup L_2$ is turing decidable or $L_1 \cup L_2$ is recursive.

Q. 3 Prove that there exists no algorithm for deciding whether a given CFG is ambiguous.

May 2006, Dec. 2007, Dec. 2008

Ans. :

The post correspondence problem can be used to prove the un-decidability of whether a given CFG is ambiguous.

Let us consider two sequences of strings over Σ.

$$A = \{u_1, u_2, u_3 \ldots u_m\}$$

$$B = \{v_1, v_2, v_3 \ldots v_m\}$$

Let us take a new set of symbols $a_1, a_2 \ldots a_m$ such that

$$\{a_1, a_2 \ldots a_m\} \cap \Sigma = \phi.$$

Symbols $a_1, a_2 \ldots a_m$ are being taken as index symbols. The index symbol a_i represents a choice of u_i from A and v_i from the list B.

A string of the form $u_i\ u_j\ u_k \ldots a_k\ a_j\ a_i$. Over alphabet $\Sigma \cup \{a_1, a_2, \ldots a_m\}$ can be defined using the set of productions :

$$G_A = \begin{Bmatrix} A \to u_1\, A\, a_1 \mid u_2\, A\, a_2 \mid \ldots \mid u_m\, Aa_m \\ u_1\, a_1 \mid u_2\, a_2 \mid \ldots \mid u_m\, a_m \end{Bmatrix}$$

Similarly a string of the form $v_i\ v_j\ v_k \ldots a_k\ a_j\ a_i$ over alphabet $\Sigma \cup \{a_1, a_2 \ldots a_m\}$ can be defined using the set of productions :

$$G_B = \begin{Bmatrix} B \to v_1\, A\, a_1 \mid v_2\, A\, a_2 \mid \ldots \mid v_m\, Aa_m \\ v_1\, a_1 \mid v_2\, a_2 \mid \ldots \mid v_m\, a_m \end{Bmatrix}$$

Finally, we can combine the languages and grammars of two lists to form a grammar G_{AB} :

A new start symbol S is added to G_{AB}

Two new productions are added to G_{AB}

$$S \to A$$

$$S \to B$$

All productions of G_A and G_B are taken.

Now, we will show that G_{AB} is ambiguous if and only if an instance (A, B) of PCP has a solution.

Assumption :

Suppose the sequence $i_1, i_2, \ldots, i_m$ is a solution to this instance of PCP. Two derivations for the above string in G_{AB} is :

$$S \Rightarrow A \Rightarrow u_{i_1} A\, a_{i_1} \Rightarrow u_{i_1} u_{i_2} A\, a_{i_1} a_{i_2} \Rightarrow \ldots \Rightarrow$$

$$u_{i1}\, u_{i2} \ldots u_{im}\, a_{i1}\, a_{i2} \ldots a_{im}$$

$$S \Rightarrow B \Rightarrow v_{i1}\, B\, a_{i1} \Rightarrow v_{i1}\, v_{i_2}\, B\, a_{i_1} a_{i_2} \Rightarrow \ldots \Rightarrow$$

$$v_{i1}\, v_{i2} \ldots v_{im}\, a_{i1}\, a_{i2} \ldots a_{im}$$

Consequently, if G_{AB} is ambiguous, then the post correspondence problem with the pair (A, B) has a solution. Conversely, if G_{AB} is unambiguous, then the post correspondence cannot have a solution.

If there exists an algorithm for solving the ambiguous problem, then there exists an algorithm for solving the post correspondence problem. But, since there is no algorithm for the post correspondence problem, the ambiguity of CFG problem is unsolvable.

Q. 4 Write short notes on post correspondence problem and Greibach Theorem.

May 2006, Dec. 2006, May 2007, Dec. 2007, May 2008, Dec. 2008, May 2009, May 2010, Dec. 2010, May 2011, Dec. 2011, May 2012, May 2016

Ans. :

Post correspondence problem

Definition : Let A and B be two non-empty lists of strings over Σ. A and B are given as below :

$$A = \{x_1, x_2, x_3 \ldots x_k\}$$
$$B = \{y_1, y_2, y_3 \ldots y_k\}$$

There is a post correspondence between A and B if there is a sequence of one or more integers i, j, k ...m such that :

The string $x_i\, x_j \ldots x_m$ is equal to $y_i\, y_j \ldots y_m$.

Example : Does the PCP with two lists :

$$A = \{a, aba^3, ab\} \text{ and}$$
$$B = \{a^3, ab, b\}$$

have a solution ?

So to find a sequence using which when the elements of A and B are listed, will produce identical strings.

The required sequence is (2, 1, 1 , 3)

$$A_2\, A_1\, A_1\, A_3 = aba^3\, a\, aab = ab\, a^6\, b$$
$$B_2\, B_1\, B_1\, B_3 = aba^3\, a^3\, b = aba^6\, b$$

Thus, the PCP has solution.

So accept the un-decidability of post correspondence problem without proof.

Example :

Determining the solution for following instance of PCP.

	List A	List B
i	ωi	xi
1	01	0
2	110010	0
3	1	1111
4	11	01

The PCP has a solution. The required sequence is (1, 3, 2, 4, 4, 3)

$$\omega_1\omega_3\omega_2\omega_4\omega_4\omega_3 = 01111001011111$$
$$x_1\, x_3\, x_2\, x_4\, x_4 x_3 = 01111001011111$$

Greibach Theorem

The Theorem states that :

"Let σ be a class of languages that is effectively closed under concatenation with regular sets and union, and for which $L = \Sigma^*$ is un-decidable for any sufficiently large fixed Σ. Let P be any non-trivial property that is true for all regular sets and that is preserved under a, where a is single symbol in Σ. Then P is un-decidable for σ".

Greibach theorem can be used to prove that many problems related to CFG are un-decidable.

Q. 5 Write short notes on : Halting problem.

Dec. 2006, Dec. 2007, May 2008, Dec. 2008, May 2011, Dec. 2011, Dec. 2015, Dec. 2016, May 2017

Ans. :

Halting Problem of a Turing Machine

The halting problem of a Turing machine states :

Given a Turing machine M and an input ω to the machine M, determine if the machine M will eventually halt when it is given input ω.

Halting problem of a Turing machine is unsolvable.

Proof :

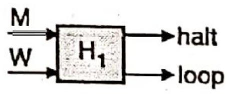
Moves of a turing machine can be represented using a binary number. Thus, a Turing machine can be represented using a string over $\Sigma^*(0,1)$. This concept has already been explained in the chapter.

Insolvability of halting problem of a Turing machine can be proved through the method of contradiction.

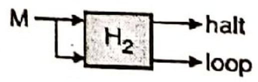
Step 1 : Let us assume that the halting problem of a Turing machine is solvable. There exists

1. A string describing M.
2. An input ω for machine M.

H_1 generates an output **"halt"** if H_1 determines that M stops on input ω; otherwise H outputs "loop". Working of the machine H_1 is shown below.

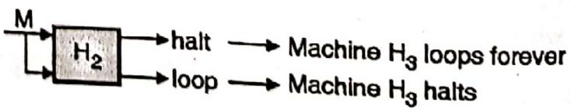


Step 2 : Let us revise the machine H_1 as H_2 to take M as both inputs and H_2 should be able to determine if M will halt on M as its input. Please note that a machine can be described as a string over 0 and 1.



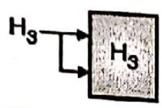
Step 3 : Let us construct a new Turing machine H_3 that takes output of H_2 as input and does the following :

1. If the output of H2 is "loop" than H3 halts.
2. If the output of H_2 is "halt" than H_3 will loop forever.



H_3 will do the opposite of the output of H_2.

Step 4 : Let us give H_3 itself as inputs to H_3.



If H_3 halts on H_3 as input then H_3 would loop (that is how we constructed it). If H_3 loops forever on H_3 as input H_3 halts (that is how we constructed it).

In either case, the result is wrong.

Hence,

H_3 does not exist.

If H_3 does not exist than H_2 does not exist.

If H_2 does not exist than H_1 does not exist.

Q. 6 Does PCP with following two list : A = (10, 011, 101) and B = (101, 11, 011) have a solution ? Justify your answer. May 2009

Ans. :

A_2 and A_3 differ from B_2 and B_3 at the first of place. Therefore, we must pick A_1 and B_1

Sequence	String
(1)	$(A_1 = 10)$ $(B_1 = 101)$

The next string to be picked up must be A_3 and B_3. Any other sequence will not lead to a solution.

Sequence	String
(1, 3)	$(A_1A_3 = 10101)$ $(B_1B_3 = 101011)$

The next string to be picked up must be A_3 and B_3. Any other sequence will not lead to a solution.

Sequence	String
(1, 3, 3)	$(A_1A_3A_3 = 10101101)$ $(B_1B_3B_3 = 101011011)$

There is only choice of next string. This choice is A_3 and B_3. This does not lead to a solution. The PCP has no solution.

Q. 7 Write short note on : Rice Theorem

Dec. 2012, May 2013, May 2014, May 2015, Dec. 2015, May 2016, Dec. 2016, May 2017, Dec. 2017

Ans. :

Rice Theorem

"Every property that is satisfied by some but not all recursively enumerable language is un-decidable". Any property that is satisfied by some recursively enumerable language but not all is known as nontrivial property. We have seen many properties of R.E. languages that are un-decidable. These properties include :

1. Given a TM M, is L(M) nonempty ?
2. Given a TM M, is L(M) finite ?
3. Given a TM M, is L(M) regular ?
4. Given a TM M, is L(M) recursive ?

The Rice's theorem can be proved by reducing some other unsolvable problem to nontrivial property of recursively enumerable language.

Theory of Computer Science

Statistical Analysis

Chapter No.	Dec. 2018	May 2019
Chapter 1	27.5 Marks	10 Marks
Chapter 2	12.5 Marks	20 Marks
Chapter 3	27.5 Marks	15 Marks
Chapter 4	-	-
Chapter 5	-	10 Marks
Chapter 6	25 Marks	10 Marks
Chapter 7	12.5 Marks	20 Marks
Chapter 8	7.5 Marks	25 Marks
Repeated questions	-	5 Marks

Dec. 2018

Chapter 1 : Introduction [Total Marks – 27.5]

Q. 1(a) Explain Chomsky Hierarchy. **(5 Marks)**

Ans. : Chomsky hierarchy

A grammar can be classified on the basis of production rules. Chomsky classified grammars into the following types :

1. Type 3 : Regular grammar
2. Type 2 : Context free grammar
3. Type 1 : Context sensitive grammar
4. Type 0 : Unrestricted grammar

Type 3 or regular grammar

– A grammar is called Type 3 or regular grammar if all its productions are of the following forms:

$A \rightarrow \varepsilon$

$A \rightarrow a$

$A \rightarrow aB$

$A \rightarrow Ba$

Where, $a \in \Sigma$ and $A, B \in V$.

– A language generated by Type 3 grammar is known as regular language.

Type 2 or context free grammar

- A grammar is called Type 2 or context free grammar if all its productions are of the following form $A \to \alpha$ where $A \in V$ and $\alpha \in (V \cup T)^*$.
- V is a set of variables and T is a set of terminals.
- The language generated by a Type 2 grammar is called a context free language, a regular language but not the reverse.

Type 1 or context sensitive grammar

- A grammar is called a Type 1 or context sensitive grammar if all its productions are of the following form:

  $\alpha \to \beta,$
- Where, β is atleast as long as α.

Type 0 or unrestricted grammar

Productions can be written without any restriction in an unrestricted grammar. If there is production of the $\alpha \to \beta$, then length of α could be more than length of β.

Every grammar also is a Type 0 grammar

A Type 2 grammar is also a Type 1 grammar

A Type 3 grammar is also a Type 2 grammar

Q. 3(b) Consider the following grammar

S→iCtS|iCtSeS|a

C→b

For the string 'ibtaeibta' find the following :

(i) Leftmost derivation

(ii) Rightmost derivation

(iii) Parse tree

(iv) Check if above grammar is ambiguous. **(10 Marks)**

Ans. :

(i) Left most derivation :

S →	iCtSeS	[using S → iCtSeS]
→	ibtSeS	[using C → b]
→	ibtaeS	[using S → a]
→	ibtaeiCtS	[using S→ iCtS]
→	ibtaeibts	[using C → b]
→	ibtaeibta	

(ii) Rightmost derivation :

S →	iCtSeS	[using S → iCtSeS]
→	iCtSeiCtS	[using S → iCtS]
→	iCtSeiCta	[using S → a]
→	iCtSeibta	[using C → b]
→	iCtaeibta	[using S → a]
→	ibtaeibta	[using C → b]

(iii) Parse tree as shown in Fig. 1-Q. 3(b).

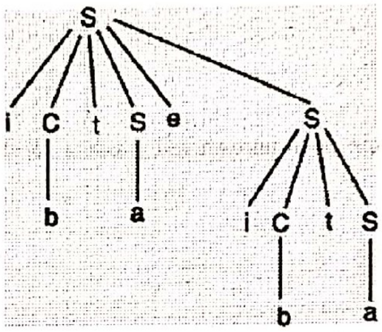


Fig. 1-Q. 3(b)

(iv) The grammar can be shown to be ambiguous by drawing two different derivation trees for the string ibtibtaea as shown in Fig. 2-Q. 3(b).

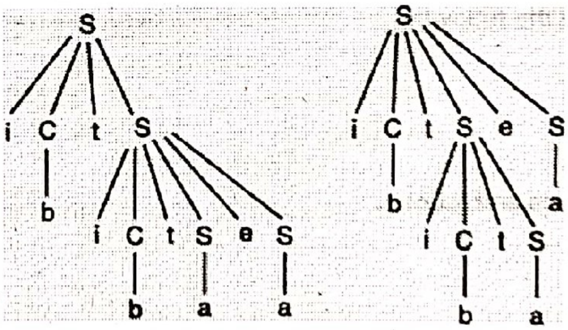


Fig. 2-Q. 3(b)

Q. 5(b) Construct Mealy and Moore Machine to convert each occurrence of 100 by 101. **(10 Marks)**

Ans. :

1. Mealy Machine

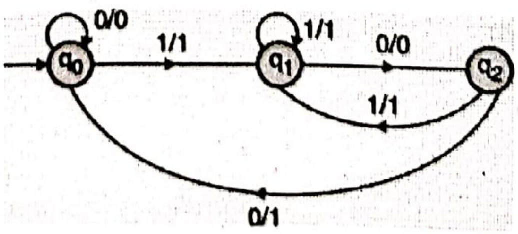


Fig. 1-Q. 5(b)

2. Moore Machine

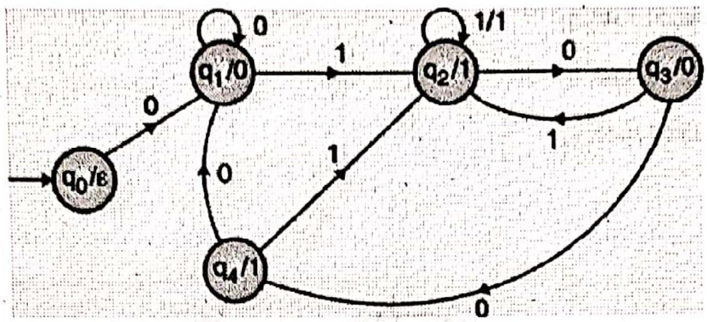


Fig. 2-Q. 5(b)

Q. 6(d) Write short note on Mealy and Moore Machine. **(2.5 Marks)**

Ans. :

Final state machines are characterised by two behaviours :

1. State transition function (δ)
2. Output function (λ)

State transition function (δ) is also known as STF.

Output function (λ) is also known as machine function (MTF).

$\delta : \Sigma \times Q \rightarrow Q$

$\lambda : \Sigma \times Q \rightarrow O$ [for Mealy machine]

$\lambda : Q \rightarrow O$ [for Moore machine]

There are two types of automata with outputs :

1. **Mealy machine :** Output is associated with transition

   $\lambda : \Sigma \times Q \rightarrow O$

   Set of output alphabet O can be different from the set of input alphabet Σ.

2. **Moore machine :** Output is associated with state

   $\lambda : Q \rightarrow O$

Chapter 2 : Finite Automata [Total Marks – 12.5]

Q. 2(a) Design a Finite State machine to determine whether ternary number (base 3) is divisible by 5. **(10 Marks)**

Ans. :

- A ternary system has three alphabets

$$\Sigma = \{0, 1, 2\}$$

- Base of a ternary number is 3.
- The running remainder could be :

$$(0)_3 = 0 \rightarrow \text{associated state, } q_0$$

$(1)_3 = 1 \rightarrow$ associated state, q_1

$(2)_3 = 2 \rightarrow$ associated state, q_2

$(10)_3 = 3 \rightarrow$ associated state, q_3

$(11)_3 = 4 \rightarrow$ associated state, q_4

↑ ↑

Ternary Decimal

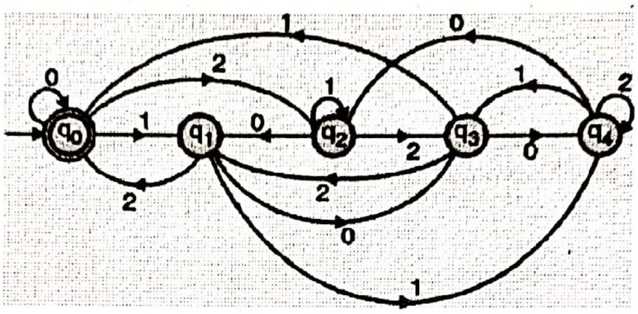


Fig. 1-Q. 2(a)

Q. 6(a) Write short note on Closure properties of Context Free Language. (2.5 Marks)

Ans. :

Closure properties of context free language

- A context free language is closed under following operations :

  1. Union 2. Concatenation 3. Kleene star

- Context free language is closed under intersection.
- The intersection of a context-free language with a regular language is a context free language.
- The CFL is closed under complementation.
- The CFL is closed under reversal.

1. CFL is closed under union

If L_2 and L_2 are context-free languages, then $L_2 \cup L_2$ is a context free language.

2. CFL is closed under concatenation

If L_2 and L_2 are context-free languages, then L_2L_2 is a context-free language.

3. CFL is closed under Kleene Star

If L is a context-free language, then L^* is a context-free language.

4. CFL is not closed under intersection

Context-free languages are closed under intersection.

5. CFL is not closed under complementation

The set of context-free languages is closed under complementation.

6. Intersection of CFL and RL

If L is a CFL and R is a regular language, then $R \cap L$ is a CFL.

7. CFL is closed under reversal

If L is a context-free language, then so is L^R.

Chapter 3 : Regular Expressions and Languages [Total Marks – 27.5]

Q. 1(c) Define Regular Expression and give regular expression for :

(i) Set of all strings over {0, 1} that end with 1 has no substring 00 **(5 Marks)**

Ans. :

Regular expression

- An expression written using the set of operators (+, · , *) and describing a regular language is known as regular expression.
- The transition graph is shown in Fig. 1-Q. 1(c).

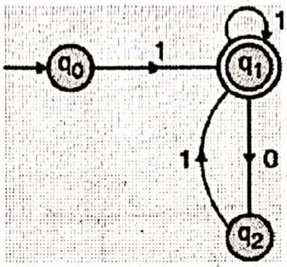


Fig. 1-Q. 1(c)

- ∴ R.E. can be written from the transition graph. The required R. E. = 1 (1 + 01)*

Q. 2(b) Give and explain formal definition of Pumping Lemma for Regular Language and prove that following language is not regular. $L = \{a^m b^{m-1} \mid m > 0\}$ **(10 Marks)**

Ans. :

Pumping Lemma for Regular Language

- Some languages are regular. There are other languages which are not regular. One can neither express a non-regular language using regular expression nor design finite automata for it.
- Pumping lemma gives a necessary condition for an input string to belong to a regular set.
- Pumping lemma does not give sufficient condition for a language to be regular.
- Pumping lemma should not be used to establish that a given language is regular.
- Pumping lemma should be used to establish that a given language is not regular.
- The pumping lemma uses the pigeonhole principle which states that if n pigeons are placed into less than n holes, some holes have to have more than one pigeon in it. Similarly, a string of length ≥ n when recognized by a FA with n states will see some states repeating.

Definition of Pumping Lemma

Let L be a regular language and $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automata with n-states. Language L is accepted by m. Let $\omega \in L$ and $|\omega| \geq n$, then ω can be written as xyz, where

(i) $|y| > 0$

(ii) $|xy| \leq n$

(iii) $xy^i z \in L$ for all $i \geq 0$ here y^i denotes that y is repeated or pumped i times.

Proving that the language $L = \{a^m b^{m-1} \mid m > 0\}$ is not regular:

Step 1 : Let us assume that the given language $L(a^n b^{n-1} \mid n > 0)$ is regular and L is accepted by an FA with n states.

Step 2 : Let us choose a string

$$\omega = a^n b^{n-1}$$

$$|\omega| = 2^{n-1} \geq n \text{ for } n > 0$$

Let us write ω as xyz, with

$$|y| > 0$$

$$\text{and } |xy| \leq n$$

$$\text{since, } |xy| \leq n, \text{ y must be of the form } a^r \mid r > 0.$$

$$\text{since } |xy| \leq n, \text{ x must be of the form } a^s.$$

Now, $a^n b^{n-1}$ can be written as

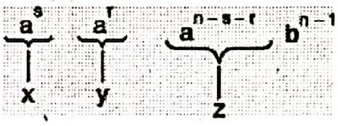


Fig. 1-Q. 2(b)

Step 3 : Let us check whether $xy^{\bar{L}}z$ for $\bar{L} = 2$ belongs to L.

$$x y^2 z = a^s (a^r)^2 a^{n-s-r} b^{n-1}$$

$$= \text{a3 a2r an} - \text{s} - \text{r bn} - 1$$

$$= \text{an} + \text{r bn} - 1$$

$$\text{Since } r > 0, a^{n+r} b^{n-1} \notin L.$$

Hence, by contradiction, we can say that the given language is not regular.

Q. 5(a) Convert (0 + 1) (10)*(0 + 1) into NFA with ∈-moves and obtain DFA. **(10 Marks)**

Ans. :

R. E. to NFA

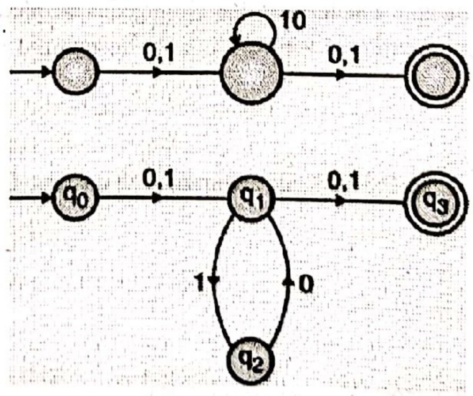


Fig. 1-Q. 5(a)

NFA to DFA using direct method

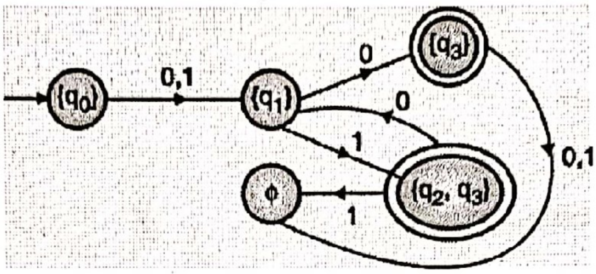


Fig. 2-Q. 5(a)

Q. 6(b) Write short note on : Applications of Regular expression and Finite automata. (2.5 Marks)

Ans. :

1. Applications of regular expression

(a) R.E. in Unix

The UNIX regular expression lets us specify a group of characters using a pair of square brackets [ ]. The rules for character classes are :

1. **[ab]** Stand for a + b
2. **[0 – 9]** Stand for a digit from 0 to 9
3. **[A – Z]** Stands for an upper-case letter
4. **[a – z]** Stands for a lower-case letter
5. **[0 – 9A-Za – z]** Stands for a letter or a digit.

The **grep** utility in UNIX scans a file for the occurrence of a pattern and displays those lines in which the given pattern is found.

For example :

$ grep president emp.txt

It will list those lines from the file emp.txt which has the pattern "president". The pattern in grep command can be specified using regular expression.

6. ***** matches zero or more occurrences of previous character.
7. **●** matches a single character.
8. **[^ pqr]** Matches a single character which is not a p, q or r.
9. **^ pat** Matches pattern **pat** at the beginning of a line
10. **pat $** Matches pattern at end of line.

Example :

(a) The regular expression **[aA] g [ar] [ar] wal** stands for either "Agarwal" or 'agrawal".

(b) **g*** stands for zero or more occurrences of g.

(c) **$grep "A · * thakur" emp.txt** will look for a pattern starting with **A.** and ending with **thakur** in the file emp.txt.

(b) Lexical analysis

Lexical analysis is an important phase of a compiler. The lexical analyser scans the source program and converts it into a steam of tokens. A token is a string of consecutive symbols defining an entity.

For example a C statement **x = y + z** has the following tokens :

x – An identifier

= – Assignment operator

y = An identifier

+ – Arithmetic operator +

z – An identifier

Keywords, identifiers and operators are common examples of tokens.

The UNIX utility **lex** can be used for writing of a lexical analysis program. Input to **lex** is a set of regular expressions for each type of token and output of **lex** is a C program for lexical analysis.

2. Applications of Finite Automata

Finite automata are used for solving several common types of computer algorithms. Some of them are :

(i) Design of digital circuit

(ii) String matching

(iii) Communication protocols for information exchange.

(iv) Lexical analysis phase of a compiler.

Finite automata can work as an algorithm for regular language. It can be used for checking whether a string $w \in L$, where L is a regular language.

Chapter 6 : Regular Grammar [Total Marks - 25]

Q. 1(b) Differentiate between PDA and NPDA. **(5 Marks)**

Ans. :

Difference between PDA and NPDA is as follows:

Sr. No.	PDA	NPDA
1.	Always a single move on a new input	Multiple moves are possible on a new input
2.	Less powerful than NPDA	More powerful than a PDA
3.	Algorithms related to PDA are simple	Algorithms related to NPDA are complex
4.	Algorithms related to PDA do not require backtracking	Algorithms related to NPDA require backtracking

Q. 3(a) Construct PDA accepting the language $L = \{a^{2n} b^n \mid n \geq 0\}$. **(10 Marks)**

Ans. :

1. For every pair of a's one x is pushed on to the stack
2. For every b, one x is popped out from the stack.
3. Finally the stack should contain the initial stack symbol Z_0.

Transition table (δ)

1. $\delta(q_0, a, Z_0) = (q_1, Z_0)$
2. $\delta(q_1, a, Z_0) = (q_0, x Z_0)$
3. $\delta(q_0, a, x) = (q_1, x)$
4. $\delta(q_1, a, x) = (q_0, xx)$
5. $\delta(q_0, b, x) = (q_2, \in)$
6. $\delta(q_2, b, x) = (q_2, \in)$
7. $\delta(q_2, \in, Z_0) = (q_2, \in)$

Accepting through empty stack

Thus, the PDA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{x, Z_0\}, \delta, q_0, Z_0, \{\phi\})$

Q. 4(b) Convert following CFG to CNF **(10 Marks)**

$S \rightarrow ASA \mid Ab$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \in$

Ans. :

1. Nullable set of symbols = (B, A)

Re-writing grammar after removing ∈-production,

we get,

$S \rightarrow AS \mid SA \mid ASA \mid aB \mid a$

$A \rightarrow B \mid S$

$B \rightarrow b$

2. Re-writing grammar after removing unit productions ($A \rightarrow B$, $A \rightarrow S$), we get

$S \rightarrow AS \mid SA \mid ASA \mid aB \mid a$

$A \rightarrow b \mid AS \mid SA \mid ASA \mid aB \mid a$

$B \rightarrow b$

3. Every symbol in α, in production of the form $A \rightarrow \alpha$ where $|\alpha| \geq 2$ should be a variable. This can be done by adding the production

$C_1 \rightarrow a$

The set of productions become,

$S \rightarrow AS \mid SA \mid ASA \mid C_1B \mid a$

$A \rightarrow b \mid AS \mid SA \mid ASA \mid C_1B \mid a$

$B \rightarrow b$

$C_1 \rightarrow a$

4. **Finding an equivalent grammar in CNF.**

$S \rightarrow AS \mid SA \mid AC_2 \mid C_1B \mid a$ [Replacing SA by C_2]

$C_2 \rightarrow SA$

$A \rightarrow b \mid AS \mid SA \mid AC_2 \mid C_1B \mid a$

$B \rightarrow b$

$C_1 \rightarrow a$

Chapter 7 : Turing Machine (TM) [Total Marks – 12.5]

Q. 4(a) Construct TM to check well-formedness of parenthesis. (10 Marks)

Ans. :

In each cycle, the left-most ')' is written as X, then the head moves left to locate the nearer '(' and it is changed to X.

The cycles of computation are shown below.

Input string is assumed to be (()())().

Cycle No.	Tape
Initial	B (()())() B
1.	B (XX())() B
2.	B (XXXX)() B
3.	B XXXXXX() B
4.	B XXXXXXXX B

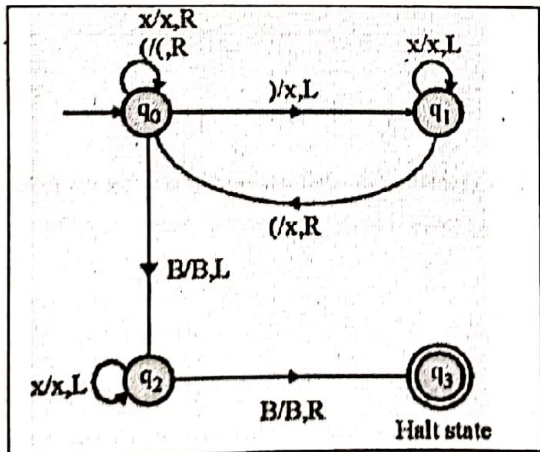


Fig. 1-Q. 4(a)(a) : State transition diagram

	(	)	x	B
q_0	$(q_0,(,R)$	(q_1,x,L)	(q_0,x,R)	(q_2,B,L)
q_1	(q_0,x,R)	–	(q_1,x,L)	–
q_2	–	–	(q_2,x,L)	(q_3,B,R)
q_3^*	q_3	q_3	q_3	q_3
⇓				
Halting state				

Fig. 1-Q. 4(a)(b) : State transition table

The Turing machine M is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where, $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{(,)\}$

$\Gamma = \{(,), x, B\}$

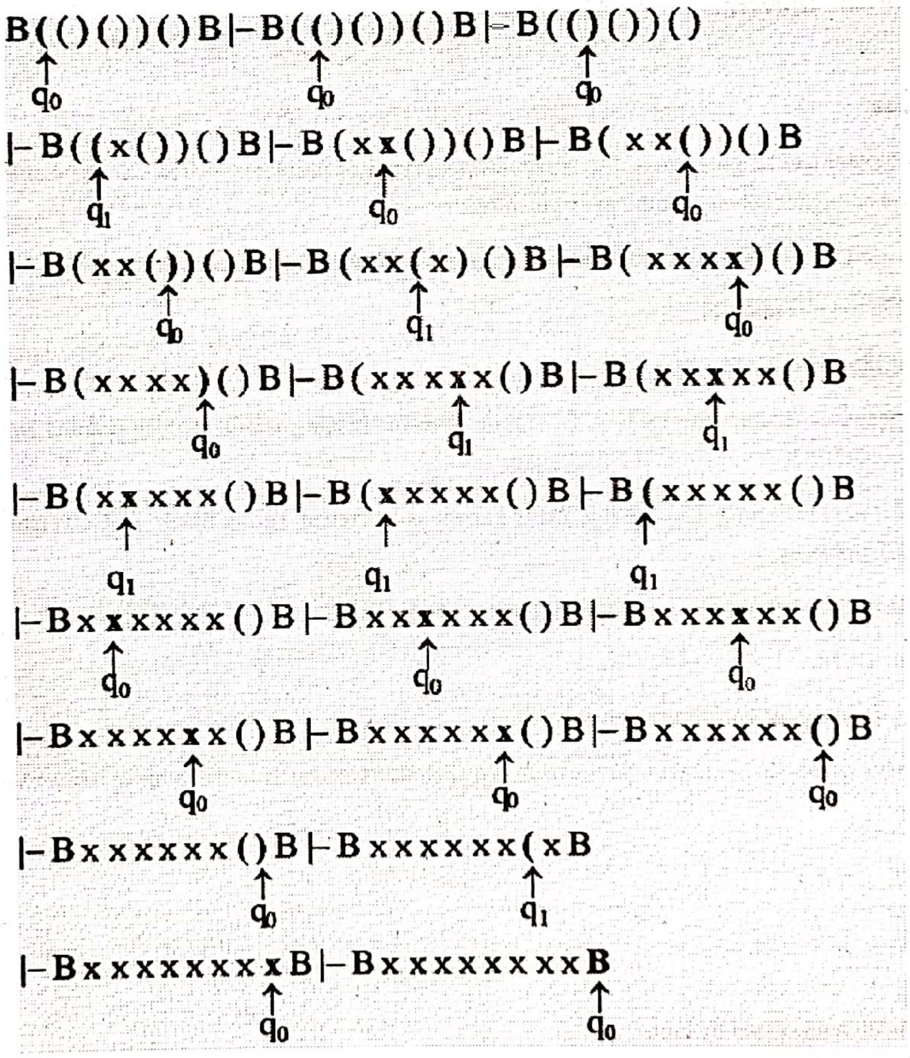
δ is given in Fig. 1-Q. 4(a)(a) or Fig. 1-Q. 4(a)(b)

q_0 = Initial state

B = Blank symbol

F = $\{q_3\}$, halting state

Making of the machine for input (()())() is given in Fig. 1-Q. 4(a)(c) :



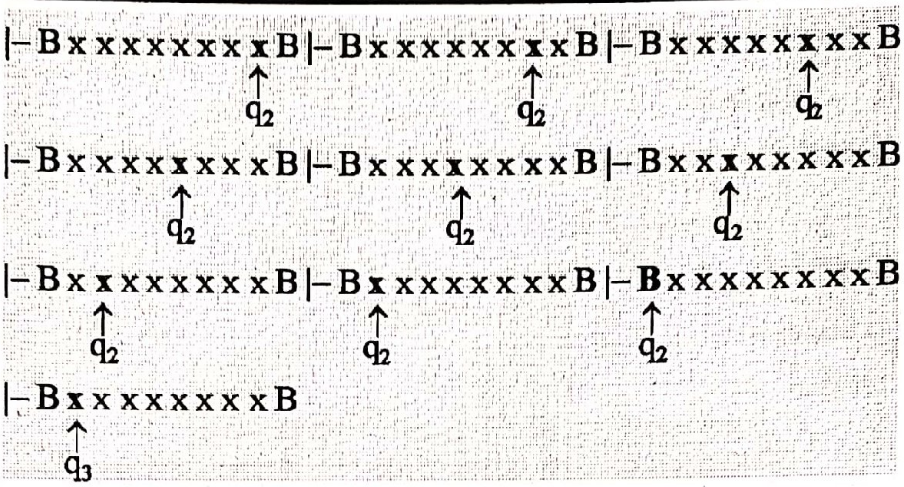


Fig. 1-Q. 4(a)(c)

Q. 6(e) Write short note on : Universal Turing Machine. **(2.5 Marks)**

Ans. :

Universal turing machine

A general-purpose computer can be programmed to solve different types of problems. A TM can also behave like a general-purpose computer. A general purpose computer solves a problem as given below :

1. A program is written in a high level language and its machine-code is obtained with the help of a complier.
2. Machine code is loaded in main memory.
3. Input to the program can also be loaded in memory.
4. Program stored in memory is executed line by line. Execution involves reading a line of code pointed by IP (instruction pointer), decoding the code and executing it.

We can follow a similar approach for a TM. Such a TM is known as Universal Turing Machine. Universal Turing Machine (UTM) can solve all sorts of solvable problems.

A Turing machine M is designed to solve a particular problem p, can be specified as :

1. The initial state q_0 of the TM M.
2. The transition function δ of M can be specified as given :

If the current state of M is q_i and the symbol under the head is a_i then the machine moves to state q_j while changing a_i to a_j. The move of tape head may be :

1. To-left,
2. To-Right or
3. Neutral

Such a move of TM can be represented by tuple

$\{(q_i, a_i, q_j, a_j, m_t) : q_i, q_j \in Q ; a_i, a_j \in \Gamma ; m_t \in$ {To- left, To-Right, Neutral}$\}$

UTM should be able to simulate every turing machine. Simulation of a Turing will involve :

1. Encoding behaviour of a particular TM as a program.
2. Execution of the above program by UTM.

A move of the form $(q_i, a_i, q_j, a_j, m_r)$ can be represented as $10^{i+1}\,10^{i}\,10^{j+1}\,10^{j}\,10^{K}$,

Where $K = 1$, if move is to the left

$K = 2$, if move is to the right

$K = 3$, if move is 'no-move'

State q_0 is represented by 0,

State q_1 is represented by 00,

State q_n is represented by 0^{n+1}.

First symbol can be represented by 0,

Second symbol can be represented by 00 and so on.

Two elements of a tuple representing a move are separated by 1.

Two moves are separated by 11.

Execution by UTM : We can assume the UTM as a 3-tape turing machine.

1. Input is written on the first tape.
2. Moves of the TM in encoded form is written on the second tape.
3. The current state of TM is written on the third tape.

The control unit of UTM by counting number of 0's between 1's can find out the current symbol under the head. It can find the current state from the tape 3. Now, it can locate the appropriate move based on current input and the current state from the tape 2. Now, the control unit can extract the following information from the tape 2 :

1. Next state 2. Next symbol to be written
3. Move of the head.

Based on this information, the control unit can take the appropriate action.

Chapter 8 : Undecidability and Recursively Enumerable Languages [Total Marks – 7.5]

Q. 1(d) Explain Halting Problem. (5 Marks)

Ans. :

Halting problem

The halting problem of a Turing machine states :

Given a Turing machine M and an input ω to the machine M, determine if the machine M will eventually halt when it is given input ω.

Halting problem of a Turing machine is unsolvable.

Proof :

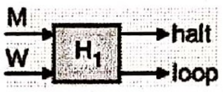
- Moves of a turing machine can be represented using a binary number. Thus, a Turing machine can be represented using a string over Σ*(0,1).
- Insolvability of halting problem of a Turing machine can be proved through the method of contradiction.

Step 1 : Let us assume that the halting problem of a Turing machine is solvable. There exists a machine H_1(say).

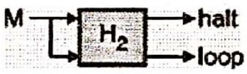
H_1 takes two inputs :

1. A string describing M.
2. An input ω for machine M.

H_1 generates an output **"halt"** if H_1 determines that M stops on input ω; otherwise H outputs "loop". Working of the machine H_1 is shown below.

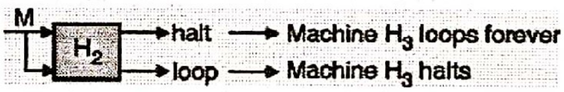


Step 2 : Let us revise the machine H_1 as H_2 to take M as both inputs and H_2 should be able to determine if M will halt on M as its input. A machine can be described as a string over 0 and 1.



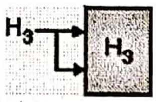
Step 3 : Let us construct a new Turing machine H_3 that takes output of H_2 as input and does the following :

1. If the output of H_2 is "loop" then H_3 halts.
2. If the output of H_2 is "halt" than H_3 will loop forever.



H_3 will do the opposite of the output of H_2.

Step 4 : Let us give H_3 itself as inputs to H_3.



If H_3 halts on H_3 as input then H_3 would loop (that is how we constructed it).

If H_3 loops forever on H_3 as input H_3 halts (that is how we constructed it).

In either case, the result is wrong.

Hence,

H_3 does not exist.

If H_3 does not exist then H_2 does not exist.

If H_2 does not exist then H_1 does not exist.

Q. 6(c) Write short note on : Rice's Theorem. (2.5 Marks)

Ans. :

Rice's theorem

"Every property that is satisfied by some but not all recursively enumerable languages is un-decidable". Any property that is satisfied by some recursively enumerable language but not all is known as non-trivial property. We have seen many properties of R.E. languages that are un-decidable. These properties include :

1. Given a TM M, is L(M) nonempty ?
2. Given a TM M, is L(M) finite ?
3. Given a TM M, is L(M) regular ?
4. Given a TM M, is L(M) recursive ?

The Rice's theorem can be proved by reducing some other unsolvable problem to non-trivial property of recursively enumerable language.

□□□

Chapter 1 : Introduction [Total Marks - 10]

Q. 5(b) Convert the following grammars to the Chomsky normal form (CNF)

S →0A0 | 1B1| BB

A →C

B →S | A

C →S | ε **(10 Marks)**

Ans. :

Step 1 : Elimination of ∈ –production.

The symbols (A, B, C, S) are nullable and hence the given granular leads to the following granular :

S → OAO | 00 | 1B1 | 11 | B | BB

A → C, B → S | A, C → S   Granular G_1

Step 2 : Resolving 2 unit productions from G_1 and also receiving non-reachable symbol C,

We get,

S → OAO | 00 | 1B1 | 11 | BB

A → OAO | 00 | 1B1 | 11 | BB   Granular G_2

B → OAO | 00 | B1 | 11 | BB′

Step 3 : All the three variables are identical and hence, the granular becomes :

S → OSO | 00 | 1S1 | 11 | SS   Granular G_3

Step 4 : Substituting A_1 for 0 and A_2 for 1, we get,

$S \rightarrow A_1SA_1 \mid A_1A_1 \mid A_2\, S\, A_2 \mid A_2\, A_2 \mid SS$

$A_1 \rightarrow 0$

$A_2 \rightarrow 1$

Step 5 : Writing productions in CNF

$S \rightarrow A_1B_1$, $B_1 \rightarrow SA_1$

$S \rightarrow A_1A_1$

$S \rightarrow A_2B_2$, $B_2 \rightarrow SA_2$

$S \rightarrow A_2A_2$

$S \rightarrow SS$

$A_1 \rightarrow 0$

$A_2 \rightarrow 1$

Chapter 2 : Finite Automata [Total Marks - 20]

Q. 1(a) Differentiate DFA and NFA. **(5 Marks)**

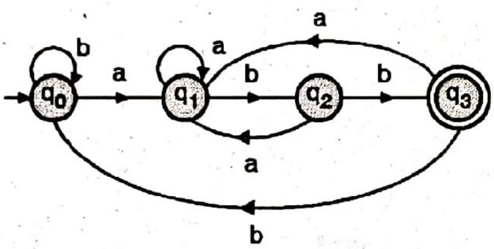
Ans. : The difference between DFA and NFA is as follows:

Sr. No.	DFA	NFA
1.	DFA stands for deterministic finite automata.	NFA stands for non-deterministic finite automata.
2.	The transition is deterministic.	The transition is non-deterministic.
3.	A deterministic finite automata is a quintuple, $M = (Q, \Sigma, \delta, q_0, F)$	A non-deterministic finite automata is a 5-tuple, $M = (Q, \Sigma, \delta, q_0, F)$
4.	The number of states is finite.	NFA can be in several states at a time.

Q. 1(b) Design a DFA to accept string of 0s and 1s ending with the string 100. **(5 Marks)**

Ans. :

The substring 'abb' should be at the end of the string. Transitions from q_3 should be modified to handle the condition that the string has to end in 'abb'.



	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3^*	q_1	q_0

(a) State transition diagram (b) State transition table

Fig. 1-Q. 1(b) : Final DFA

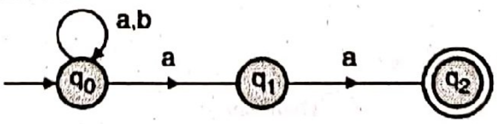
q_3 to q_1 on input a : An input of a in q_3 will make the previous four characters as 'abba'. Out of the four characters as 'abba' only the last character 'a' is relevant to 'abb'.

q_3 to q_0 on input b : An input of b in q_3 will make the previous four characters 'abbb'. Out of the four characters 'abbb', nothing is relevant to 'abb'.

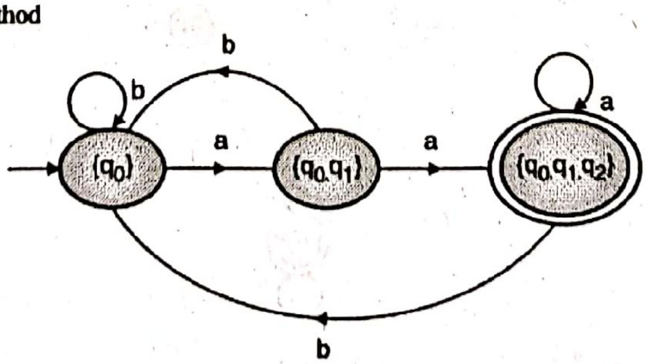
Q. 2(a) Design NFA for recognizing the strings that end in "aa" over Σ = {a,b} and convert NFA to DFA. **(10 Marks)**

Ans. :

(i) NFA for strings ending in "aa" is given below :



(ii) NFA to DFA using the direct method

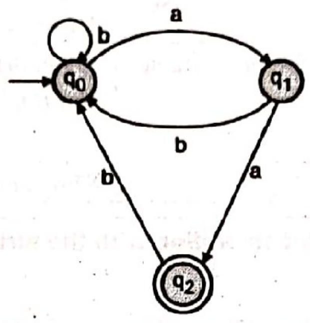


Chapter 3 : Regular Expressions and Languages [Total Marks - 15]

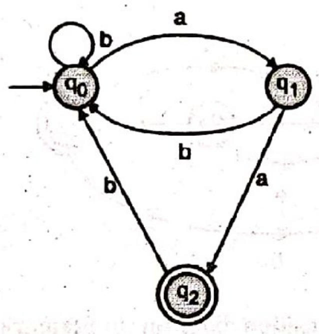
Q. 1(c) Explain the applications of regular expressions. **(5 Marks)**

Ans. : Please refer Q. 6(b) of Dec. 2018.

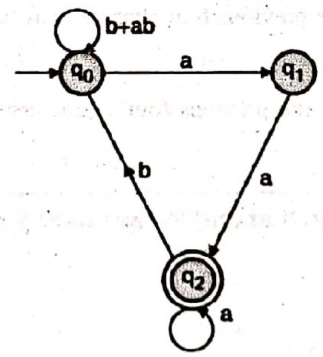
Q. 3(a) Obtain a regular expression for the FA shown below : **(10 Marks)**



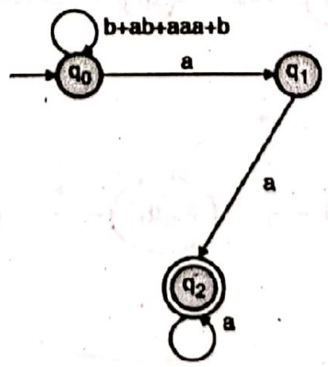
Ans. : Given FA :



Step 1 : Receiving loop between the states q_0 and q_1, we get



Step 2 : Receiving the loop among q_0, q_1 and q_2, we get



Required R. E. = $(b + ab + aaa^*b)^* \; aaa^*$

Chapter 5 : Pushdown Automata (PDA) [Total Marks - 10]

Q. 4(b) State and explain pumping lemma for context free languages. **(10 Marks)**

Ans. :

Let G be a context free grammar. Then there exists a constant n such that any string $w \in L(G)$ with $|w| \geq n$ can be rewritten as $w = uvxyz$, subject to the following conditions :

1. $|vxy| \leq n$, the middle portion is less than n.
2. $vy \neq \epsilon$, strings v and y will be pumped.
3. For all $i \geq 0$, uv^ixy^iz is in L. The two strings v and y can be pumped zero or more times.

Proof :

Let us assume that the grammar

G is given by (V, T, P, S).

$\Phi(G)$ denotes that largest number of symbols on the right-hand side of a production in P.

In pumping lemma, it is a requirement that the constant n should satisfy the following condition

$n \geq \Phi(G)^{|V-T|}$

Let us take a string $w \in L(G)$, such that $|w| \geq n$. Let us construct a parse tree T with root as S. The parse tree T generates w with smallest number of leaves.

The tree T will have a path length of at least $|V - T| + 1$. This path will have $|V - T| + 2$ nodes with the last node labelled as terminal and remaining non-terminals.

Fig. 1-Q. 4(b) shows paths in detail.

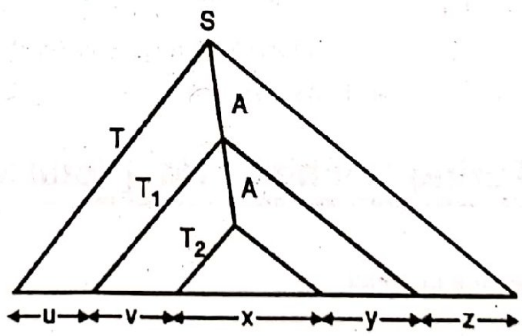


Fig. 1-Q. 4(b) : Paths in the parse tree

x is generated by T_2

v is generated by T1

u is generated by T

T_1 excluding T_2 can be repeated any number of times.

This will yield a string of the form uv^ixy^iz where $i \geq 0$

Chapter 6 : Regular Grammar [Total Marks - 10]

Q. 5(a) Design PDA for the following language :
L(M) = {wcw I w {a,b}*} where w^R is reverse of w & c is a constant. **(10 Marks)**

Ans. :

W^R stands for reverse of W. A string of the form WcW^R is an odd length palindrome with the middle character as c.

Algorithm :

If the length of the string is 2n + 1, then the first n symbols should be matched with the last n symbols in the reverse order. A stack can be used to reverse the first n input symbols.

Status of the stack and state of the machine is shown in Fig. 1-Q. 5(a). Input applied is abbcbba.

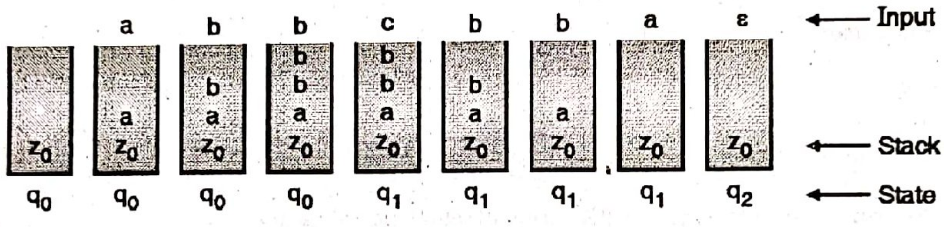


Fig. 1-Q. 5(a) : A PDA on input abbcbba

The PDA accepting through final state is given by

$M = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_2\})$

Where the transition function δ is given below :

1. $\delta(q_0, a, \varepsilon) = (q_0, a)$		First n symbols are pushed onto the stack
2. $\delta(q_0, b, \varepsilon) = (q_0, b)$		
3. $\delta(q_0, c, \varepsilon) = (q_1, \varepsilon)$		[State changes on c]
4. $\delta(q_1, a, a) = (q_1, \varepsilon)$		Last n symbols are matched with first n symbols in reverse order
5. $\delta(q_1, b, b) = (q_1, \varepsilon)$		
6. $\delta(q_1, \varepsilon, z_0) = (q_2, z_0)$		[Accepted through final state]

A transition of the form $\delta(q_0, a, \varepsilon) = (q_0, a)$ implies that always push a, irrespective of stack symbol.

Chapter 7 : Turing Machine (TM) [Total Marks - 20]

Q. 3(b) Explain the types of Turing machine in detail. **(10 Marks)**

Ans. :

The types of Turing machine are as follows :

1. Two-way infinite Turing machine

In a standard turing machine number of positions for leftmost blanks is fixed and they are included in instantaneous description, where the right-hand blanks are not included.

In the two way infinite Turing machine, there is an infinite sequence of blanks on each side of the input string. In an instantaneous description, these blanks are never shown.

2. Turing machine with multiple heads

A turing machine with single tape can have multiple heads. Let us consider a turing machine with two heads H_1 and H_2. Each head is capable of performing read/write /move operation independently.

BabaabbaBBB

↑ ↑

H_1 H_2

Fig. 1-Q. 3(b) : A Turing machine with two heads

The transition behavior of 2-head one tape Turing machine can be defined as given below :

δ (State, Symbol under H_1, Symbol under H_2) = (New state, (S_1, M_1), (S_2, M_2))

Where,

S_1 is the symbol to be written in the cell under H_1.

M_1 is the movement (L, R, N) of H_1.

S_2 is the symbol to be written in the cell under H_2.

M_2 is the movement (L, R, N) of H_2.

3. Multi-tape Turing machine

Multi-tape turing machine has multiple tuples with each tape having its own independent head. Let us consider the case of a two tape turing machine. It is shown in Fig. 2-Q. 3(b).

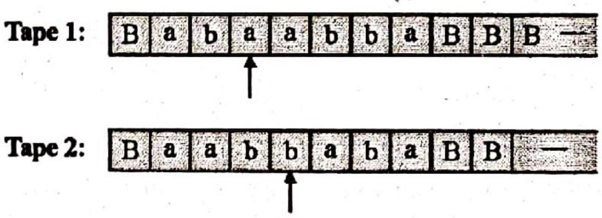


Fig. 2-Q. 3(b) : A two-tape turing machine

The transition behavior of a two-tape Turing machine can be defined as :

$$\delta(q_1,a_1,a_2) = (q_2,(S_1,M_1),(S_2,M_2))$$

Where,

q_1 is the current state,

q_2 is the next state,

a_1 is the symbol under the head on tape 1,

a_2 is the symbol under the head on tape 2,

S_1 is the symbol written in the current cell on tape 1,

S_2 is the symbol written in the current cell on tape 2,

M_1 is the movement (L, R, N) of head on tape 1,

M_2 is the movement (L, R, N) of head on tape 2.

4. Non-deterministic Turing machine

- Non-deterministic is a powerful feature. A non-deterministic TM machine might have, on certain combinations of state and symbol under the head, more than one possible choice of behaviour.
- Non-deterministic does not make a TM more powerful.
- For every non-deterministic TM, there is an equivalent deterministic TM.
- It is easy to design a non-deterministic TM for certain class of problems.
- A string is said to be accepted by a NDTM, if there is at least one sequence of moves that takes the machine to final state.
- An example of non-deterministic move for a TM is shown in Fig. 3-Q. 3(b).

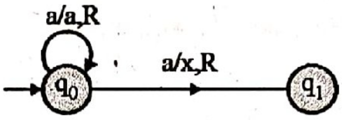


Fig. 3-Q. 3(b) : A sample move for NDTM

The transition behaviour for state q_0 for TM of Fig. 3-Q. 3(b) can be written as

$\delta(q_0, a) = \{(q_0, a, R)\ (q_1, x, R)\}$

5. Universal Turing machine

A general-purpose computer can be programmed to solve different types of problems. A TM can also behave like a general-purpose computer. A general purpose computer solves a problem as given below :

1. A program is written in a high level language and its machine-code is obtained with the help of a complier.
2. Machine code is loaded in main memory.
3. Input to the program can also be loaded in memory.
4. Program stored in memory is executed line by line. Execution involves reading a line of code pointed by IP (instruction pointer), decoding the code and executing it.

We can follow a similar approach for a TM. Such a TM is known as **Universal Turing Machine.** Universal Turing Machine (UTM) can solve all sorts of solvable problems.

A Turing machine M is designed to solve a particular problem p, can be specified as :

1. The initial state q_0 of the TM M.
2. The transition function δ of M can be specified as given :

If the current state of M is q_i and the symbol under the head is a_i then the machine moves to state q_j while changing a_i to a_j. The move of tape head may be :

1. To-left,
2. To-Right or
3. Neutral

Such a move of TM can be represented by tuple

$\{(q_i, a_i, q_j, a_j, m_r) : q_i, q_j \in Q ; a_i, a_j \in \Gamma ; m_r \in \{\text{To- left, To-Right, Neutral}\}\}$

UTM should be able to simulate every turing machine. Simulation of a Turing will involve :

1. Encoding behaviour of a particular TM as a program.

2. Execution of the above program by UTM.

A move of the form $(q_i, a_i, q_j, a_j, m_r)$ can be represented as $10^{i+1}\ 10^{i}\ 10^{j+1}\ 10^{j}\ 10^{K}$,

Where $K = 1$, if move is to the left

$K = 2$, if move is to the right

$K = 3$, if move is 'no-move'

State q_0 is represented by 0,

State q_1 is represented by 00,

State q_n is represented by 0^{n+1}.

First symbol can be represented by 0,

Second symbol can be represented by 00 and so on.

Two elements of a tuple representing a move are separated by 1.

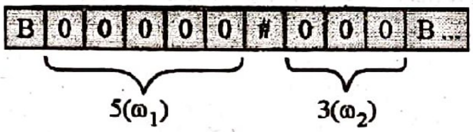
Two moves are separated by 11.

Q. 4(a) Design a turing machine that computes a function f(m,n) = m+n i.e. addition of two integers. (10 Marks)

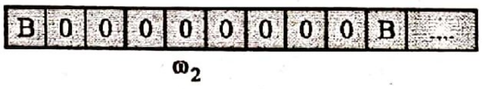
Ans. :

Addition of two unary numbers can be performed through append operation. To add two numbers 5 (say ω_1) and 3 (say ω_2) will require following steps :

1. Initial configuration of tape :



2. ω_1 is appended to ω_2.



While every '0' from ω_1 is getting appended to ω_2, '0' from ω_1 is erased. ω_2 contains 8 0's, which is sum of 5 and 3.

Chapter 8 : Undecidability and Recursively Enumerable Languages [Total Marks - 25]

Q. 1(d) What are recursive and recursively enumerable languages? (5 Marks)

Ans. :

Recusive language

A language over an alphabet Σ can be described recursively. A recursive definition has three steps :

1. Specify some basic objects in the set.
2. Specify the rules for constructing more objects from the objects already known.
3. Declaration that no objects except those constructed as given above are allowed in the set.

Recursively enumerable language

There is a difference between recursively enumerable (Turing Acceptable) and recursive (Turing Decidable) language.

Following statements are equivalent :

1. The language L is **Turing acceptable.**
2. The language L is **recursively enumerable.**

Following statements are equivalent

1. The language L is **Turing decidable.**
2. The language L is **recursive.**
3. There is an algorithm for recognizing L.

Every Turing decidable language is Turing acceptable.

Every Turing acceptable language need not be Turing decidable.

Q. 6 Write detailed note on (any two):-

(a) Post correspondence problem

(b) Halting problem

(c) Rice's theorem **(20 Marks)**

Ans. :

(a) Post correspondence problem

Let A and B be two non-empty lists of strings over Σ. A and B are given as below :

$$A = \{x_1, x_2, x_3 \dots x_k\}$$

$$B = \{y_1, y_2, y_3 \dots y_k\}$$

We say, there is a post correspondence between A and B if there is a sequence of one or more integers i, j, k ...m such that :

The string $x_i\, x_j \dots x_m$ is equal to $y_i\, y_j \dots y_m$.

Example : To check whether

$$A = \{a, aba^3, ab\} \text{ and}$$

$$B = \{a^3, ab, b\}$$

has a solution.

We will have to find a sequence using which when the elements of A and B are listed, will produce identical strings.

The required sequence is (2, 1, 1 , 3)

$$A_2\, A_1\, A_1\, A_3 = aba^3\; a\; aab = ab\; a^6\; b$$

$$B_2\, B_1\, B_1\, B_3 = aba^3\; a^3\; b = aba^6\; b$$

Thus, the PCP has solution.

We are accepting the un-decidability of post correspondence problem without proof.

(b) Halting problem

The halting problem of a Turing machine states :

Given a Turing machine M and an input ω to the machine M, determine if the machine M will eventually halt when it is given input ω.

Halting problem of a Turing machine is unsolvable.

Proof :

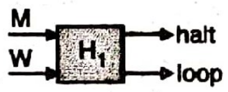
Moves of a turing machine can be represented using a binary number. Thus, a Turing machine can be represented using a string over Σ*(0,1).

Insolvability of halting problem of a Turing machine can be proved through the method of contradiction.

Step 1 : Let us assume that the halting problem of a Turing machine is solvable. There exists a machine H_1(say). H_1 takes two inputs :

1. A string describing M.
2. An input ω for machine M.

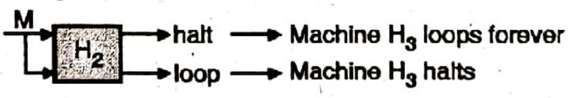
H_1 generates an output "halt" if H_1 determines that M stops on input ω; otherwise H outputs "loop". Working of the machine H_1 is shown below.



Step 2 : Let us revise the machine H_1 as H_2 to take M as both inputs and H_2 should be able to determine if M will halt on M as its input. A machine can be described as a string over 0 and 1.

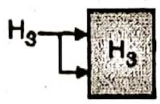
Step 3 : Let us construct a new Turing machine H_3 that takes output of H_2 as input and does the following :

1. If the output of H_2 is "loop" than H_3 halts.
2. If the output of H_2 is "halt" than H_3 will loop forever.



H_3 will do the opposite of the output of H_2.

Step 4 : Let us give H_3 itself as inputs to H_3.



If H_3 halts on H_3 as input then H_3 would loop (that is how we constructed it).

If H_3 loops forever on H_3 as input H_3 halts (that is how we constructed it).

In either case, the result is wrong.

Hence,

H_3 does not exist.

If H_3 does not exist than H_2 does not exist.

If H_2 does not exist than H_1 does not exist.

(c) Rice's theorem

Every property that is satisfied by some but not all recursively enumerable language is un-decidable. Any property that is satisfied by some recursively enumerable language but not all is known as nontrivial property. We have seen many properties of R.E. languages that are un-decidable. These properties include :

1. Given a TM M, is L(M) nonempty?
2. Given a TM M, is L(M) finite?
3. Given a TM M, is L(M) regular?
4. Given a TM M, is L(M) recursive?

The Rice's theorem can be proved by reducing some other unsolvable problem to nontrivial property of recursively enumerable language.

Dec. 2018

Q. 1 (a) Explain Chomsky Hierarchy. **(5 Marks)**

(b) Differentiate between PDA and NPDA. **(5 Marks)**

(c) Define Regular Expression and give regular expression for :

(i) Set of all strings over {0, 1} that end with 1 has no substring 00 **(5 Marks)**

(d) Explain Halting Problem. **(5 Marks)**

Q. 2 (a) Design a Finite State machine to determine whether ternary number (base 3) is divisible by 5. **(10 Marks)**

(b) Give and explain formal definition of Pumping Lemma for Regular Language and prove that following language is not regular. $L = \{a^m b^{m-1} \mid m > 0\}$ **(10 Marks)**

Q. 3 (a) Construct PDA accepting the language $L = \{a^{2n} b^n \mid n \geq 0\}$. **(10 Marks)**

(b) Consider the following grammar

S → i C t S | i C t S e S | a

C → b

For the string 'ibtaeibta' find the following :

(i) Leftmost derivation

(ii) Rightmost derivation

(iii) Parse tree

(iv) Check if above grammar is ambiguous. **(10 Marks)**

Q. 4 (a) Construct TM to check well-formedness of parenthesis. **(10 Marks)**

(b) Convert following CFG to CNF **(10 Marks)**

S → ASA | Ab

A → B | S

B → b | ∈

Q. 5 (a) Convert $(0 + 1)(10)^*(0 + 1)$ into NFA with ∈-moves and obtain DFA. **(10 Marks)**

(b) Construct Mealy and Moore Machine to convert each occurrence of 100 by 101. **(10 Marks)**

Q. 6 Write short note on (any four) **(10 Marks)**

(a) Closure properties of Context Free Language.

(b) Applications of Regular expression and Finite automata.

(c) Rice's Theorem.

(d) Mealy and Moore Machine

(e) Universal Turing Machine

May 2019

Q. 1 (a) Differentiate DFA and NFA. **(5 Marks)**

(b) Design a DFA to accept string of 0's and 1's ending with the string 100. **(5 Marks)**

(c) Explain the applications of Regular Expressions. **(5 Marks)**

(d) What are Recursive and Recursively Enumerable Languages? **(5 Marks)**

Q. 2 (a) Design NFA for recognizing the strings that end in "aa" over Σ ={a,b} & convert above NFA to DFA. **(10 Marks)**

(b) Design moore m/c for following :

If input ends in '101' then output should be A, if input ends in '110' output should be B, otherwise output should be C and convert it into mealy m/c. **(10 Marks)**

Q. 3 (a) Obtain a regular expression for the FA shown below : **(10 Marks)**

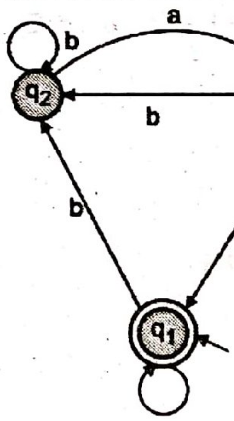


Fig. 1 Q. 3(a)

(b) Explain the types of Turing machine in detail. **(10 Marks)**

Q. 4 (a) Design a turing machine that computes a function f(m,n) = m + n i.e. addition of two integers. **(10 Marks)**

(b) State and explain pumping Lemma for Context Free Languages. Find out whether the language $L = \{x^n y^n z^n \mid n \geq 1\}$ is context free or not. **(10 Marks)**

Q. 5 (a) Design PDA for the following language :

$L(M) = \{wcw^R \mid w \{a,b\}^*\}$ where w^R is reverse of w & c is a constant. **(10 Marks)**

(b) Convert the following Grammars to the Chomsky normal form (CNF).

S →0A0 | 1B1| BB

A →C

B →S | A

C →S | ε **(10 Marks)**

Q. 6 Write detailed note on (any two) : **(20 Marks)**

(a) Post Correspondence Problem

(b) Halting Problem.

(c) Rice's Theorem.

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