08/05/13 Sem III
IT. Maths (III)
Applied Maths - III

1st Half-13-Mina - (b)-69

Con. 6414-13. **GS-6132**

(3 Hours) [**Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** questions.

1. (a) If $L\{f(t)\} = \overline{F}(s)$, then prove that $L\left\{\int_0^t f(u)\,du\right\} = \frac{\overline{F}(s)}{s}$ hence find : **5**

$$L\left\{\int_0^t \sin u \cos 2u \, du\right\}.$$

(b) Expand Fourier series for $f(x) = |x|$ in $(-1, 1)$. **5**

(c) Evaluate $\int |z|\,dz$ along the left half of the unit circle $|z| = 1$ from $z = -i$ to i. **5**

(d) Show that every square matrix can be uniquely expressed as the sum of a Harmitian matrix and Skew-Harmitian matrix. **5**

2. (a) Find half range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$ hence deduce that : **6**

$$\sum \frac{(-1)^n}{(2n-1)^3} = \frac{\pi^3}{32}.$$

(b) Find the rank of the matrix by reducing it to normal form : **6**

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 3 & 1 & 2 \\ 1 & 0 & 3 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix}.$$

(c) Find Laplace transform of the following :— **8**

(i) $\frac{e^{-2t} \cos 2t \sin 3t}{t}$

(ii) $t\int_0^u e^{-2u} \cos^2 u \, du$

[TURN OVER

3. (a) Show that the set of functions cos nx, n = 1, 2, 3... is orthogonal on $(0, 2\pi)$. 6

(b) Use adjoint method to find the inverse of 6

$$\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}, \text{ if exist.}$$

(c) Find inverse Laplace transform of :— 8

(i) $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$

(ii) $e^{-4s}\frac{s}{(s+4)^3}$.

4. (a) If $V = 3x^2y + 6xy - y^3$, show that V is harmonic and find its corresponding analytic function. 6

(b) Find a, b, c, if $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ is orthogonal. 6

(c) Find Fourier series for $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$ hence deduce that : 8

$$\frac{1}{2} = \sum_{1}^{\infty}\frac{1}{4n^2-1}.$$

5. (a) Solve the system of equations, if consistent, 6

$$x - 2y + 3t = 2$$
$$2x + y + z + t = 4$$
$$4x - 3y + z + 7t = 8.$$

(b) Find inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ by using Convolution theorem. 6

(c) Obtain Laurentz and Taylor's series expansion of $f(z) = \frac{(z+1)}{(z+2)(z+3)}$ indicating region of convergence. 8

6. (a) Evaluate $\oint_c \frac{(z+1)}{(z^2-1)(z-3)} \cdot dz$ where : **6**

   (i) $|z - 1| = 1$
   (ii) $|z + 1| = 1$,
   (iii) $|z - 3| = 1$.

   (b) Solve $(\theta^2 - 2\theta + 3)y = \cos 2t$ with $y(0) = 1$, $y'(0) = 0$ by using Laplace transform. **6**

   (c) Expand Fourier series for $f(x) = \begin{cases} x & 0<x<1 \\ 1-x & 1<x<2 \end{cases}$ **8**

7. (a) For the following matrix A, find non-singular matrices P and Q such that PAQ is a normal form and hence find Rank of A, **6**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 2 \\ 3 & 0 & 7 & 3 \end{bmatrix}.$$

   (b) Find Laplace transform of $f(t) = \begin{cases} \sin t & 0<t<\pi \\ \cos t & \pi<t<2\pi \end{cases}$. **6**

   (c) Find real part of of Analytic function whose imaginary part is : **8**

$$V = x^2 + \frac{x}{x^2 + y^2} - y^2.$$