

Roll No.

67011

**M.C.A. Ist Sem. w.e.f. Dec. 2011
(with old notes)**

Examination-December, 2014

**Mathematical Foundation
of Computer Science**

Paper-MCA-101

Time : 3 hours

Max. Marks : 80

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all, selecting at least **one** question from each unit. All questions carry equal marks.

Unit-I

1. (a) Define properties of relation. Consider a relation R on the set $A = \{a, b, c, d\}$
 $R = \{(a, b), (b, b), (b, c), (c, a), (c, c), (d, a)\}$. Find 8

(i) Reflexive closure of R

(ii) Symmetric closure of R

- (b) Let $f(x) = \frac{ax}{x+1}$, $x \neq -1$. If $(f \circ f)(x) = x$, find the value of a . 8

2. (a) Let Z be the set of integers, show that the operation $*$ on Z , defined by $a*b = a + b + 1$ for all $a, b \in Z$ satisfies the closure property, associative law and the commutative law. Also find the identity element and inverse of an integer. 8

- (b) State and prove Lagrange's theorem. 8

Unit-II

3. (a) Let P, Q, R be the prepositions 4

P: Today is Monday

Q: It is raining

R: It is cold

Convert the following formula in good English sentences :

(i) $\sim Q \rightarrow (R \wedge P)$

(ii) $\sim (P \vee Q) \leftrightarrow R$

(b) Show that the formula $(P \wedge \sim Q) \wedge (P \leftrightarrow Q)$ is contradiction or not. 4

(c) Using logical equivalence formulas show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology. 4

- (d) Represent the following argument symbolically and determine whether the argument is valid or not. 4

If today is Pooja birthday, then today is July 12.

Today is Pooja birthday.

\therefore Today is July 12.

4. (a) Using principle of mathematical induction, prove that for all $n \in \mathbb{N}$, $3^{2n+2} - 8n - 9$ is divisible by 64. 8
- (b) Show that $((P \rightarrow Q) \rightarrow R) \leftrightarrow ((P \rightarrow R) \vee (Q \rightarrow R))$ is a tautology or not. 8

Unit-III

5. (a) Determine whether each of the poset $\{1, 2, 3, 4, 5\}$ and $\{1, 2, 4, 8, 16\}$ is a lattice under the relation 'divides'. Also draw the Hasse diagram. 8

- (b) Define complemented lattice. Find out whether the lattice $(D_{12}, \text{gcd}, \text{lcm})$ is complemented, where D_{12} is the set of all factors of 12 under divisibility. 8

6. (a) In the Boolean algebra $(B, +, \cdot, /)$, show that

(i) $a + b + c.a' = a + b + c$ for all $a, b, c \in B$ 4

(ii) $a.b + c.(a' + b') = a.b + c$ for all $a, b, c \in B$ 4

(b) (i) Prove that in Boolean algebra unit element is unique. 4

(ii) Simplify the Boolean expression $a[b + c(ab + ac)']$ 4

Unit-IV

7. (a) Given the string $u = a^2b$ and $v = bab^2$, find the strings uv , v^2 , $\lambda \cup v$, $u \cup v$. Also find their length. 4
- (b) Let $A = \{a, b\}$. Describe the following language over A 4
- (i) $L_1 = \{a, ab, ab^2, ab^3, \dots\}$
- (ii) $L_2 = \{a^m b^n : m > 0, n > 0\}$
- (c) Write short note on regular expression. 4
- (d) Explain Chomsky Hierarchy with an example. 4
8. (a) Consider a Non-deterministic Finite State Automation (NFA) whose transition function is given in the table. Let $S = \{s_0, s_1, s_2\}$, $F = \{s_1\}$, $\Sigma = \{0, 1\}$ 8

Transition Function Table

		δ	
		0	1
S	Σ		
	$\rightarrow s_0$	{s1}	{s0}
	s1	{s2}	{s1, s2}
	s2	{s2}	{s2}

Construct a transition diagram for NFA and DFA equivalent to NFA.

- (b) Explain Mealy machine with the help of an example. 8