

#### **Time: 3hours**

Max. Marks: 80

## Answer any FIVE questions All Questions Carry Equal Marks

1. Describe Source Panel Method for the numerical solution of two-dimensional incompressible flow over arbitrary bodies. Explain how boundary conditions are implemented and velocity field and pressure distribution obtained from the method.

[16]

- 2. Describe Vortex Panel Method for computing velocity field and pressure distribution of two-dimensional incompressible flow over an airfoil. Elaborate on how Kutta condition is implemented at trailing edge. [16]
- 3. Consider the full velocity potential equation for the steady, two-dimensional supersonic flow given by

$$(1-u^2/a^2) \Phi_{xx} - 2uv/a^2 \Phi_{xy} + (1-v^2/a^2) \Phi_{yy} = 0$$

where a and  $\Phi$  are the speed of sound and full-velocity potential, and u,v are velocity components. Derive the compatibility equations  $\theta + \nu = K_{-}$  (along the C<sub>-</sub> characteristic) and  $\theta - \nu = K_{+}$  (along the C<sub>+</sub> characteristic) [16]

4. (a) Explain explicit MaCormack Technique for a steady, two-dimensional, supersonic, inviscid flowfield in (x,y) space using the following generic conservation form without source terms

 $\partial F/\partial x = - \partial G/\partial y$ 

where F and G represent flux vectors formed from the governing equations.(b) Discuss the advantages and disadvantages of explicit and implicit methods. [10+6]

5. (a) Explain Area Rule and Supercritical Airfoil and illustrate through a schematic their effects on drag rise in transonic flows

(b)Derive the Transonic Small Disturbance equation

 $(1-M_{\infty}^{2})\phi_{xx} + \phi_{yy} + \phi_{zz} = M_{\infty}^{2} [(\gamma+1) (\phi_{x} / V_{\infty})] \phi_{xx}$ where  $\phi$  is perturbed velocity potential and  $M_{\infty}$  is free stream Mach number. [8+8]

- 6. Describe Blasius solution for incompressible two-dimensional boundary layer flow over a flat plate. Discuss on similarity transformation used, boundary conditions adopted and procedure used for the numerical solution of Blasius equation. [16]
- 7. Explain explicit Euler's Forward Time and Backward Space (FTBS) scheme for the wave equation  $\partial u/\partial t + a \partial u/\partial x = 0$ , a > 0 and conduct its stability analysis using Von Neumann stability method. [16]
- 8. Discuss MacCormack predictor-corrector scheme for one-dimensional wave equation  $\partial u/\partial t + a \partial u/\partial x = 0$ , a > 0 and comment on its stability and indicate its accuracy. [16]



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- What are lower order and higher order panel methods? Discuss how source panel method 1. can be used to study incompressible potential flow past arbitrary bodies. [16]
- 2. Describe Vortex Panel Method for the numerical solution of two-dimensional incompressible flow over an airfoil. Discuss on how Kutta condition is implemented at the trailing edge and how the inherently over-determined system of algebraic equations for vortex panel strengths are solved to get velocity field and pressure distribution.

[16]

(a) What are characteristic lines? Explain the philosophy of the Method of 3. Characteristics.

(b) Consider the full velocity potential equation for the steady, two-dimensional supersonic flow given by (1-u<sup>2</sup>/a<sup>2</sup>)  $\Phi_{xx}$  -2uv/a<sup>2</sup>  $\Phi_{xy}$  +(1-v<sup>2</sup>/a<sup>2</sup>)  $\Phi_{yy}$  = 0

where a and  $\Phi$  are the speed of sound and full-velocity potential, and u,v are velocity components. Determine the equation for characteristic curves in the physical xy space.and classify the nature of above velocity potential equation based on Mach number. [6+10]

- 4. Explain through a schematic diagram how to design minimum length rocket nozzle using method of characteristics. [16]
- 5. Explain the following with illustrations
  - Shock fitting versus shock capturing (i)
  - (ii) Conservation versus non-conservation forms of governing equations of fluid mechanics. [8+8]
- 6. Derive the boundary layer equations from the Navier-Stokes equations in the following form and explain the boundary conditions. State the assumptions made.

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{v} \frac{\partial \mathbf{u}}{\partial \mathbf{y}} = \frac{\partial \mathbf{U}}{\partial \mathbf{t}} + \mathbf{U} \frac{\partial \mathbf{U}}{\partial \mathbf{x}} + \mathbf{v} \frac{\partial^2 \mathbf{u}}{\partial \mathbf{y}^2}$$

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} = 0$$
[16]

- 7. Describe Forward-Time/Central-Space scheme (FTCS) for the unsteady diffusion equation  $\partial u/\partial t$  - a  $\partial^2 u/\partial x^2 = 0$ , a > 0 and conduct its stability analysis using Von Neumann stability method. [16]
- 8. Discuss Lax-Wendroff splitting scheme for one-dimensional wave equation  $\partial u/\partial t + a$  $\partial u/\partial x = 0$ , a > 0 and comment on its stability and indicate its accuracy. [16]



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# Answer any FIVE questions All Questions Carry Equal Marks

- 1. What are panel methods and what are their advantages and disadvantages? How they are used to study non-lifting incompressible potential flows past bodies? [16]
- 2. Explain Vortex Panel Method for the numerical solution of two-dimensional incompressible flow over an airfoil. Discuss how the inherently over-determined system of algebraic equations for vortex panel strengths are solved to get velocity field and pressure distribution [16]
- 3. (a) Describe Unit Processes associated with Method of Characteristics using the compatibility equations θ + v = K<sub>-</sub> (along the C<sub>-</sub> characteristic) and θ v = K<sub>+</sub> (along the C<sub>+</sub> characteristic).
  (b) Explain the Domain of Dependence and Pange of Influence with reference to

(b) Explain the Domain of Dependence and Range of Influence with reference to supersonic flows [10+6]

4. (a) Explain explicit MaCormack Technique for a steady, two-dimensional, supersonic, inviscid flowfield in (x,y) space using the following generic conservation form without source terms

#### $\partial F/\partial x = - \partial G/\partial y$

where F and G represent flux vectors formed from the governing equations.(b) Discuss the advantages and disadvantages of explicit and implicit methods. [10+6]

- (a) What are transonic flows and why they are important for engineering applications? What are the reasons for drag rise in transonic flows?
  (b) Explain different ways of delaying and reducing the extent of drag rise in transonic flows. [8+8]
- 6. Explain the concept of boundary layer as propounded by Ludwig Prandtl. Derive boundary layer equations for two-dimensional incompressible flow over a flat plate and comment on the validity of these equations at the leading edge. [16]
- 7. Explain explicit Euler's Forward Time and Backward Space (FTBS) scheme for the wave equation  $\partial u/\partial t + a \partial u/\partial x = 0$ , a > 0 and conduct its stability analysis using Von Neumann stability method. [16]
- 8. Show that Alternating Direction Implicit (ADI) formulation is an approximate factorization of the Crank-Nicolson scheme  $\partial u/\partial t = a (\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2) = 0.$  [16]



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## Answer any FIVE questions All Questions Carry Equal Marks

1. What are source panel methods and where they are applied? Describe the steps involved in applying source panel method to solve incompressible potential flow past bodies.

[16]

- 2. Discuss Vortex Panel Method for computing velocity field and pressure distribution of two-dimensional incompressible flow over an airfoil. Explain how Kutta condition is implemented at trailing edge. [16]
- 3. (a) Describe Unit Processes associated with Method of Characteristics using the compatibility equations θ + ν = K<sub>-</sub> (along the C<sub>-</sub> characteristic) and θ ν = K<sub>+</sub> (along the C<sub>+</sub> characteristic).
  (b) Explain the Shock capturing and Shock fitting techniques used to handle shocks in the numerical solution of inviscid supersonic flows. [10+6]
- 4. Describe with the help of a schematic diagram the design process for the nozzle contour used in supersonic wind tunnels by the method of characteristics. [16]
- 5. (a) Explain different ways of controling transonic wave drag rise (b) Derive the Transonic Small Disturbance equation  $(1-M_{\infty}^{2})\phi_{xx} + \phi_{yy} = M_{\infty}^{2} [(\gamma+1) (\phi_{x} / V_{\infty})] \phi_{xx}$

where  $\varphi$  is perturbed velocity potential and  $M_{\infty}$  is free stream Mach number. [8+8]

- 6. Starting with the Navier-Stokes equations for two-dimensional incompressible flow along a wall, derive the boundary layer equations and state the boundary conditions to be satisfied. [16]
- 7. Explain Lax method and mid-point leapfrog method for the wave equation  $\partial u/\partial t + a$  $\partial u/\partial x = 0$ , a > 0 and comment on the stability of these schemes [16]
- 8. Discuss Lax-Wendroff splitting scheme for one-dimensional wave equation  $\partial u/\partial t + a$  $\partial u/\partial x = 0$ , a > 0 and comment on its stability and indicate its accuracy. [16]