GRAPH THEORY.

* Definition of a Graph

A Graph G is a finite non-empty set of vertices V with a set of edges E which consists of pairs of vertices.

i.e., $G = (V, E)$ where

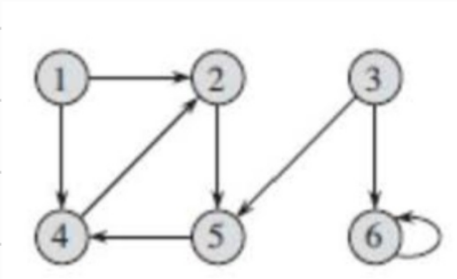
$V \rightarrow$ Set of vertices

$E \rightarrow$ set of edges of the form (x, y) such that $x, y \in V$.

* Directed Graph :-

A graph in which edges have directions. Thus the edge set is a set of ordered pairs of vertices.

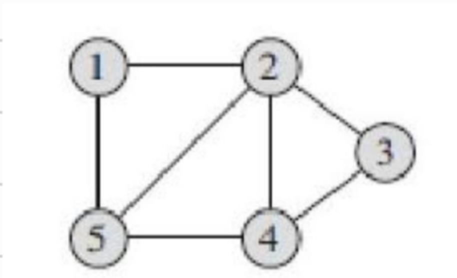
In this example, the vertex set $V = \{1, 2, 3, 4, 5, 6\}$, edge set $E = \{(1,2), (1,4), (2,5), (3,5), (3,6), (4,2), (5,4), (6,6)\}$



* Undirected Graph :-

A graph in which edges do not have directions

In this example, vertex set $V = \{1, 2, 3, 4, 5\}$, edge set $E = \{(1,2), (1,5), (2,3), (2,4), (2,5), (3,4), (4,5)\}$



Note :- i) Since directions are not given we can write an edge as $(1,2)$ or $(2,1)$.

ii) Vertices are also called as Nodes.

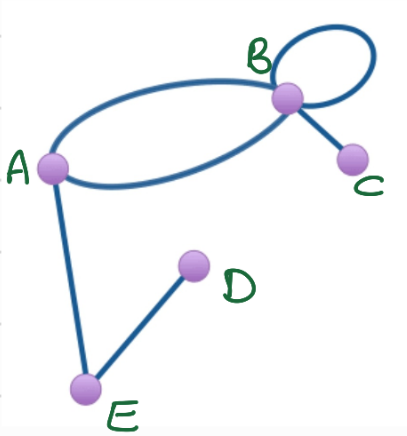
* Adjacent vertices
The vertices with which a given vertex forms an edge.

* Degree of a vertex
→ In undirected graph, degree of a vertex is the number of edges formed by it
→ In directed graph, if we have an edge from A to B, then A is called as the Source and B is called as Sink.
Here, indegree of a vertex is the no. of edges coming towards it (sink) and outdegree is the no. of edges going away from it. (source vertex)

* Parallel Edges :-
multiple edges between same pair of vertices

* Loops :-
an edge from a vertex to itself.

Ex :- Here, we have parallel edges between A and B and there is a loop at vertex B.

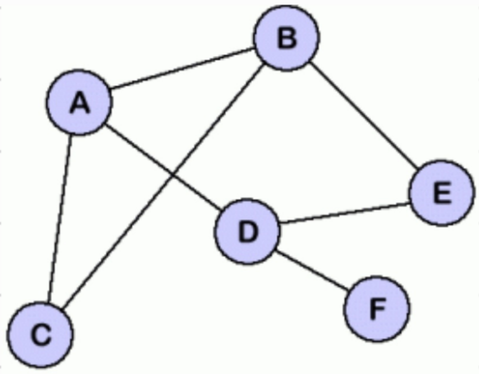


Degrees of the vertices :-

deg(A) = 3, deg(D) = 3

deg(B) = 3, deg(E) = 2

deg(C) = 2, deg(F) = 1



Indegree → d^+ , outdegree → d^-

$d^+(1) = 0$ $\quad$ $d^-(1) = 1$

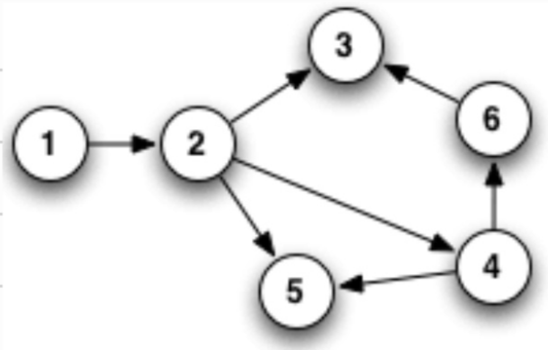
$d^+(2) = 1$ $\quad$ $d^-(2) = 3$

$d^+(3) = 2$ $\quad$ $d^-(3) = 0$

$d^+(4) = 1$ $\quad$ $d^-(4) = 2$

$d^+(5) = 2$ $\quad$ $d^-(5) = 0$

$d^+(6) = 1$ $\quad$ $d^-(6) = 1$



Theorem 1 :- Handshaking Theorem

In a graph, the sum of degrees of all vertices is equal to twice the no. of edges.

Note :- i) Sum of degrees of all vertices is always even.

ii) The no. of odd vertices in a graph is even

Types of Vertices :-

1. Isolated Vertex :- vertex with degree 0.
2. Pendant Vertex :- Vertex with degree 1
3. Odd Vertex :- vertex with odd degree

4. Even Vertex :- vertex with even degree

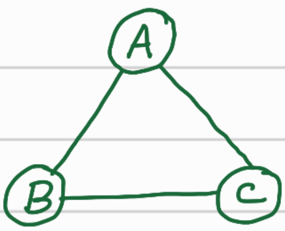
5. Source/Sink vertex :- In a directed edge, A is the source vertex and B is the sink vertex.

Types of Graphs :-

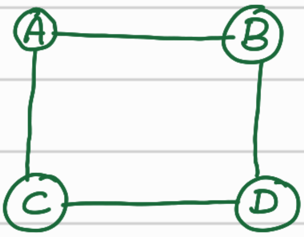
1. <u>Simple Graph</u> : A graph without parallel edges and loops.

2. <u>Point Graph</u> : A graph with only one vertex & 0 edges

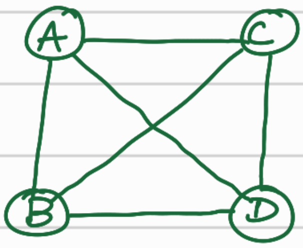
3. <u>Regular Graph</u> : A graph in which degree of all vertices are equal to r. It is called r-regular graph.



2-regular graph



2-regular graph.



3-regular graph.

4. <u>Connected Graph</u> :- A graph in which we have a path from one vertex to any other vertex.

5. <u>Complete Graph</u> :- A graph in which each vertex is connected with every other vertex. (or)
It is a graph with maximum no. of edges. It is represented using K_n.

Note :- In a simple graph with n vertices, the maximum no. of edges is $\frac{n(n-1)}{2}$

6. Planar Graph: A graph which can be drawn in a plane such that there are no intersecting edges.

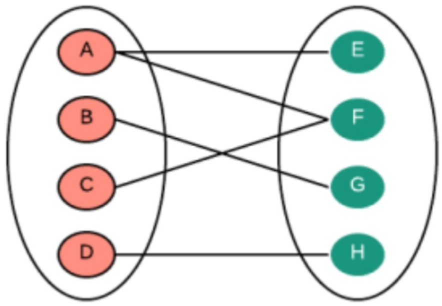
7. Bipartite Graph: A graph in which vertex set V is divided into two sub-vertex sets V_1 & V_2 and the edges are formed with one vertex from each V_1 & V_2.

In this example,

$V = \{A, B, C, D, E, F, G, H\}$

$V_1 = \{A, B, C, D\}$ & $V_2 = \{E, F, G, H\}$

edges are formed one vertex each from sets V_1 & V_2.

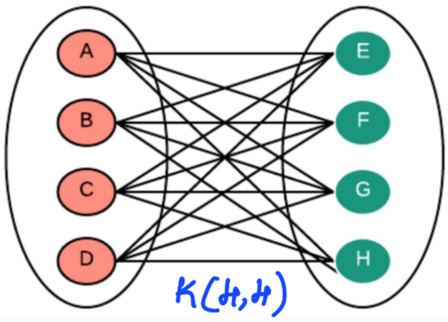


8. Complete Bipartite Graph: A bipartite graph in which each vertex from one sub-vertex set is connected with every vertex from second sub-vertex set.

It is represented as $K_{(p,q)}$ where

$p \rightarrow$ no. of vertices in V_1

$q \rightarrow$ no. of vertices in V_2.



For Video explanation, Subscribe to our YouTube Channel: ScholarD Career Solutions.

- Compiled By Usha.