Unit - IV

Scheduling by PERT and CPM

Introduction :-

A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal. A large and complex project involves usually a number of interrelated activities requiring men, machines and materials. It is impossible for the management to make and execute an optimum schedule for such a project just by intuition, based on the organisational capabilities and work experience. A systematic "scientific approach has become a necessity for such projects. So a number of methods applying network scheduling techniques has been developed.

Programme Evaluation Review Technique (PERT) and Critical Path Method (CPM) are two of the many network techniques which are widely used for planning, scheduling and controlling.

Phases of project Management:-

The three main managerial functions for any project are:

1. Planning
2. Scheduling
3. Control.

Planning:- This phases involves listing of tasks or jobs that must be performed to complete a project under consideration. In this phase, men, machines and material required for the project in addition to the estimates of costs and duration of various activities of the project are also determined.

Scheduling:- This phase involves the laying out of actual activities of the projects in a logical sequence of time in which they have to be performed. Also, start and finish times for each activity, critical path on which activities require special attention and slack (or) float for the non-critical paths are determined.

Control: This consists of reviewing the progress of the project and whether the actual performance is according to the planned schedule and finding the reasons for the difference, if any, between the schedule and the performance. The basic aspect of control is to analyse and correct this difference by taking remedial action wherever possible.

Differences between PERT and CPM

PERT

1. Since PERT was developed in connection with an R & D work, therefore, it had to cope with the uncertainties which are associated with such activities. In PERT, the total project is regarded as a random variables and therefore, associated probabilities are calculated so as to characterise it.

2. It is an event — oriented network because in the analysis of network, emphasis is given to important stages of completion of task rather than to the activities required to be performed to reach a particular event or task.

3. PERT is normally used for projects involving activities of non-repetitive nature in which time estimates are uncertain.

4. It helps in pinpointing critical areas in a project so that necessary adjustments can be made to meet the scheduled completion date of the project.

CPM:

1. Since CPM was developed in connection with a construction project which consisted of routine tasks whose resource requirements and duration were known with certainty, it is basically deterministic.
2. CPM is suitable for establishing a trade-off for optimum balancing between schedule time and costs of the project
3. CPM is used for projects involving activities of a repetitive nature.

Network construction

Some basic Definitions:

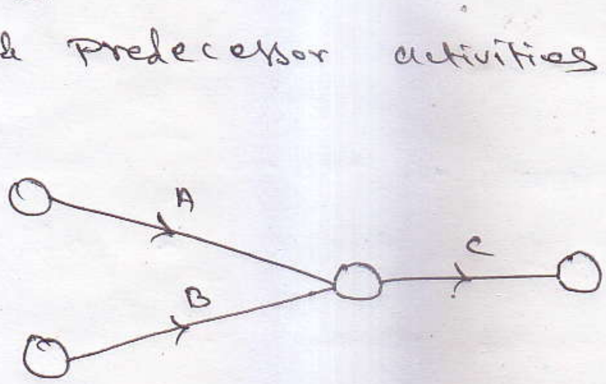
Activity An activity is a task or an item of work to be done in a project. It consumes resources like time, money and labour. etc

An activity is represented by an arrow with a node at the begining and a node at the end indicating the start and termination of the activity. Nodes are denoted by circles.

If A is the activity whose initial node as i and the terminal node is j, ~~then its~~

Predecessor activity:

Activities that must be completed immediately prior to the start of another activity are called predecessor activities



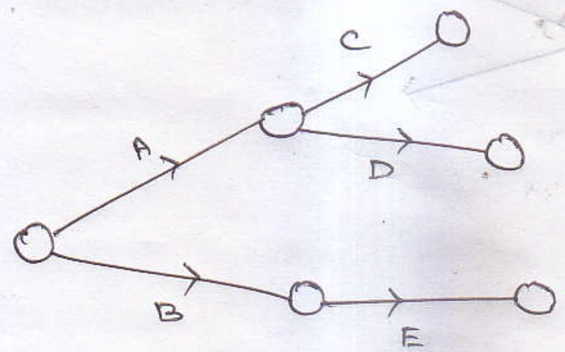
Here, A and B are predecessors of C and these are denoted by $A < C$; $B < C$.

Successor activity:-

Activities that cannot be started until one or more of other activities are completed, but immediately succeed them are called successor activities. See the above figure. C is a successor of A and B. this is denoted by $C > A$ and $C > B$.

Concurrent or Parallel activities:-

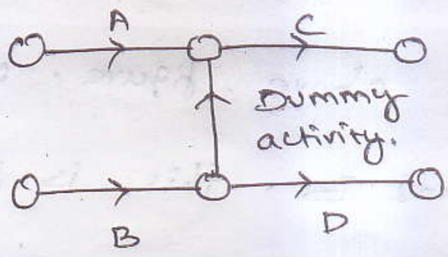
Activities which can be accomplished concurrently are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of the other activities.



A and B are parallel activities; C and D are parallel activities.

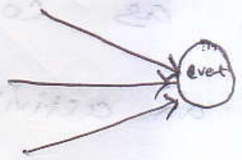
Dummy activity:-

An activity which does not consume any kind of resource but merely serves the purpose of indicating the predecessor or successor relationship clearly in network is called as a dummy activity.



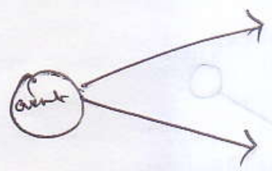
Merge event:-

When more than one activity joins an event. is known as it merge event.



Burst event:-

When more than one activity leaves an event, such an event is known as burst event



Some important Tips for Drawing a Good Network.

1. Avoid crossing of arrows.

2. Use straight arrows.

3. Do not attempt to represent the duration of an activity by its arrow length.

4. Use arrows from left to right. Avoid mixing two directions; vertical and horizontal arrows may be used if necessary.

5. Use dummies freely in rough draft but final network should not have any redundant dummies.

6. The network has only one entry point called the start event, and one point of emergence, called the end event.

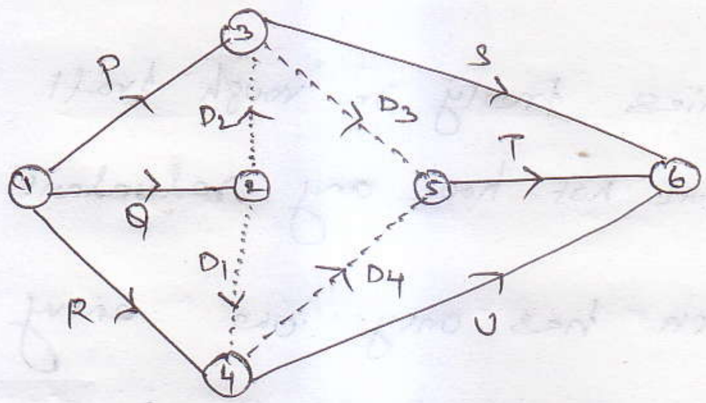
Errors in a Network

1. Looping

2. Dangling. To disconnect an activity before the completion of all activities in a network diagram is known as dangling.

(1) Draw the network for the project whose activities and their precedence relationships are given below.

Activity :	P	Q	R	S	T	U
predecessor :	–	–	–	P, Q	P, R	Q, R



D1, D2, D3, D4 are dummy activities.

Critical Path method (CPM)

Critical activity: Since the difference between the latest start time and earliest start time of an activity is usually called a total float, the activities with zero total float are known as critical activities.

Critical Path :-

Path, connecting the first initial node to the very last terminal node, of longest duration. In any project network is called the critical path.

Prob 1

Compute the earliest start, earliest finish latest start and latest finish of each activity of the project given below.

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days)	8	4	10	2	5	3

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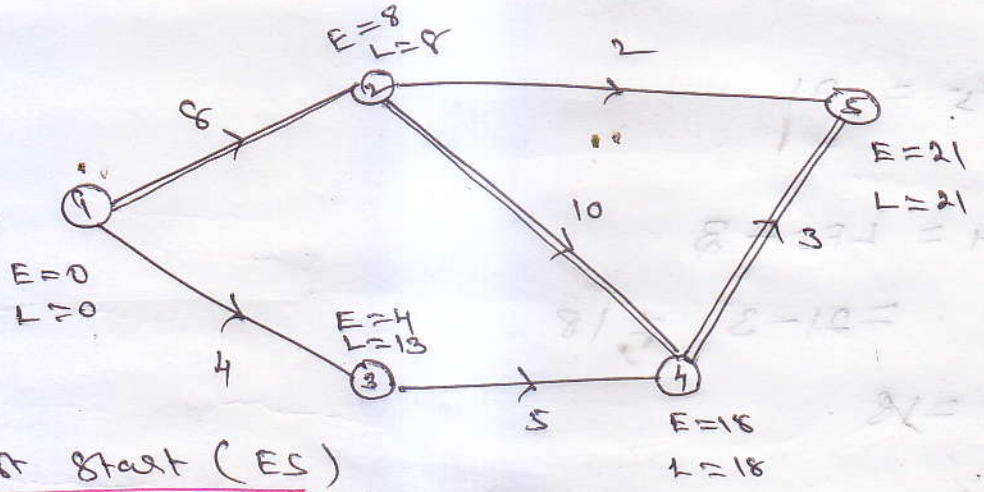


Earliest Start (ES)

$ES_1 = 0$

$ES_2 = ES_1 + 8 = 0+8 = 8$

$ES_3 = ES_1 + 4 = 0+4 = 4$

$$ES_4 = \max\{ES_2 + 10,\ ES_3 + 5\}$$

$$= \max\{8 + 10,\ 4 + 5\}$$

$$= \max\{18,\ 9\}$$

$$ES_4 = 18$$

$$ES_5 = \max\{ES_2 + 2,\ ES_4 + 3\}$$

$$= \max\{8 + 2,\ 18 + 3\}$$

$$= \max\{10,\ 21\}$$

$$ES_5 = 21$$

latest finish:

$$LF_5 = 21$$

$$LF_4 = LF_5 - 3$$

$$= 21 - 3 = 18$$

$$LF_4 = 18$$

$$LF_3 = LF_4 - 5$$

$$LF_3 = 18 - 5 = 13$$

$$LF_3 = 13$$

$LF_2 = \min\{LF_5 - 2,\ LF_4 - 10\}$

$= \min\{21 - 2,\ 18 - 10\}$

$= \min\{19,\ 8\}$

$LF_2 = 8$

$LF_1 = \min\{LF_2 - 8,\ LF_3 - 4\}$

$= \min\{8 - 8,\ 13 - 4\}$

$= \min\{0,\ 9\}$

$LF_1 = 0$

Activity	Duration days	Earliest Start	Earliest Finish	Latest Start	Latest Finish
1-2	8	0	8	0	8
1-3	4	0	4	9	13
2-4	10	8	18	8	18
2-5	2	8	10	19	21
3-4	5	4	9	13	18
4-5	3	18	21	18	21

∴ critical activities, 1-2-4-5

Earliest start:

Activity i-j in a project network is given by

$$ES_j = Max\{ES_i + t_{ij}\}$$ where

ES_i denotes the earliest start time of all the activities emanating from node i and t_{ij} is the estimated duration of the activity i-j.

latest start

latest start time of all the activities emanating from the event i of the activity i-j.

$$LS_i = Min[LS_j - t_{ij}]$$ for all defined i-j activities where t_{ij} is the estimated duration of the activity i-j.

Total float:

Total float of an activity (T.F) is defined as the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Total float of an activity i-j = (LF) − (EF)
or (LS) − (ES)

Free Float:-

Free float of an activity (FF) is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity.

Free float of an activity $i-j$ = Total float of $i-j$ − (L−E) of the event j

Independent float:- (IF) of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity

Independent float of an activity $i-j$ =

Free float of $i-j$ − (L−E) of event i.

Interfering Float (or) Interference float:- of an activity $i-j$ is nothing but the slack of the head event j.

Interfering Float of $i-j$ = Total float of $i-j$ − Free float of $i-j$

(8)

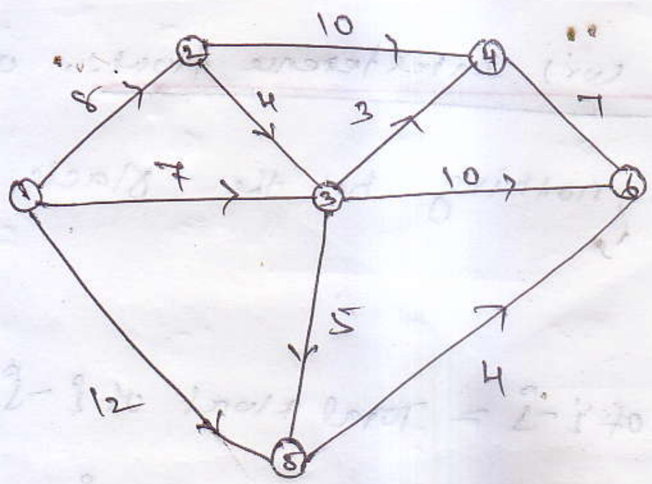
(1) Calculate the total float, free float and independent float for the project whose activities are given below.

Activity	1-2	1-3	1-5	2-3	2-4
Duration	8	7	12	4	10

Activity	3-4	3-5	3-6	4-6	5-6
Duration	3	5	10	7	4

<u>Soln</u>

Draw Network



Activity	Duration	Earliest		Latest		Floats		
		Start	Finish	Start	Finish	TF	FF	IF
1-2	8	0	8	0	8	0	0	0
1-3	7	0	7	8	15	8	5	5
1-5	12	0	12	9	21	9	5	5
2-3	4	8	12	11	15	3	0	0
2-4	10	8	18	8	18	0	0	0
3-4	3	12	15	15	18	3	3	0
3-5	5	12	17	16	21	4	0	-3
3-6	10	12	22	15	25	3	3	0
4-6	7	18	25	18	25	0	0	0
5-6	4	17	21	21	25	4	4	0

∴ Critical path is $8 + 10 + 7$

$= 25$

Critical activity is 1-2, 2-4, 4-6.

H.W

① Consider the following data for the activities of a project

Activity:	A	B	C	D	E	F
Immediate predecessors:	–	A	A	B, C	–	E
Duration (days):	2	3	4	6	2	8

Draw the network and find the critical path and various floats.

Project Evaluation and Review Technique:-

Optimistic (least) time estimate: (t_o or a) is the duration of any activity when everything goes on very well during the project. ie labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate: (t_p or b) is the duration of any activity when almost every things goes against our will and a lot of difficulties is faced while doing a project.

Most likely time estimate (t_m or m) is the duration of any activity when sometimes things go on very well, sometimes things go on very bad while doing the project.

Note: (i) To compute the expected duration of each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$

(ii) Expected variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

(ii) Calculate the standard normal deviate

$\frac{T_S - T_E}{\sigma_c}$ where

T_S = Scheduled time to complete the project

T_E = Normal expected project duration

σ_c = Expected standard deviation of the project length.

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(1) Three time estimates (in months) of all activities of a project are as given below.

Activity	a	m	b
1 – 2	0.8	1.0	1.2
2 – 3	3.7	5.6	9.9
2 – 4	6.2	6.6	15.4
3 – 4	2.1	2.7	5.1
4 – 5	0.8	3.4	3.6
5 – 6	0.9	1.0	1.1

a) Find the expected duration and standard deviation of each activity.

b) Construct the project network.

c) Determine the critical path, expected project length and expected variance of the project length.

d) what is the probability that the project will be completed i) two months later than expected

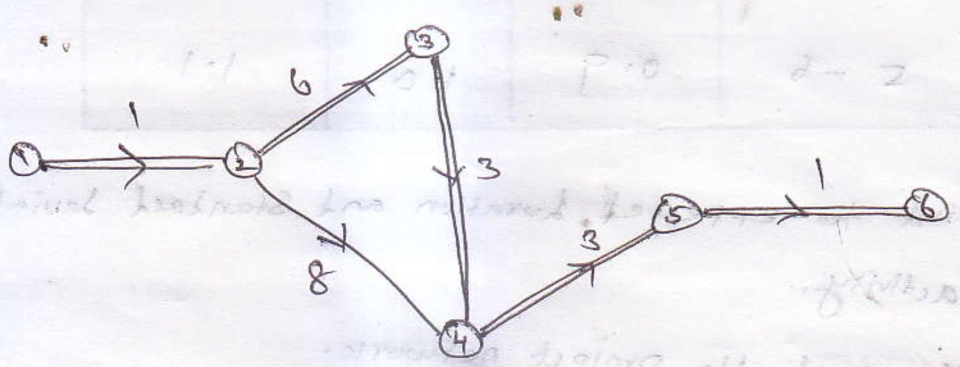
(ii) not more than 3 months earlier than expected

(iii) what due date has about 90% chance of being met?

Soln (a)

Activity	a	m	b	te	σ
1-2	0.8	1.0	1.2	1	0.067
2-3	3.7	5.6	9.9	6	1.03
2-4	6.2	6.6	15.4	8	1.53
3-4	2.1	2.7	5.1	3	0.5
4-5	0.8	3.4	3.6	3	0.47
5-6	0.9	1.0	1.1	1	0.033

b,



(c) critical path: 1-2-3-4-5-6

Expected Project Length = 14 months

Expected variance $= (0.067)^2 + (1.03)^2 + (0.5)^2 + (0.47)^2 + (0.033)^2$

$= 1.5374$

$\therefore \sigma_c = \sqrt{1.5374} = 1.2399$

d) i) $T_s = 16, \quad T_E = 14, \quad \sigma_c = 1.2399$

$$Z = \frac{16-14}{\sigma_c} = \frac{2}{1.6023}$$

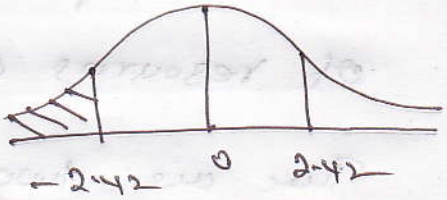
$= 1.61$

$P(T_s \leq 13) = 0.9463$

$P(T_s \leq 13) = 94.63\%$ = Required probability

ii) $T_s = 11, \quad T_E = 14, \quad \sigma_c = 1.2399$

$$Z = \frac{T_s - T_E}{\sigma_c} = \frac{11-14}{1.2399}$$



$$= -\frac{3}{1.2399} = -2.42$$

$P(T_s \leq 11) = 0.5 - 0.4922 = 0.0078$

Required probability $= 0.78\%$

(iii) Z value for 90% area in the table = 1.28 nearly

Let Ts be the required due data

then $Z = \frac{T_S - T_E}{\sigma_c}$

i.e, $1.28 = \frac{T_S - 14}{1.6023}$

$T_S = 14 + 1.28 \times 1.2399 = 15.59$ months nearly

Resource Analysis in Network sheduling.

In the earlier discussions on PERT and CPM, it was assumed that the only constraint for an activity was the starting date and the completion date. Availability of resources and cost aspects were not considered. There are two kinds of costs which can influence project schedules - direct costs and indirect costs.

Direct costs are the costs directly associated with each activity such as machine costs, labour costs etc for each activity.

Indirect costs are the costs due to management services, rentals, insurance including allocation of fixed expenses, cost of security etc.

Crashing: The process of reducing an activity time by putting extra effort is called crashing the activity.

Note:

i) compute the cost slope for each activity by using the formula

$$\text{cost slope} = \frac{\text{crash cost} - \text{Normal cost}}{\text{Normal duration} - \text{crash duration}}$$

ii) Total cost = New direct cost + current indirect cost.

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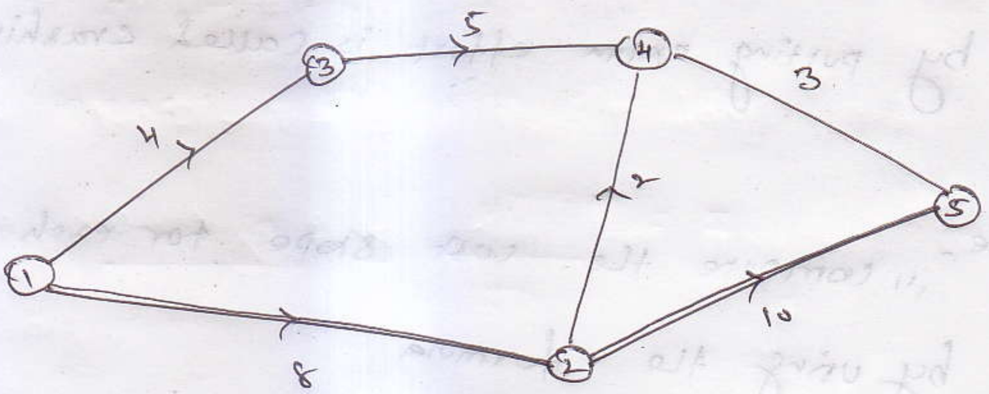
(1) The following data is pertaining to a Project with normal time and crash time

Jobs	Normal		Crash	
	Time	cost	Time	cost
1-2	8	100	6	200
1-3	4	150	2	350
2-4	2	50	1	90
2-5	10	100	5	400
3-4	5	100	1	200
4-5	3	80	1	100

a) If the indirect cost is Rs. 100 per day. find the least cost schedule

b, whate is the minimom duration?

Soln



critical path 1-2-5. Normal duration 18 days

Total cost = Indirect cost + direct cost

= 1800 + 580

= RS. 2380

cost - slope table.

Activity	Cost slope
1-2	50
1-3	100
2-4	40
2-5	60
3-4	25
4-5	10

Stage 1   1-2 is the critical activity of least cost slope. crash 1-2 by 2 days.

current critical path: 1-2-5

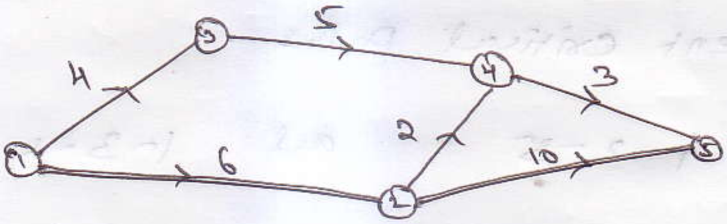
current duration = 18-2 = 16 days

current Total cost = 16×100 + 580 + 100

= 1600 + 680

= 2280

Stage 1



Stage 2:   critical activities 1-2 and 2-5

Crash 2-5 by 4 days

Since the duration of the path 1-3-4-5 is 12 days

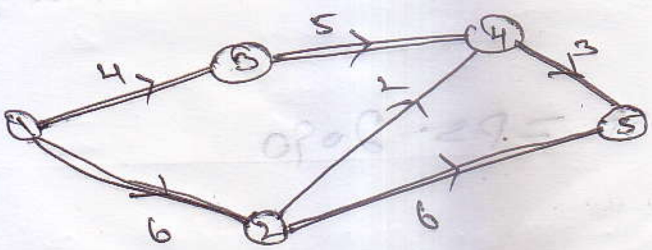
∴ current critical path i) 1-2-5 ii) 1-3-4-5

current duration = 16-4 = 12 days

Current total cost = RS. 12×100 + RS. 680 + RS. 240

= RS. 2120

Stage 2



Stage 3:-

critical activities 1-2, 1-3, 2-5, 3-4 and 4-5.

Crash the critical activities 2-5 and 4-5 by 1 day each since the duration of the path 1-2-4-5 is 11 days and also the activity 2-5 can be crashed only by one day.

Current critical paths is 1-2-5 and 1-3-4-5

current duration 12-1 = 11 days

Current total cost = Rs. 11×100 + Rs. 920 + 1×60 + 1×10 = Rs. 2090

No further crashing is possible since all the activities on the critical path 1-2-5 have been crashed to the maximum extent

Hence the optimum duration = 11 days

Least cost = Rs. 2090

b, Least or minimum duration is also 11 days

crashing schedule can be tabulated as follows:

Stage	crash	current duration	Direct cost	Indirect cost	Total cost
0	0	18	580	1800	2380
1	1-2 by 2 days	16	680	1600	2280
2	2-5 by 4 days	12	920	1200	2120
3	2-5 & 4-5 by 1 days	11	990	1100	2090