



Third Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Expand the function $f(x) = x - x^2$ in the interval $-\pi < x < \pi$. Deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$ (07 Marks)
- b. Find the half-range cosine series for the function $f(x) = (x - 1)^2$ in $0 < x < 1$. (07 Marks)
- c. The following table gives the variations of periodic current over a period

t (sec) :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the 1st harmonic. (06 Marks)

- 2 a. Express the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. (07 Marks)
- b. Find Fourier sine transform of $\frac{1}{x} e^{-ax}$. (07 Marks)
- c. Use convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+s^2)^2}$ given that $\frac{2}{1+s^2}$ is the Fourier transform of $e^{-|x|}$. (06 Marks)
- 3 a. Form the partial differential equation by eliminating the arbitrary function from $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (07 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial x} = -2 \sin y$, when $x = 0$; and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. (07 Marks)
- c. Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (06 Marks)
- 4 a. Derive the one dimensional heat equation in the standard form. (07 Marks)
- b. Obtain the various solutions of the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables. (07 Marks)
- c. A string stretched between the two fixed points $(0, 0)$ and $(1, 0)$ and released at rest from the position $y = \lambda \sin(\pi x)$. Show that the formula for its subsequent displacement $y(x, t)$ is $\lambda \cos(c\pi t) \sin(\pi x)$. (06 Marks)

PART - B

5. a. Show that a real root of the equation $\tan x + \tan hx = 0$ lies between 2 and 3. Then apply the regula falsi method to find the third approximation. (07 Marks)

- b. Apply Gauss - Jordan method to solve the system of equations:

$$2x + 5y + 7z = 52; 2x + y - z = 0; x + y + z = 9.$$

- c. Use power method to find the dominant eigen value and the corresponding eigen vector of

the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigen vector as $[1, 1, 1]^T$.

(06 Marks)

6. a. Under the suitable assumptions find the missing terms in the following table:

x :	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
f(x) :	2.6	-	3.4	4.28	-	14.2	29

- b. Use Newton's divided difference formula to find f(4) given :

x :	0	2	3	6
f(x) :	-4	2	14	158

(07 Marks)

- c. Using Simpson's $\frac{3}{8}$ th rule, evaluate $\int_0^{0.3} \sqrt{1-8x^3} dx$, by taking 7 ordinates. (06 Marks)

7. a. Solve the variational problem $\delta \int_0^{\pi/2} [(y)^2 - (y')^2] dx$ under the conditions $y(0) = 0, y\left(\frac{\pi}{2}\right) = 2$. (07 Marks)

- b. Find the curve on which the function $\int_0^{\pi/2} [(y)^2 - (y')^2 - y \sin x] dx$ under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$, can be extremised. (07 Marks)

- c. Prove that the catenary is the plane curve which when rotated about a line (x - axis) generates a surface of revolution of minimum area. (06 Marks)

8. a. Find the Z - transform of i) n^2 ; ii) $n e^{-an}$. (07 Marks)

- b. Prove that: i) $Z(\cos n\theta) = \frac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$; ii) $Z(\sin n\theta) = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$. (07 Marks)

- c. Find the inverse Z - transform of $\frac{Z}{(Z-1)(Z-2)}$. (06 Marks)

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