

- N.B. (1) Question No. 1 is compulsory.  
 (2) Attempt any four out of remaining six questions.  
 (3) Make suitable assumptions if required and justify the same.  
 (4) A figure to right indicates the full marks.

1. (a) Find  $L(t \sin^3 t)$  05

(b) Use the adjoint method to find the inverse of 05

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

(c) Determine the constants  $a, b, c, d$  if 05

$$f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2) \text{ is analytic.}$$

(d) Find complex form of Fourier Series for  $f(x) = e^{-x}$  in  $(-1, 1)$  05

2. (a) Show that  $v = e^x \sin y$  is harmonic function. Find its harmonic conjugate and corresponding analytic function. 08

(b) Show that the set of functions  $\frac{\cos x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\cos 3x}{\sqrt{\pi}}, \dots$  from a orthonormal set in the interval  $(-\pi, \pi)$ . 06

(c) Using Green's theorem evaluate  $\int_c (x^2 y dx + x^2 dy)$  where  $c$  is the boundary described counter clockwise of the triangle with vertices  $(0,2), (2,0)$  and  $(4,2)$  06

3. (a) Find the Laplace transform of each of the following:-

(i)  $\int_0^t u \cos^2 u du$       (ii)  $te^{3t} \sin 3t$  06

(b) Find half range sine series for the function

$$f(x) = \frac{2x}{3}, \quad 0 \leq x \leq \frac{\pi}{3}$$

$$= \frac{\pi - x}{3}, \quad \frac{\pi}{3} \leq x \leq \pi$$

(c) Find non-singular matrices  $P$  &  $Q$  such that  $PAQ$  is normal form. Hence find its rank where  $A$  is given by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

08

4. (a) Solve the system of equations  $3x + 3y - z = 11$ ,  $2x - y + 2z = 9$ ,  $4x + 3y + z = 25$  06

- (b) Find the inverse Laplace transform of the following

(i)  $\cot^{-1}(as)$  (ii)  $\frac{8e^{-3s}}{(s+4)^3}$  06

- (c) Expand the function  $f(x)$  with period 2 into a Fourier Series.

$$f(x) = \pi x, \quad 0 \leq x \leq 1$$

$$= 0, \quad 1 \leq x \leq 2$$

08

5. (a) Using Convolution theorem, Find the inverse Laplace transform of the following

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

06

- (b) Find the analytic function and its imaginary part if real part is

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

06

- (c) Prove that  $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$  is a conservative field. Find (i) scalar potential (ii) the work done in moving an object in this field from  $(0, 1, -1)$  to  $(\frac{\pi}{2}, -1, 2)$ .

08

6. (a) Using Laplace transformation, solve the following equation.

$$(D^2 + 3D + 2)y = 2(t^2 + t + 1), \text{ with } y(0) = 2 \text{ \& } y'(0) = 0$$

06

- (b) Find the orthogonal trajectories of the family of curves

$$x^3 y - xy^3 = c$$

06

- (c) Find the inverse Z - transform of

$$F(z) = \frac{1}{(z-3)(z-2)}$$

If ROC is (i)  $|z| < 2$  (ii)  $2 < |z| < 3$  (iii)  $|z| > 3$

08

7. (a) Evaluate the following integral by using Laplace transform

$$\int_0^{\infty} \frac{\cos 4t - \cos 3t}{t} dt$$

06

- (b) Find the bilinear transformation which maps the points  $2, i, -2$  onto points  $1, i, -1$  and also find the fixed points.

08

- (c) Find Fourier integral representation of

$$f(x) = e^{ax}, \quad x \leq 0, a > 0$$

$$= e^{-ax}, \quad x \geq 0, a > 0$$

06