CBCS SCHEME

USN | | | | | | | | | | **18CS36**

Third Semester B.E. Degree Examination, July/August 2022

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define Tautology. Prove that for any propositions p, q, r the compound proposition $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(06 Marks)**

b. Test the validity of the arguments using rules of inference.

$$\begin{array}{l} (\neg p \vee q) \to r \\ r \to s \vee t \\ \neg s \wedge \neg u \\ \neg u \to \neg t \\ \hline \therefore p \end{array}$$

(06 Marks)

c. Give an indirect proof and proof by contradiction for, "If m is an even integer, then m + 7 is odd". **(08 Marks)**

OR

2 a. Prove the following logical equivalences using laws of logic:
$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

b. Consider the following open statements with the set of all real numbers as the universe:
$p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4 = 0$
$s(x): x^2 - 3 > 0$. Determine the truth values of the following statements.

(i) $\exists x, p(x) \wedge q(x)$
(ii) $\forall x, p(x) \to q(x)$
(iii) $\forall x, q(x) \to s(x)$
(iv) $\forall x, r(x) \vee s(x)$
(v) $\exists x, p(x) \wedge r(x)$
(vi) $\forall x, r(x) \to p(x)$ **(06 Marks)**

c. Establish the validity of the following :

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \exists x, \neg p(x) \\ \forall x, [\neg q(x) \vee r(x)] \\ \forall x, [s(x) \to \neg r(x)] \\ \hline \therefore \exists x, \neg s(x) \end{array}$$

(08 Marks)

Module-2

3 a. Prove by mathematical induction $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part? **(06 Marks)**

c. Determine the coefficient of,
(i) xyz^2 in $(2x - y - z)^4$ (ii) x^9y^3 in the expansion of $(2x - 3y)^{12}$. **(08 Marks)**

OR

4 a. Prove by mathematical induction, $1.3 + 2.4 + 3.5 + \ldots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$. **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four A's are together? How many of them begin with S? **(06 Marks)**

c. In how many ways can we distribute eight identical white balls into four distinct containers so that,
 (i) no container is left empty?
 (ii) the fourth container has an odd number of balls in it? **(08 Marks)**

Module-3

5 a. State pigeonhole principle. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that atleast two of these points are such that the distance between them is less than $\frac{1}{2}$ cm. **(08 Marks)**

b. If $A = A_1 \cup A_2 \cup A_3$ where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$ and $A_3 = \{5\}$, define a relation R on A by xRy if x and y are in the same subset A_i for $1 \le i \le 3$. Is R an equivalence relation. **(06 Marks)**

c. Let $f, g : R \to R$ where $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$ determine a, b. **(06 Marks)**

OR

6 a. Prove that if $f : A \to B$, $g : B \to C$ are invertible functions, then $gof : A \to C$ in invertible and $(gof)^{-1} = f^{-1} o g^{-1}$. **(06 Marks)**

b. For $A = \{a, b, c, d, e\}$ the Hasse diagram for the poset (A, R) is shown in Fig. Q6 (b).
 (i) Determine the relation matrix for R.
 (ii) Construct the directed graph G that is associated with R. **(06 Marks)**

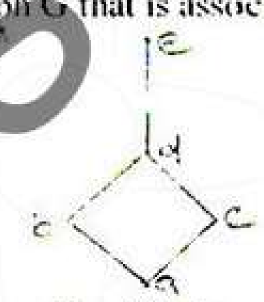

Fig. Q6 (b)

c. If R is an equivalence relation on a set A and $x, y \in A$ then prove
 (i) $x \in [x]$
 (ii) xRy if and only if $[x] = [y]$ and
 (iii) if $[x] \cap [y] \neq \varphi$ then $[x] = [y]$. **(08 Marks)**

Module-4

7 a. Find the number of permutations of a, b, c ......x, y, z in which none of the patterns spin, game, path or net occurs. **(08 Marks)**

b. For the positive integers 1, 2, 3, ...n there are 11660 derangements where 1, 2, 3, 4 and 5 appear in the first five positions. What is the value of n? **(06 Marks)**

c. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ where $n \ge 2$ and $a_0 = -1$, $a_1 = 8$. **(06 Marks)**

OR

8 a. Determine the number of integers between 1 and 300 (inclusive) which are, (i) divisible by exactly two of 5, 6, 8 …. (ii) divisible by atleast two of 5, 6, 8. **(06 Marks)**

b. Describe the expansion formula for Rook polynomials. Find the Rook polynomial for 3×3 board using expansion formula. **(08 Marks)**

c. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(06 Marks)**

Module-5

9 a. Define with examples. (i) Subgraph, (ii) Spanning subgraph (iii) Complete graph (iv) Induced subgraph (v) Complement of a graph (vi) path. **(06 Marks)**

b. Merge sort the list.
$-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3$ **(06 Marks)**

c. Define isomorphism of two graphs. Determine whether the following graphs G_1 and G_2 are isomorphic or not.

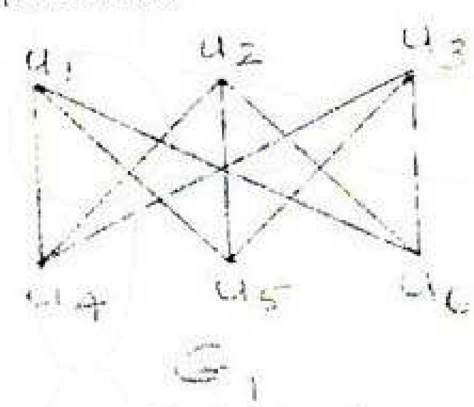

Fig. Q9 (c) – i

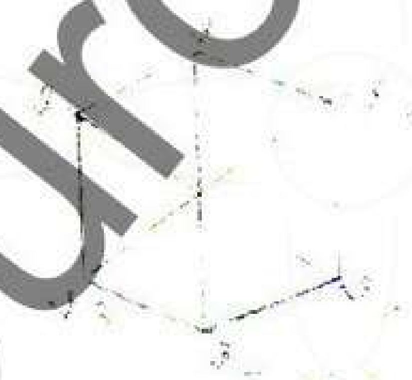

Fig. Q9 (c) - ii

(08 Marks)

OR

10 a. Let G = (V, E) be the undirected graph in Fig. Q10 (a). How many paths are there in G from a to h? How many of these paths have length 5? **(06 Marks)**

Fig. Q10 (a)

b. Prove that in every tree $T = (V, E)$, $|V| = |E| + 1$ **(06 Marks)**

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(08 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | | **18CS36**

Third Semester B.E. Degree Examination, Jan./Feb. 2021

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Verify that, for any three propositions p, q, r the compound proposition $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology or not. **(06 Marks)**

b. Test for validity of following argument.

If Ravi goes out with friends, he will not study
If Ravi do not study, his father becomes angry
His father is not angry
$\therefore$ Ravi has not gone out with friends **(07 Marks)**

c. Give direct and indirect proof of following statement "Product of two odd integers is an odd integer". **(07 Marks)**

OR

2 a. For any three propositions p, q, r, prove that $[\sim p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

b. Check for validity of following argument,

If a triangle has two equal sides then it is isosceles. If a triangle is isoceles than it has two equal angles.
A certain triangle ABC does not have two equal angles
$\therefore$The triangle ABC does not have two usual sides **(07 Marks)**

c. Consider the following open statement on set of all real numbers as universe:
$p(x) : x \geq 0$ $q(x) : x^2 \geq 0$ $r(x) : x^2 - 3x - 4 = 0$ $s(x) : x^2 - 3 > 0$
Then find truth value of i) $\exists x\ p(x) \wedge q(x)$ ii) $\forall x, p(x) \to q(x)$ iii) $\forall x, q(x) \to s(x)$
iv) $\forall x, r(x) \vee s(x)$ **(07 Marks)**

Module-2

3 a. By mathematical induction prove that

$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{2}\, n\,(2n-1)\,(2n+1)$$ **(06 Marks)**

b. Find coefficient of i) x^0 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$

ii) $x^{11}\, y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ **(07 Marks)**

c. A total amount of Rs.1500 is to be distributed to three students A, B, C. In how many ways distribution can be done in the multiples of Rs.100 if

i) Every students sets at least Rs.300
ii) A must get at least Rs.500, B and C must set at least Rs.400 each. **(07 Marks)**

OR

4 a. By mathematical induction prove that for any positive integer n the number $11^{n+2} + 12^{2n+1}$ is divisible by 133 **(06 Marks)**

b. How many positive integers n can be formed from the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000. **(07 Marks)**

c. A certain question paper has 3 parts A, B, C with four questions in Part A. Five in B and Six in C. It is required to answer seven questions by selecting at least two from each part. In how many different ways student can answer seven questions. **(07 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4, 5, 6}, B = {6, 7, 8, 9, 10} and f be a function from A to B defined by f = {(1, 7) (2, 7), (3, 8) (4, 6) (5, 9) (6, 9)}. Then find $f^{-1}(6)$, $f^{-1}(9)$. If B_1 = {7, 8}, B_2 = {8, 9, 10} find $f^{1}(B_1)$, $f^{1}(B_2)$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and R be a relation on A defined by xRy if and only if x divides y. Then i) Write R as ordered pairs ii) Draw diagram iii) Write matrix of R. **(07 Marks)**

c. If f, g, h are functions from R to R defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$. Then verify that f o(goh) = (fog) oh **(07 Marks)**

OR

6 a. If 30 dictionaries in a library contain total 61,327 pages then prove that at least one of the dictionaries must have at least 2045 pages. **(06 Marks)**

b. For any three nonempty sets A, B, C prove that

i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

ii) $AX(B \cap C) = (A \times B) \cap (A \times C)$ **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 8, 12} define a partial order R on A by xRy if and only if x divides y. Draw Hasse diagram of R. **(07 Marks)**

Module-4

7 a. For the integers 1, 2, …n, there are 11660 derangements where 1, 2, 3, 4, 5 appear in first five positions then find value of n. **(06 Marks)**

b. Determine number of integers between 1 and 300 which are i) divisible by exactly two of 5, 6, 8 ii) at least two of 5, 6, 8. **(07 Marks)**

c. Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given $a_0 = 1$, $a_1 = 2$ **(07 Marks)**

OR

8 a. Out of 30 students of a hostel 15 study history, 8 study economics, 6 study geography and 3 study all the three subjects. Show that 7 or more study none of the subjects. **(06 Marks)**

b. An apple, a banana, a mango, and an orange to be distributed to 4 boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish apple, B_3 does not want banana or mango B_1 refuses orange. In how many ways distribution can be made so that all of them are happy. **(07 Marks)**

c. Solve $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given $a_0 = 2$. **(07 Marks)**

Module-5

9 a. Show that following graphs in the Fig.Q.9(a)(i) and Fig.Q.9(a)(ii) are isomorphic

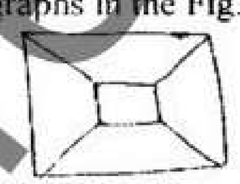

Fig.Q.9(a)(i)

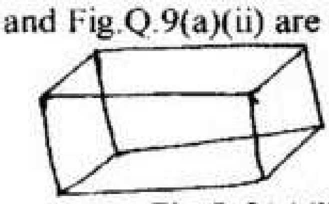

Fig.Q.9(a)(ii)

(06 Marks)

b. Define with an example to each i) Complement of a graph ii) Vertex degree iii) Rooted tree iv) Prefix code **(07 Marks)**

c. Apply merge sort to the list
-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3 **(07 Marks)**

OR

10 a. Prove that a tree with n vertices has (n – 1) edges. **(06 Marks)**

b. Determine number of vertices in following graph G:

i) G has 9 edges and all vertices have degree 3

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18CS36

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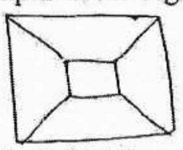

Fig.Q.9(a)(i)

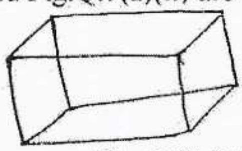

Fig.Q.9(a)(ii)

(06 Marks)

b. Define with an example to each i) Complement of a graph ii) Vertex degree iii) Rooted tree iv) Prefix code **(07 Marks)**

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c. Obtain optimal prefix code for the message ROAD IS GOOD. **(07 Marks)**

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THIRD SEMESTER B.E DEGREE EXAMINATION FEB 202

DISCRETE MATHEMATICAL STRUCTURES

(18CS36)

Time: 3 hrs. MAX. Marks: 10

Solutions / Answers

Module - 1.

1a) $[P \to (q \to r)] \to [(P \to q) \to (P \to r)]$ is tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$p \to (q \to r)$ A	$(p \to q) \to (p \to r)$ B	$A \to$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

From the truth table, we can see that the given proposition is always True for all possible values. $\therefore$ It is tautology.

1b) Let p : Ravi goes out with friends.

q : Ravi will study.

r : Ravi's father becomes angry.

Given argument is

$$\begin{array}{l} p \to \neg q \\ \sim q \to r \\ \sim r \\ \hline \therefore \sim p \end{array} \Rightarrow \begin{array}{l} p \to r \\ \sim r \\ \hline \therefore \sim p \end{array}$$

$\because$ Rule of Syllogism

$\because$ Modus Tollens Rule.

$\therefore$ This is a valid argument.

1C) Given a statement is,

"If x is odd and y is odd then xy is odd"

Let p : x is odd

q : y is odd

r : xy is odd.

Given statement in symbolic form : $(p \wedge q) \rightarrow r$

Direct Proof : Let $p \wedge q$ be true.

$\Rightarrow$ p is true and q is true.

$\Rightarrow$ x is odd & y is odd

$\Rightarrow$ $x = 2k+1$ & $y = 2l+1$ $\quad k, l \in \mathbb{Z}$.

$\Rightarrow$ $xy = (2k+1)(2l+1)$

$= 4kl + 2k + 2l + 1$

$= 2(2kl + k + l) + 1$

$= 2m + 1$ where $m = 2kl + k + l \in \mathbb{Z}$.

$\therefore$ xy is odd.

Indirect Proof :

We know that

$(p \wedge q) \rightarrow r \Leftrightarrow \neg r \rightarrow \neg(p \wedge q)$

Let $\neg r$ be true $\Rightarrow$ xy is not odd.

$\Rightarrow$ xy is even.

$\Rightarrow$ x is even and y is odd is true and.

$\Rightarrow \neg p$ is true & q is true

$\Rightarrow$ $\neg(p \wedge q)$ is true.

or x is odd & y is even

p is true & $\neg q$ is true

$\Rightarrow \neg p \vee \neg q$ is true

or x is even & y is even

$\neg p$ is true

$\neg q$ is true

$\neg q \vee \neg q$ is true

$\Rightarrow \neg(p \wedge q)$ is true.

$\therefore$ $\neg r \rightarrow \neg(p \wedge q)$ is true.

so by $(p \wedge q) \rightarrow r$ is true.

2a) Consider

LHS $= [\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

First consider.

$\Leftrightarrow -p \wedge (-q \wedge r) \Leftrightarrow (-p \wedge -q) \wedge r)$ Associative law

$\Leftrightarrow [\neg(p \vee q)] \wedge r \Leftrightarrow \boxed{r \wedge [\neg(p \vee q)]}$ Demorgan's law & Commutative law

and.

$(q \wedge r) \vee (p \wedge r) \Leftrightarrow (r \wedge q) \vee (r \wedge p)$ commutative law

$\Leftrightarrow r \wedge (q \vee p)$ Distributive law

$\Leftrightarrow \boxed{r \wedge (p \vee q)}$ Commutative law

$\therefore \{r \wedge (\sim p \vee q) \vee (r \wedge (p \vee q)\}$

$\Leftrightarrow r \wedge \{[\neg(p \vee q)] \vee (p \vee q)\}$ Distributive law.

$\Leftrightarrow r \wedge T_0$ Inverse law.

$\Leftrightarrow r.$ //.

2b. Let $p(x)$: x has two equal sides.

$q(x)$: x is isosceles.

$r(x)$: x has two equal angles.

a : triangle ABC.

Given $\forall x,\ p(x) \rightarrow q(x)$

$\forall x,\ q(x) \rightarrow r(x).$

$\sim r(a)$

$\therefore \sim p(a)$

$\Rightarrow$ Universal Specification

$p(a) \rightarrow q(a)$

$q(a) \rightarrow r(a)$

$\sim r(a)$

$\therefore \sim p(a)$

$p(a) \rightarrow q(a)$

$q(a) \rightarrow r(a)$

$\therefore p(a) \rightarrow r(a)$ $\because$ law of Syllogism.

$\sim r(a)$

$\therefore \sim p(a)$ This is valid argument. in view of Modus Tollens ..

2c) i) $\exists x, P(x) \wedge q(x)$.

we know that, there exists a real no $x=1$, for which both $P(x)$ and $q(x)$ are true.

$\exists x, P(x) \wedge q(x)$ is a true statement.

It's truth value is 1.

ii) $\forall x, P(x) \rightarrow q(x)$.

for every real no x, $q(x)$ is true.

$\therefore \forall x, P(x) \rightarrow q(x)$ is true

$\therefore$ It's truth value is 1.

iii) $\forall x, q(x) \rightarrow s(x)$

WKT, $s(x)$ is false, and

$q(x)$ is true for $x=1$

Thus $\forall x, q(x) \rightarrow s(x)$ is false.

$\therefore$ It's truth value is 0

iv) $\forall x, r(x) \vee s(x)$

$r(x)$ is true only for $x=4$ & $x=-1$

$r(x)$ and $s(x)$ are false for $x=1$

Thus, $r(x) \vee s(x)$ is not always true.

$\therefore \forall x, r(x) \vee s(x)$ is false.

$\therefore$ It's truth value is 0

3a) Let $S(n) = 1^2 + 3^2 + \dots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$

Basis step: we know that,

$S(1): 1^2 = \frac{1}{3} \times 1 \times 3$.

$1 = 1$ which is true.

Inductive step: we assume that $S(n)$ is true for $n=k$, where $k \geq 1$

then, $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{1}{3} k(2k-1)(2k+1)$

Adding $(2k+1)^2$ on both sides,

$1^2 + 3^2 + 5^2 + \ldots\ldots + (2k-1)^2 + (2k+1)^2 = \frac{1}{3} k(2k-1)(2k+1)$.

$= \frac{1}{3}(2k+1)\left[k(2k-1) + 3(2k+1)\right]$

$= \frac{1}{3}(2k+1)\left[2k^2 - k + 6k + 3\right]$

$= \frac{1}{3}(2k+1)(2k^2 + 5k + 3)$

$= \frac{1}{3}(2k+1)(k+1)(2k+3)$

This stmt is true for $S(k+1)$.

Thus $S(k+1)$ is true for whenever $S(k)$ is true where $k \geq 1$. Hence By Mathematical Induction, $S(n)$ is true for $\forall n \geq 1$.

3b) i) x^0 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$

ii) $x^{11} y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$

i) $\left(3x^2 - \frac{2}{x}\right)^{15} = \sum_{r=0}^{15} \binom{15}{r} (3x^2)^r \cdot \left(-\frac{2}{x}\right)^{15-r}$

$= \sum_{r=0}^{15} \binom{15}{r} 3^r \cdot (-2)^{15-r} \cdot x^{2r} \cdot \left(\frac{1}{x}\right)^{15-r}$

$= \sum_{r=0}^{15} \binom{15}{r} 3^r \cdot (-2)^{15-r} \cdot x^{2r} \cdot x^{r-15}$

$= \sum_{r=0}^{15} \binom{15}{r} \cdot 3^r \cdot (-2)^{15-r} \cdot x^{3r-15}$

Taking $r=5$ in the above expansion, the co-efficient of x^0 is $= \binom{15}{5} 3^5 \cdot (-2)^{10}$

$= 15C_5 \cdot 3^5 (-2)^{10}$

$= 74,72,42,496.$

ii) The general term in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ is

$= \binom{6}{n_1\, n_2\, n_3} (2x^3)^{n_1} (-3xy^2)^{n_2} (z^2)^{n_3}$

$= \binom{6}{n_1\, n_2\, n_3} 2^{n_1} (-3)^{n_2} \cdot x^{3n_1} \cdot x^{n_2} \cdot y^{2n_2} \cdot z^{2n_3}$

For $n_3 = 0$, $n_2 = 2$, $n_1 = 3$ we have

$$\binom{6}{3,2,0} \; 2^3 . (-3)^2 . x^{11} y^4$$

$\therefore$ Co-eff of $x^{11} y^4$ is $2^3 . (-3)^2 . \binom{6}{3,2,0}$

$$= 8 \times 9 \times \frac{6!}{3! . 2! \; 0!}$$

$$= 4,320 \; //$$

3c) There are 15 objects. (15 hundred Rs notes), to be distributed among 3 students A, B, C.

i) Every student gets atleast Rs. 300.

Distribute Rs. 300 to every student. $(300 \times 3) = 900$

Remaining '6' notes should be distributed among 3 students.

$r = 6$, $n = 3$.

This canbe done in $n+r-1 \, C_r$ ways.

$$= {}^{3+6-1} C_6$$

$$= 8 C_6 \text{ ways.}$$

$$= \frac{8!}{6! . (8-6)!}$$

$$= \frac{\cancel{8}^4 \times 7 \times \cancel{6!}}{\cancel{6!} \; \cancel{2!}}$$

$$= 28 \; //$$

ii)

	A	B	C
a)	500	400	600
b)	500	500	500
c)	500	600	400
d)	600	400	500
e)	600	500	400
f)	700	500	400

6 - ways.

By Direct method.

By using combination with Repitition,

Distribute Rs 500 to A, Rs 400 to B, C each.

Remaining 2 notes of 100 should be distributed among 3-students A, B, C.

$r = 2, \quad n = 3.$

No of ways of distributing $= {}^{n+r-1}C_r$

$= {}^{3+2-1}C_2 = {}^4C_2$

$= \frac{4!}{2! \times 2!}$

$= \frac{\cancel{4}^2 \times 3 \times \cancel{2}1}{\cancel{2}1 \times 2}$

$= 6$ ways //.

4a) We know that, $11^{n+2} + 12^{2n+1}$

$A_1 = 11^{1+2} + 12^{2+1}$

$= 11^3 + 12^3$

$= 1331 + 1728 = 3059$

Thus, An is divisible by 133 for $n=1$

Induction step:

Assume that An is divisible by 133. for $n = k \geq 1$

Now we find that,

$A_{k+1} = 11^{k+3} + 12^{2(k+1)+1}$

$= (11^{k+2} \times 11) + (12^{2k+1} \times 12^2)$

$= (11^{k+2} \times 11) + (12^{2k+1} \times 144)$

$= (11^{k+2} \times 11) + \{12^{2k+1} \times (11 + 133)\}$

$= (11^{k+2} + 12^{2k+1}) \times 11 + (12^{2k+1} \times 133)$

$= (A_k \times 11) + (12^{2k+1} \times 133)$

This representation shows that A_{k+1} is divisible by 133, when A_k is divisible by 133.

$\therefore$ By induction, the given result is true

4b) Here 'n' must be of the form, with 7-digits formed by, 3, 4, 4, 5, 5, 6, 7

$n = x_1 x_2 x_3 x_4 x_5 x_6 x_7$

If 'n' wants to exceed 5,000,000

then $x_1 = 5, 6$ or 7.

Suppose $x_1 = 5$, Then its arrangement of 6-digits which contains two 4's and one each of 3, 5, 6, 7

$\therefore$ The no of such arrangements $= \dfrac{6!}{2! \times 1! \times 1! \times 1! \times 1!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$

$= \underline{360}$

Next suppose $x_1 = 6$,

then its arrangement of 6-digits, which contains two 4's & 2 5's & each of 3, 7

$\therefore$ The no of such arrangements $= \dfrac{6!}{2! \cdot 2! \cdot 1! \cdot 1! \times 1!}$

$= \dfrac{6 \times 5 \times \overset{2}{4} \times 3 \times \cancel{2!}}{2 \times \cancel{2!}}$

$= \underline{180}$

Next suppose $x_1 = 7$,

$\therefore$ then no of arrangements, $= \dfrac{6!}{2! \cdot 2! \times 1! \cdot 1! \cdot 1!}$

$= \underline{180}$

According By the sum rule

$\therefore$ The no of arrangements of which n-exceeds 5,000,000 $= 360 + 180 + 180$

$= \underline{720 \text{ ways}}$

4c) Question paper has 3-parts. A, B, C
with 4- Questions - in Part A
5- Questions - in Part B
6- Questions - in Part C

It is required to answer - 7- Questions, by selecting atleast ~~a~~ two questions from each part.

$\therefore$ Different possible ways in which a student can make a selection are

i) 2 questions from part A, 2 from B, 3 from C
ii) 2 " from A, 3 from B, 2 from C
iii) 3 " from A, 2 from B, 2 from C.

i) The no of selection $= 4C_2 \times 5C_2 \times 6C_3$
$= \underline{\underline{1200 \text{ ways}}}$

ii) No. of selection $= 4C_2 \times 5C_3 \times 6C_2$
$= \underline{\underline{900 \text{ ways}}}$

iii) No of Selection $= 4C_3 \times 5C_2 \times 6C_2$
$= \underline{\underline{600 \text{ ways}}}$

$\therefore$ The total no of possible Selections
$= 1200 + 900 + 600 = \underline{\underline{2700}}$ ways

MODULE -3

5a)

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$

$f: A \to B$, defined by $f = \{(1,7)(2,7)(3,8)(4,6),$
$(5,9)(6,9)\}$.

$f^{-1}(6) = \{x \in A \mid f(x) = 6\} = \{4\}$.
$f^{-1}(9) = \{x \in A \mid f(x) = 9\} = \{5, 6\}$.

For $B_1 = \{7, 8\}$.

$f(x) \in B_1$ when $f(x) = 7$, and $f(x) = 8$

Here $f(x) = 7$ when $x = 1$, $x = 2$

$f(x) = 8$ when $x = 3$.

$\therefore f^{-1}(B_1) = \{1, 2, 3\}$.

Illy $B_2 = \{8, 9, 10\}$.

$f(x) = 8$ when $x = 3$

$f(x) = 9$ when $x = 5, 6$.

$f(x) = 10$ for no values of 'x'

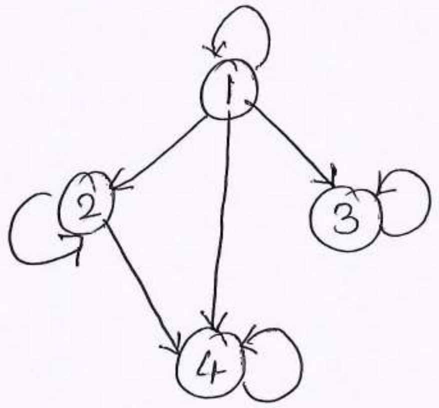
$\therefore f^{-1}(B_2) = \{x \in A \mid f(x) \in B_2\} = \{3, 5, 6\}$.

5b) Let $A = \{1, 2, 3, 4\}$ and 'R' be a relation on A defined by xRy, iff x divides y.

$\therefore R = \{1/1, 1/2, 1/3, 1/4, 2/2, 2/4, 3/3, 4/4\}$

i) $R = \{(1,1), (1,2) (1,3) (1,4) (2,2) (2,4) (3,3) (4,4)\}$

ii)



iii)

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5(c)

$f(x) = x^2$, $g(x) = x+5$, $h(x) = \sqrt{x^2+2}$

i) $f \circ (g \circ h)$

$g \circ h = g[h(x)] = g(\sqrt{x^2+2}) = \sqrt{x^2+2} + 5$

$f \circ (g \circ h)(x) = f((g \circ h)(x$

$= f(\sqrt{x^2+2} + 5)$

$= (\sqrt{x^2+2} + 5)^2$

$= (x^2+2) + 25 + 10\sqrt{x^2+2}$

$= x^2 + 27 + 10\sqrt{x^2+2}$ //.

ii) $g \circ (f \circ h)$

$(f \circ g)(x) = f(g(x))$

$= f(x+5)$

$= (x+5)^2 = x^2 + 25 + 10x$.

$(f \circ g) \circ h(x) = (f \circ g)\, h(x)$

$= [h(x)]^2 + 25 + 10(h(x))$

$= (\sqrt{x^2+2})^2 + 25 + 10\sqrt{x^2+2}$

$= x^2 + 2 + 25 + 10\sqrt{x^2+2}$

$= x^2 + 27 + 10\sqrt{x^2+2}$ //.

$\therefore f \circ (g \circ h)(x) = (f \circ g) \circ h(x)$.

6a)

30 - dictionaries. Total No. of pages = 61,237

Treating the pages as pigeons and dictionaries as pigeonholes, we find by using the generalized pigeonhole principle, that atleast one of the dictionaries, must contain p+1 or more pages.

where $p = \left\lfloor \frac{61327 - 1}{30} \right\rfloor = \lfloor 2044.2 \rfloor = 2044.$

6b) i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Let LHS $(A \cup B) \times C$

Let $(x, y) \in (A \cup B) \times C$

$\Rightarrow x \in A \cup B$ and $y \in C$

$\Rightarrow x \in A$ or $x \in B$ and $y \in C$

$\Rightarrow \{(x \in A)$ and $(y \in C)\}$ or $\{(x \in B)$ and $(y \in C)\}$

$\Rightarrow (x, y) \in (A \times C)$ or $(x, y) \in (B \times C)$

$\Rightarrow (x, y) \in (A \times C) \cup (B \times C)$ R.H.S.

$\therefore (A \cup B) \times C = (A \times C) \cup (B \times C).$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(x, y) \in [A \times (B \cap C)]$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B)$ or $(y \in C)$

$\Rightarrow \{(x \in A)$ and $(y \in B)\}$ or $\{x \in A$ and $y \in C\}$

$\Rightarrow \{(x, y) \in (A \times B)\}$ or $\{(x, y), (A \times C)\}$

$\Rightarrow \{x, y \in (A \times B) \cap (A \times C)\}$

$\Rightarrow (A \times B) \cap (A \times C)$ R.H.S.

$\therefore A \times (B \cap C) = (A \times B) \cap A \times C$

6c) $A = \{1, 2, 3, 4, 6, 8, 12\}$. Define partial order R

$R = \{ xRy$ iff x divides $y \}$.

$R = \{ (1,1) (1,2) (1,3) (1,4), (1,6) (1,8) (1,12), (2,2)$
$(2,4) (2,6) (2,8) (2,12) (3,6) (3,12), (4,8)$
$(4,12) (6,12) (12,12), (3,3) (4,4) (6,6) (8,8) \}$

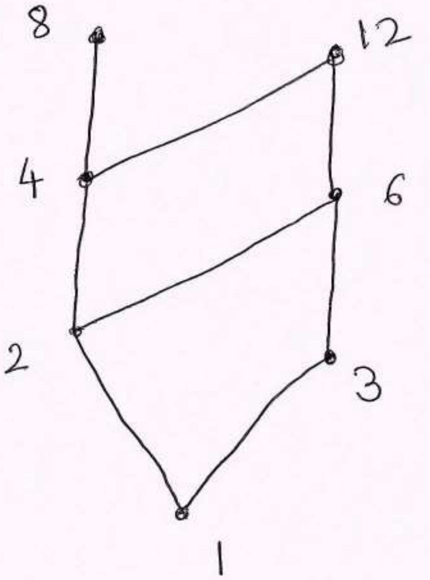
'R' is partial-order on set A.
If 'R' is reflexive, antisymmetric, and transitive.

i) Reflexive : $\forall a \in A, (a,a) \in R$.
$(1,1) (2,2) (3,3) (4,4) (6,6) (8,8) (12,12) \in R$
Hence 'R' is Reflexive.

ii) AntiSymmetric :
if $(a,b) \in R$ and $a \neq b$, then we see that
$(b,a) \notin R$, $\forall a,b \in A$.
$\therefore$ R is antisymmetric.

iii) Transitive :
if $(a,b) \in R$ and $(b,c) \in R$ then we that $(a,c) \in R$
$\therefore$ R is transitive.

Thus 'R' is partial order on A. i.e (A, R) is poset



Hasse Diagram.

7a) For integers $1, 2 \ldots \ldots n$, $d_n = 11,660$.

$n = 1, 2, 3, 4, 5$.

The integers $1, 2, 3, 4, 5$ can be deranged in the first five places in d_5 ways;

The last $n-5$ integers in d_{n-5} ways.

Hence, the no of derangements

$$d_n = d_5 \times d_{n-5}.$$

$$11660 = d_5 \times d_{n-5}., \text{ So that}$$

$$d_{n-5} = \frac{11660}{d_5} = 265$$

$$= \frac{11660}{44.} = 265$$

But $265 = d_6$, Thus $n-5 = 6$, So that $n = 11$

$\therefore \quad n = 11$ //

7b) $S = \{1, 2 \ldots \ldots 300\}$.

Let A_1, A_2, A_3 be subsets of 'S' whose elements are divisible by 5, 6, 8 resply.

$S_0 = |S| = 300$

$|A_1| = \left\lfloor \frac{300}{5} \right\rfloor = 60$, $|A_2| = \left\lfloor \frac{300}{6} \right\rfloor = 50$ $|A_3| = \left\lfloor \frac{300}{8} \right\rfloor = 37$

$|A_1 \cap A_2| = \left\lfloor \frac{300}{30} \right\rfloor = 10$ $|A_1 \cap A_3| = \left\lfloor \frac{300}{40} \right\rfloor = 7$

$|A_2 \cap A_3| = \left\lfloor \frac{300}{24} \right\rfloor = 12$

$|A_1 \cap A_2 \cap A_3| = \left\lfloor 300/120 \right\rfloor = 2.$

$S_1 = |A| + |B| + |C|$
$= 147$

$S_2 = |A \cap B| + |B \cap C| + |A \cap C|$
$= 29.$

$S_3 = |A \cap B \cap C| = 2$

i) $E_2 = S_2 - 3C_1 \; S_3$

$= 29 - 3C_1 \times 2 = 23.$

ii) $L_2 = S_2 - 2C_1 S_3$.

$= 29 - (2 \times 2) = 25$

7c) Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$,

given $a_0 = 1$, $a_1 = 2$

Characteristic eqn is. $k^2 - 2k + 2 = 0$

$$k = 1 \pm i$$

$\therefore$ The general solution is :

$$a_n = r^n [A \cos n\theta + B \sin n\theta]$$

where $r = \sqrt{1^2 + 1^2} = \sqrt{2}$ &

$$\theta = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\therefore \quad a_n = (\sqrt{2})^n \left[A \cos \frac{n\pi}{4} , B \sin \frac{n\pi}{4}\right]$$

Given $a_0 = 1$ & $a_1 = 2$.

$\Rightarrow$ $1 = A$, $2 = (\sqrt{2}) \left[A \cos \frac{\pi}{4} + B \sin \frac{\pi}{4}\right] = A + B$

$\Rightarrow$ $A = 1$, & $B = 1$ —— (1)

$\therefore$ Soln is

$$a_n = (\sqrt{2})^n \left[\cos \frac{n\pi}{4} + \sin \frac{n\pi}{4}\right] \text{ By using (1)}$$

8a)

Let 'S' denote the set of all students in a hostel.

A_1, A_2, A_3, who study History, Economics, Geography respectively. $|S| = 30$, $|A_1| = 15$, $|A_2| = 8$, $|A_3| = 6$

$$\therefore \quad S_1 = \Sigma |A_i| = A_1 + A_2 + A_3$$

$$= 15 + 8 + 6 = 29.$$

$$S_3 = |A_1 \cap A_2 \cap A_3| = 3.$$

$$|\bar{A_1} \cap \bar{A_2} \cap \bar{A_3}| = |S| - \Sigma |A_i| + \Sigma |A_i \cap A_j| - \Sigma |A_1 \cap A_2 \cap A_3|$$

$$= |S| - S_1 + S_2 - S_3$$

$$= 30 - 29 + S_2 - \cancel{S}3$$

$$= S_2 - 2$$

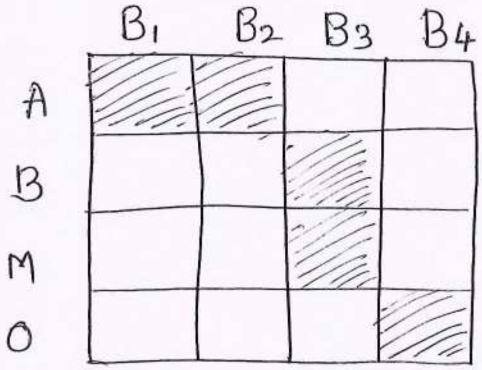
$|A_1 \cap A_2 \cap A_3| \subseteq (A_i \cap A_j)$ for $i,j = 1,2,3 \ldots$

$S_2 = \Sigma |A_i \cap A_j| \geq 3|A_1 \cap A_2 \cap A_3| = 9$

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 9 - 2 \geq 7.$

$\therefore$ 7 or more study none of the subjects.

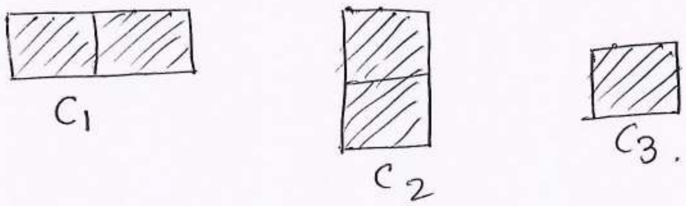
8b)



Let B_1, B_2, B_3, B_4 represent 4 - boys.

Let A, B, M, O represent Apple, Banana & Mango & Orange.

Let



$$r(C, x) = r(C_1, x) \times r(C_2 . x) \times r(C_3, x)$$

$$= (1+2x) \times (1+2x) \times (1+x)$$

$$= 1 + 5x + 8x^2 + 4x^3.$$

Here $r_1 = 5$, $r_2 = 8$, $r_3 = 4$.

$S_0 = n! = 4! = 24.$ $\qquad S_k = (n-k) . k!$

$S_1 = (4-1)! \times r_1 = 30$

$S_2 = (4-2)! \times r_2 = 16$

$S_3 = (4-3)! \times r_3 = 4.$

$$\therefore \bar{N} = S_0 - S_1 + S_2 - S_3$$

$$= 24 - 30 + 16 - 4 = 6.$$

$\therefore$ '6' ways of distribution, can be made, so that all of them

8c) $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$, given $a_0 = 2$

Given: $a_n = 3a_{n-1} + (5 \times 3^n)$ ——— (1)

is a non-homogeneous relation with $c = 3$.

$f(n) = 5 \times 3^n$.

General Solution is given by,

$$a_n = 3^n a_0 + \sum_{k=1}^{n} 3^{n-k} \cdot f(k).$$

$$a_n = 3^n a_0 + 3^{n-1} f(1) + 3^{n-2} f(2) + \dots + 3^0 f(n).$$

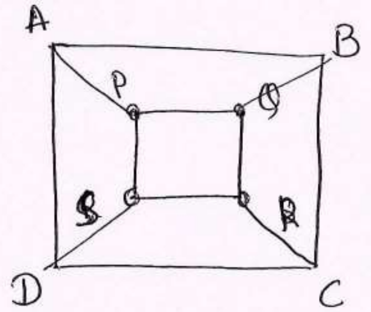
$$\Rightarrow a_n = 3^n \times 2 + 3^{n-1}(5 \times 3^1) + 3^{n-2}(5 \times 3^2) + \dots + 3^0 \times (5 \times 3^n)$$

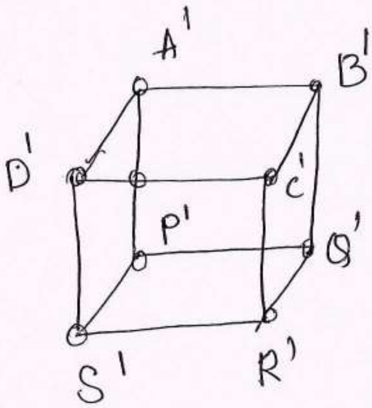
$$= 2 \times 3^n + 5[3^n + 3^n + \dots + 3^n] \text{ n times.}$$

$$= 2 \times 3^n + 5 \times n \times 3^n$$

$\therefore a_n = 3^n(2 + 5n)$ is the required soln.

9a)





Let us consider the one-to-one correspondence between the vertices of the two graphs under which the vertices A, B, C, D, P, Q, R, S of the first graph correspond to the vertices A', B', C', D', P', Q', R', S' respectively of the second graph and vice-versa.

$A \leftrightarrow A'$
$B \leftrightarrow B'$
$C \leftrightarrow C'$
$D \leftrightarrow D'$

$P \leftrightarrow P'$
$Q \leftrightarrow Q'$
$R \leftrightarrow R'$
$S \leftrightarrow S'$

The edges determined by corresponding vertices

$AB \leftrightarrow A'B'$ $\quad$ $BQ \leftrightarrow B'Q'$

$AP \leftrightarrow A'P'$ $\quad$ $BC \leftrightarrow B'C'$

$AD \leftrightarrow A'D'$ $\quad$ $CD \leftrightarrow C'D'$ ....and so on

Edges determined by corresponding vertices correspond so that the adjacency of vertices is retained.

Both graphs have 8-vertices & 12-edges and are cubic graphs.

$\therefore$ The two graphs are isomorphic.

9b) i) <u>Complement of a graph:</u>

If 'G' is a simple graph of order 'n', then the Complement of G in K_n is called the complement of G. It is denoted by $\bar{G}$

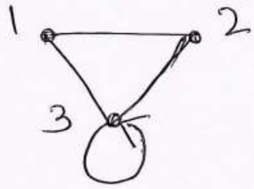
Ex:

'G' $\qquad$ $\bar{G}$

ii) <u>vertex degree:</u>

Let $G = (V, E)$ be a graph and 'v' be a vertex of G, then the <u>no</u> of edges of 'G', that are incident on V with loops counted twice is called "vertex degree".

Ex:



$\deg(v_1) = 2$

$(v_2) = 2$

$(v_3) = 3.$

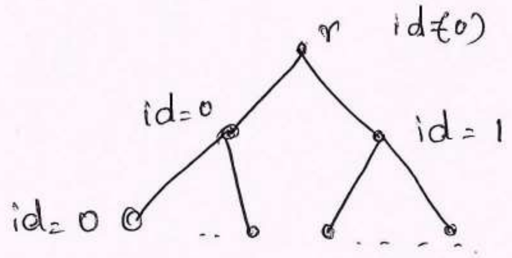
iii) <u>Rooted tree:</u>

A directed tree 'T' is called a rooted tree if

(i) T contains a unique vertex called the root whose indegree is equal to '0'

(ii) The in-degree of all other vertices of T are equal to '1'.

Ex:



iv) prefix code:

Let 'p' be a set of binary set of binary sequences that represent a set of symbols. Then 'p' is called a prefix code, if no sequence in 'p' is the prefix of any other sequence in P.

Ex. $P_1 = \{10, 0, 1101, 111, 1100\}$ is a prefix code.

$A_1 = \{01, 0, \underline{10}1, \underline{10}, 1\}$. is not a prefix code

coz 10 is sequence of other sequence.

9c) Apply merge sort.

−1, 7, 4, 11, 5, −8, 15, −3, −2, 6, 10, 3.

{−1, 7, 4, 11, 5, −8} {15, −3, −2, 6, 10, 3}.

{−1, 7, 4} {11, 5, −8} {15, −3, −2} {6, 10, 3}

{−1, 7} {4} {11, 5} {−8} {15, −3} {−2} {6, 10} {3}

{−1} {7} {4} {11} {5} {−8} {15} {−3} {−2} {6} {10} {3}

~~{−1, 7}~~

To merge.

−1, 7 4 11 5 −8 15 −3 −2 6 10 3

To merge

$-1 \quad 7 \quad 4 \quad 11 \quad 5 \quad -8 \quad 15 \quad -3 \quad -2 \quad 6 \quad 10$

$\{-1, 7\} \quad 4 \quad \{5, 11\} \quad -8 \quad \{-3, 15\} \quad \{-2\} \quad \{6, 10\}$

$\{-1, 4, 7\}, \quad \{-8, 5, 11\} \quad \{-3, -2, 15\} \quad \{3, 6,$

$\{-8, -1, 4, 5, 7, 11\} \quad \{-3, -2, 3, 6, 10, 15\}$

$\{-8, -3, -2, -1, 3, 4, 5, 6, 7, 10, 11, 15\}$

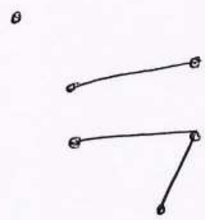
This is the sorted version of the given list.

10a) Prove that a tree with 'n' vertices has $n-1$ edges.

Pf:→ $n=1$ (proof by M I)

$n=2$

$n=3$



Result is true for $n = 1, 2, 3$.

Consider a tree with $k+1$ vertices. Remove an edge 'e' from tree. Now there are two components, both of which are trees T_1 & T_2.

Let the no of edges in T_1 & T_2 be k_1 & k_2 resply.

$k_1, k_2 \le k+1$.

No of edges in $T_1 = k_1 - 1$

" $T_2 = k_2 - 1$

Total no of edges in T_1 & T_2 $= k_1 + k_2 - 2$

$$k_1 + k_2 - 2$$
$$= (k+1) - 2$$
$$= (k-1).$$

keeping the edge e, back in its place

No of edges in $T = (k-1)+1 = k$.

So the result is true for k+1 also.

∴ Hence by Mathematical Induction, the result is true for all +ve in n.

10 b) Determine the no of vertices in G.

i) G has 9 edges & all vertices have degree 3.

Let no of vertices be 'n'

Sum of degrees of all vertices = 3n.

Since 'G' has 9 edges, we have $3n = 2 \times E$

$$3n = 2 \times 9$$
$$n = 6.$$

∴ order of $G = 6$.

ii) G has 10 edges, with 2 vertices of degree 4 and all other have degree 3.

∴ the sum of deg of all vertices $= (2 \times 4) + (n-2) \times 3$

$$\Rightarrow 2 \times 4 + (n-2) \times 3 = 2 \times 10$$
$$\Rightarrow 8 + (n-2) \times 3 = 20$$
$$(n-2) = \frac{12}{3} = 4$$
$$\underline{n = 6.}$$

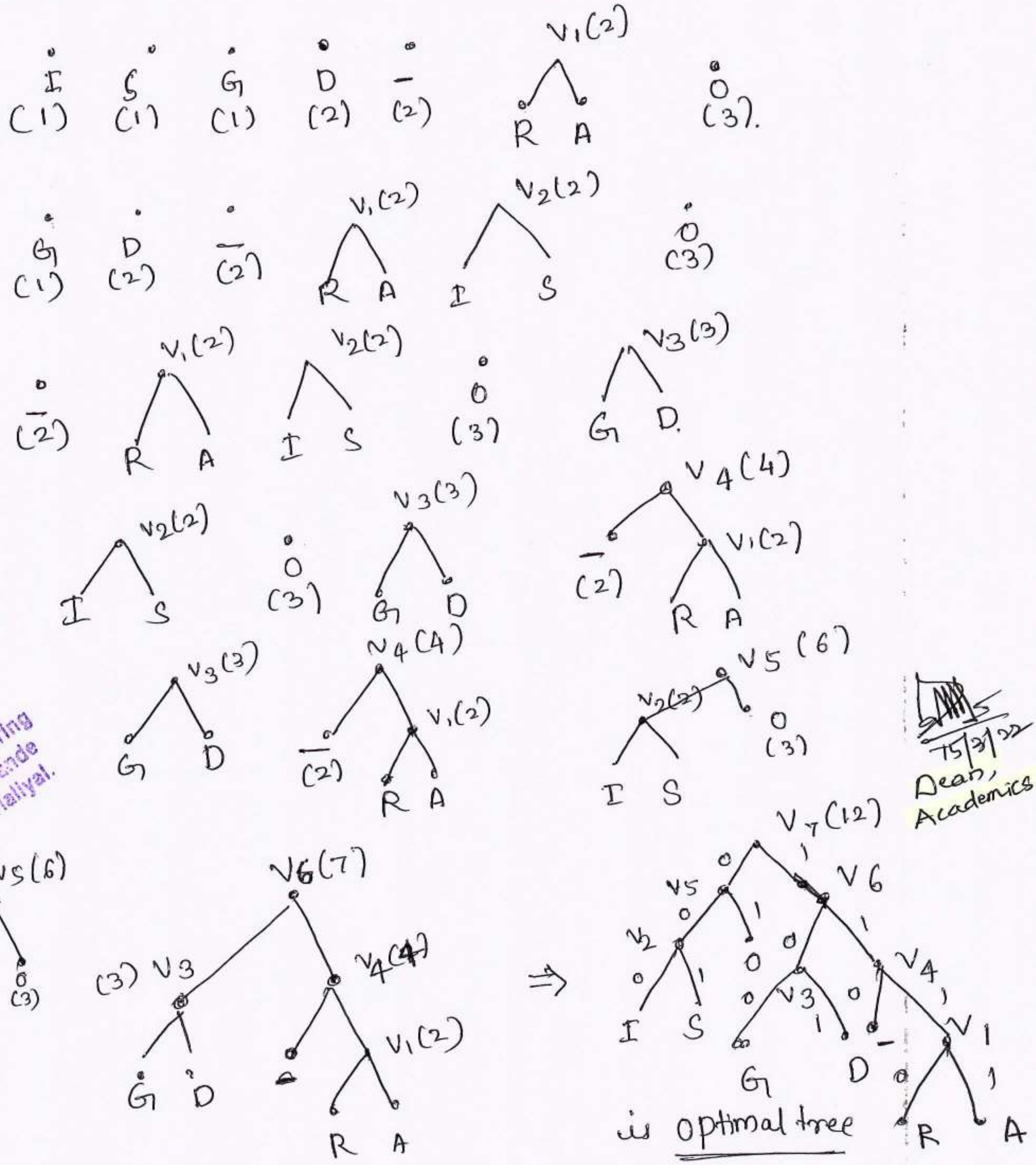
∴ order of $G = 6$.

10 c) ROAD IS GOOD.

The given message consists of the letters R, O, A, D, I, S, G with frequencies 1, 3, 1, 2, 1, 1, 1 resp'ly. Also there are 2 blank spaces (_) betn two words.

Arranging all these in non-decreasing order.

R	A	I	S	G	D	_	O
(1)	(1)	(1)	(1)	(1)	(2)	(2)	(3).



is optimal tree

R : 1111	O : 01	A : 1111	D : 101
_ : 110	I : 000	S : 001	G : 100

Code : 1111 01 1111 101 110 000 001 110 100 01 01 101

CBCS SCHEME

USN | | | | | | | | | | |

18CS36

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r) \to (p \to r)]$ is a tautology. **(06 Marks)**

b. Test the validity of the following argument.

If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
―――――――――――――――
∴ I must have watched TV in the evenings **(07 Marks)**

c. Let $p(x) : x^2 - 7x + 10 = 0$, $q(x) : x^2 - 2x + 3 = 0$, $r(x) : x < 0$. Find the truth or falsity of the following statements, when the universe U contains only the integers 2 and 5.

(i) $\forall x, p(x) \to \sim r(x)$ (ii) $\forall x, q(x) \to r(x)$
(iii) $\exists x, q(x) \to r(x)$ (iv) $\exists x, p(x) \to r(x)$ **(07 Marks)**

OR

2 a. Prove that, for any three propositions p, q, r
$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$ **(06 Marks)**

b. Prove that, the following are valid arguments:

(i)
$$\begin{array}{l} p \to (q \to r) \\ \sim q \to \sim p \\ p \\ \hline \therefore r \end{array}$$

(ii)
$$\begin{array}{l} \sim p \leftrightarrow q \\ q \to r \\ \sim r \\ \hline \therefore p \end{array}$$
(07 Marks)

c. Give :
(i) a direct proof
(ii) an indirect proof.
(iii) proof by contradiction for the following statement.
"If n is an odd integer, then n+9 is an even integer". **(07 Marks)**

Module-2

3 a. Prove that for each $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$. **(06 Marks)**

b. Determine the coefficient of,
(i) xyz^2 in the expansion of $(2x - y - z)^4$.
(ii) $x^2y^2z^3$ in the expansion of $(3x - 2y - 4z)^7$. **(07 Marks)**

c. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
(i) There is no restriction on the choice.
(ii) Two particular persons will not attend separately.
(iii) Two particular persons will not attend together. **(07 Marks)**

OR

4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and / or 7's. **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four A's are together? How many of them begin with S? **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers, so that, (i) No containers is left empty.
(ii) The fourth container gets an odd number of balls. **(07 Marks)**

Module-3

5 a. For any non empty sets A, B, C prove that,
(i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$
(ii) $(A\times(B-C))=(A\times B)-(A\times C)$ **(06 Marks)**

b. Let $f: R \to R$ be defined by $f(x)=\begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x\leq 0 \end{cases}$.
(i) Determine f(0), $f\left(\frac{5}{3}\right)$ (ii) Find $f^{-1}([-5,5])$. **(07 Marks)**

c. Let f, g, h be functions form z to z defined by $f(x)=x-1$, $g(x)=3x$, $h(x)=\begin{cases} 0 & \text{if x is even} \\ 1 & \text{if x is odd} \end{cases}$. Verify that $(f\circ g)\circ h(x)=f\circ(g\circ h)(x)$. **(07 Marks)**

OR

6 a. Let $A=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if "a is a multiple of b". Represent the relation R as a matrix and draw its diagraph. **(06 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(07 Marks)**

c. Let $A=\{1,2,3,4,5\}$, define a relation R on $A\times A$, by $(x_1,y_1)R(x_2,y_2)$ if and only if $x_1+y_1=x_2+y_2$
(i) Verify that R is an equivalence relation.
(ii) Find the partition of $A\times A$ induced by R. **(07 Marks)**

Module-4

7 a. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. **(06 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(07 Marks)**

c. By using the expansion formula, obtain the rook polynomial for the board C. **(07 Marks)**

		1
	2	3
4	5	6
7	8	

OR

8 a. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple. The boy B_3 does not want banana or mango, and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(06 Marks)**

b. If $a_0=0$, $a_1=1$, $a_2=4$ and $a_4=37$ satisfy the recurrence relation $a_{n+2}+ba_{n+1}+ca_n=0$, for $n\geq 0$, find the constants b and c, and solve the relation a_n. **(07 Marks)**

c. How many integers between 1 and 300 (inclusive) are,
(i) Divisible by at least one of 5, 6, 8?
(ii) Divisible by none of 5, 6, 8? **(07 Marks)**

Module-5

9 a. Show that the following two graphs shown in Fig. Q9 (a) – (i) and Fig. Q9 (a) – (ii) are isomorphic. **(06 Marks)**

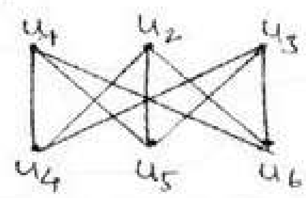


Fig. Q9 (a) – (i)

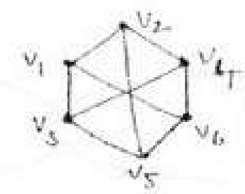


Fig. Q9 (a) – (ii)

b. Define the following with example of each.
   (i) Simple graph (ii) Sub graph
   (iii) Compliment of a graph (iv) Spanning sub graph **(07 Marks)**

c. Construct an optimal profix code for the symbols a, o, q, u, y, z that occurs with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

OR

10 a. Prove that two simple graphs G_1 and G_2 are isomorphic if and only if their complements are isomorphic. **(06 Marks)**

b. Let $G = (V, E)$ be a simple graph of order $|V| = n$ and size $|E| = m$, if G is a bipartite graph. Prove that $4m \le n^2$. **(07 Marks)**

c. Construct an optimal prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(07 Marks)**

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CBCS SCHEME

USN 18CS36

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that for any propositions p, q, r the compound proposition
$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology. **(08 Marks)**

b. Prove the logical equivalence without using truth table:
$p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$ **(05 Marks)**

c. Find whether the following argument is valid. No engineering student of first or second semester studies logic.
Anil is an Engineering student who studies logic
$\therefore$ Anil is not in second semester **(07 Marks)**

OR

2 a. Give a direct proof and an indirect proof for the given statement. "If 'n' is an odd integer, then n + 9 is an even integer". **(06 Marks)**

b. Prove the given logical equivalence problem using laws of logic.
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(07 Marks)**

c. Verify the given argument is valid or not?

$$\begin{array}{l} p \to (q \to r) \\ p \vee \neg s \\ q \\ \hline \therefore\ s \to r \end{array}$$
(07 Marks)

Module-2

3 a. Prove that for each $n \in z^+$
$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = 1/6\, n(n + 1)(2n + 1)$ **(07 Marks)**

b. Find the number of permutation of the letter of the word "MASSASAUGA". In how many of there all four 'A's are together? How many of them begin with 'S'? **(06 Marks)**

c. Find how many distinct four digit integers one can make from the digit 1, 3, 3, 7, 7, 8. **(07 Marks)**

OR

4 a. Determine the co-efficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(06 Marks)**

b. In how many ways can 10 identical pencils be distributed among 5 children in following cases:
i) There are no restrictions.
ii) Each child gets atleast one pencil.
iii) The youngest child gets at least two pencils. **(07 Marks)**

c. Find the number of arrangements of all the letters in "TALLAHASSEE"? How many of these arrangement have no adjacent 'A's? **(07 Marks)**

Module-3

5 a. Let $f : R \rightarrow R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(-6)$, $f^{-1}([-5, 5])$. **(07 Marks)**

b. On the set Z^{-1} a relation 'R' is defined by aRb if and only if "a divides b (exactly)" verify that 'R' is equivalence relation. **(06 Marks)**

c. Draw the Hasse diagram representing the positive divisor of 36. **(07 Marks)**

OR

6 a. Let A = {1, 2, 3, 4, 5} define relation 'R' on A×A by (X_1Y_1) R (X_2Y_2) if and only if $X_1 + Y_1 = X_2 + Y_2$.

i) Verify 'R' is a equivalence relation on A×A

ii) Determine the partition of A×A induced by R. **(07 Marks)**

b. Let A = {1, 2, 3, 4, 6} and 'R' be a relation on 'A' defined by aRb if and only if "a is multiple of b" represent the relation 'R' as a matrix, draw its diagraph and relation R. **(06 Marks)**

c. Let f, g, h be a function from R to R defined by $f(x) = x + 2$, $g(x) = x - 2$, $h(x) = 3x$ for $\forall x \in R$ find gof, fog, fof, gog, foh, fohog. **(07 Marks)**

Module-4

7 a. How many integers between 1 and 300 (inclusive) are

i) Divisible by atleast one of 5, 6, 8

ii) Divisible by none of 5, 6, 8. **(07 Marks)**

b. Find the rook polynomial for the 3 × 3 board by using the expansion formula. **(07 Marks)**

c. Solve the recurrence relation

$a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(06 Marks)**

OR

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the recurrence relation

$a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given that $a_0 = 1$ and $a_1 = 2$. **(07 Marks)**

c. Compute derangement of d_4, d_5, d_6, d_7. **(07 Marks)**

Module-5

9 a. Define Isomorphism. Verify the given two graphs are Isomorphic (Fig.Q.9(a)). **(07 Marks)**

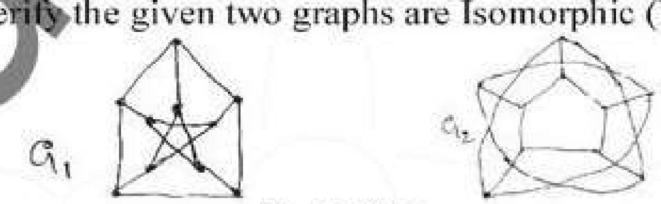

Fig.Q.9(a)

b. "A tree with 'n' vertices has n – 1 edges". Prove this. Define a tree. **(06 Marks)**

c. Construct an optimal prefix code for the given set of frequencies, 20, 28, 4, 17, 12, 7. **(07 Marks)**

OR

10 a. Explain complete graph, Bipartite graph, subgraph, regular graph, spanning subgraph, minimally connected graph, with example for each. **(07 Marks)**

b. Apply merge sort to the given list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. **(06 Marks)**

c. Obtain an optimal prefix code for the message "LETTER RECEIVED" indicate the code. **(07 Marks)**

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CBCS SCHEME

USN 18CS36

Third Semester B.E. Degree Examination, Aug./Sept.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define proposition, tautology, contradiction. Determine whether the following compound statement is a tautology or not.

$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$ **(06 Marks)**

b. Using the laws of logic, show that

$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ **(07 Marks)**

c. Establish the validity of the following argument :

$\forall x, p(x) \vee q(x)$
$\exists x, \neg p(x)$
$\forall x, \neg q(x) \vee r(x)$
$\underline{\forall x, s(x) \rightarrow \neg r(x)}$
$\therefore \exists x, \neg s(x)$ **(07 Marks)**

OR

2 a. Define Converse, Inverse and Contrapositive of a conditional. Find converse, inverse and contrapositive of $\forall x, (x > 3) \rightarrow (x^2 > 9)$, where universal set is R. **(06 Marks)**

b. Test the validity of the following arguments:

(i) If there is a strike by students, the exam will be postponed but the exam was not postponed

$\therefore$ there was no strike by students

(ii) If Ram studies, then he will pass in DMS. If Ram doesn't play cricket, then he will study. Ram failed in DMS.

$\therefore$ Ram played cricket **(06 Marks)**

c. Let $p(x) : x \geq 0$

$q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$, then

for the universe completing of all real numbers, find the truth value of

(i) $\exists x \{p(x) \wedge q(x)\}$ (ii) $\forall x \{p(x) \rightarrow q(x)\}$ (iii) $\exists x \{\phi(x) \wedge r(x)\}$ **(06 Marks)**

d. Define dual of logical statement. Write the dual of the statement

$(p \vee T_o) \wedge (q \vee F_o) \vee (r \wedge s \wedge T_o)$ **(02 Marks)**

Module-2

3 a. Define well ordering principle and prove the following by mathematical induction.

(i) $1^2 + 3^2 + 5^2 + \ldots\ldots(2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$

(ii) $1*3 + 2*4 + 3*5 + \ldots\ldots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$ **(12 Marks)**

b. Find the coefficients of

(i) x^9y^3 in the expansion of $(2x-3y)^{12}$

(ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ **(08 Marks)**

Module-5

9 a. Merge sort the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(06 Marks)**

b. Determine whether the following graphs are isomorphic or not. [Refer Fig.Q9(b)]

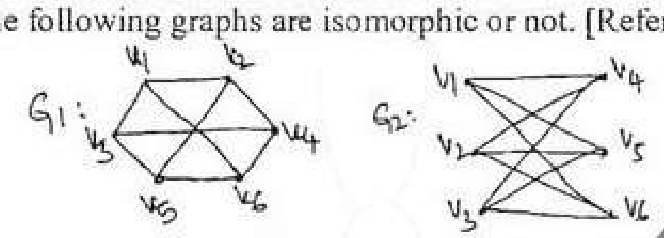


Fig.Q9(b) **(06 Marks)**

c. Define the following with an example to each :
(i) Simple graph (ii) Complete graph (iii) Tree (iv) Regular graph
(v) Spanning subgraph (vi) Induced sub graph (vii) Complete Bipartite graph
(viii) Complement of graph. **(08 Marks)**

OR

10 a. Let G : (V, E) be a connected undirected graph, what is the largest possible value for |V| if $|E| = 19$ and $\deg(V) \geq 4$ for all $v \in V$? **(06 Marks)**

b. Construct an optional prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(08 Marks)**

c. T : (V, E) is a complete m-ary tree with $|V| = n$, if T has ℓ leaves and i internal vertices, prove the following results:
(i) $n = mi + 1$
(ii) $\ell = (m-1)i + 1$
(iii) $i = \dfrac{\ell - 1}{m-1} = \dfrac{n-1}{m}$ **(06 Marks)**

GCEM **CSE** **DMS (18CS36)** **Semester: 3**

Important Questions for VTU Examination (Dec.-2019)

MODULE-1

(I) DEFINITIONS with example for each:

Proposition, Tautology, Contradiction, Open sentence, Disjunction (OR), Conjunction (AND), Negation, Quantifier, compound statement, Converse, Inverse, and Contra Positive. etc.,

(II) Problems on TAUTOLOGY, CONTRADICTION, CONTINGENCY:

Examples:

(1) Prove that, for any propositions p, q, r , the compound propositions, are tautologies.

i) $\{(p \to q) \wedge (q \to r)\} \to \{(p \to r)\}$ ii) $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$

(2) Determine whether the following statement is tautology or not.

$(p \to (q \vee r)) \leftrightarrow ((p \wedge \neg q) \to r)$

(III) Problems on LOGICALLY EQUIVALENT STATEMENTS: (using truth tables)

Examples:

(1) By constructing the truth table, show that the compound propositions $p \wedge (\neg q \vee r)$ and $p \vee (\ \wedge \neg r)$ are not logically equivalent.

(2) Use truth tables to verify,

(i) $[p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$ (ii) $[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$

(iii) $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \to q) \wedge (q \to r) \wedge (r \to p)]$

(IV) Problems on LOGICALLY EQUIVALENT STATEMENT USING Laws of Logic:

Examples:

(1) Define logical equivalence of two propositions. Prove the following logical equivalences without using the truth tables (using laws of logic):

i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$ ii) $[(\neg p \vee \neg q) \longrightarrow (p \wedge q \wedge r) \Leftrightarrow \ \wedge q$

(iii) $(p \to q) \wedge (\neg q \wedge (r \vee \neg q)) \Leftrightarrow \neg[q \vee p]$

(2) Show that $[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is tautology using laws of logic.

(V) Problems on TRUTH TABLES AND INDEPENDENTS OF ITS COMPONENTS:

Examples:

(1) Find the possible truth values of p, q and r if (i) $p \to (q \vee r)$ is FALSE

(ii) $p \wedge (q \to r)$ is TRUE

(2) Show that $(p \wedge (p \to q)) \to q$ is independent of its components.

(3) Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following: (i) $p \wedge$ (ii) $\neg p \vee q$ (iii) $q \to p$

(4) Let p, q, r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions: i) $(p \wedge q) \to r$ ii) $p \to (q \wedge r)$ iii) $p \wedge (r \to q)$ iv) $\to (q \to \neg r)$.

(5) Show that the truth values of the following statements are independent of their components: i) $[p \wedge (p \to q)] \to q$ ii) $(p \to q) \leftrightarrow [\neg p \vee q]$

(VI) Problems on DUAL AND PRINCIPLE OF DUALITY:

Examples:

(1) Verify the principle of duality for the logical equivalence:

$$\sim(p \wedge q) \rightarrow \sim p \vee (\sim p \vee q) \Leftrightarrow \sim p \vee q.$$

(2) Define dual of logical statement. Write the dual of the following logical statements:

i) $(p \vee T_0) \wedge (q \vee r) \vee (r \wedge s \wedge T_0)$ **(**ii)$(p \wedge q) \vee T_0$ (iii) $[\neg(p \vee q) \wedge \{p \vee \neg(q \wedge \neg s)\}]$

(VII) Problems on DIRECT PROOF, PROOF BY CONTRADICTION, INDIRECT PROOF:

Examples:

(1) Give : (i) A direct proof (ii) An indirect proof and (iii) Proof by contradiction, for the following statement: "If n is an even integer, then (n + 7) is an even integer".

(2) Give a direct proof for each of the following. (i) For all integers k and l, if k, l are both even, then $k + l$ is even. (ii) For all integers k and l, if k, l are even, then $k.\ l$ is even.

(3) Prove that for every integer n, n^2 is even if and only if n is even.

(4) Give a direct proof of the statement "the square of an odd integer is an odd integer".

(5) Give i) direct proof ii) indirect proof iii) proof by contradiction for the following statement:

" if n is an odd integer then n+9 is an even integer".

(VIII) Problems on QUANTIFIERS, TRUTH VALUES OF QUANTIFIED STATEMENTS:

(1) Determine the truth value of each of the following quantified statements for the set of all non-zero integers: i) $\exists x, \exists[xy = 1]$ (ii) $\forall x, \exists y[xy = 1]$ (iii) $\exists x, \exists y[(2x + y = 5) \wedge (x - 3y = -8)]$ iv) $\exists x, \exists y, [(3x - \quad = 17) \wedge (2x + 4y = 3)]$ (v) $\exists x, \forall y, [xy = 1]$

(2) Determine the truth value of each of the following quantified statements for the set of all non-zero integers: i) $\exists x, \exists y, [xy = 2]$ (ii) $\forall x, \exists y, [xy = 2]$ iii) $\exists x, \forall y, [xy = 2]$ (iv) $\exists x, \exists y, [(3x + \quad = 8) \wedge (2x - y = 7)]$ (v) $\exists x, \exists y, [(4x + 2y = 3) \wedge (x - y = 1)]$

(3) Consider the following open statements with the set of all real numbers as the universe,

$p(x)$: $x \geq 0$, $q(x) : x^2 \geq 0$, q(x): $x^2 - 3x - 4 = 0$ and $s(x)$: $x^2 - 3 > 0$, then find the truth values of (i) $\exists x, [(x) \wedge r(x)]$ (ii) $\forall x, [p(x) \rightarrow q(x)]$ (iii) $\forall x, [q(x) \rightarrow s(x)]$

(4) Let (x): $x \geq 0$, q(x): $x^2 \geq 0$ and $r(x)$: $x^2 - 3x - 4 = 0$. Then for the universe comprising of all real numbers, find the truth values of, (i) $\exists x, [(x) \wedge q(x)]$ (ii) $\forall x, [p(x) \rightarrow q(x)]$ (iii) $\exists x, [p(x) \wedge r(x)]$ (iv) $\forall x, [(x) \rightarrow s(x)]$ (v) $\forall x, [r(x) \rightarrow p(x)]$ (vi) $\forall x, [r(x) \vee q(x)]$

(IX) Problems on CONVERSE, INVERSE, CONTRA POSITIVE PROBLEMS:

Examples:

(1) Find converse and contra positive of the statement: p → (q∧r).

(2) Consider the sentence ," if 5x-1=9 then x=2" i.e., : p → q . Find converse, inverse and contra positive statement.

(X) Problems on VALIDITY OF THE ARGUMENT:

(1) Prove that the following argument is valid**.**

$$\begin{array}{l} \forall x, [p(x) \rightarrow q(x)] \\ \underline{\forall x, [q(x) \rightarrow r(x)]} \\ \therefore \forall x, [p(x) \rightarrow r(x)] \end{array}$$

(2) Establish the validity of the following Argument

$$\begin{array}{l} p \rightarrow r \\ \sim p \rightarrow q \\ \underline{q \rightarrow s} \\ \therefore \sim r \rightarrow s \end{array}$$

------------------------------------End of 1st Module------------------------------------

NOTE: -For PASSING MARKS, prepare Q.No. (II), (III), (IV), (V),(VII) and (VIII).
For good marks, prepare (I) to (X) ,(all) .

MODULE-2

(I) Problems on MATHEMATICS INDUCTION:

Examples:

(1) By the principle of .Mathematical Induction, prove that, $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(2) Define Principle of Mathematical Induction. For the Fibonacci sequence $F_0, F_1, F_2, F_3, \ldots\ldots,$ Prove that, $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$

(3) By Mathematical Induction, Prove that, for any positive integer n, the number $A_n = 5^n + 2 \times 3^{n-1} + 1$ is a multiple of 8.

[OR] Prove by mathematical induction, for every integer 8 divides $5^n + 2 \times 3^{n-1} + 1$.

(4) Prove the following: [or] Prove by using principle of mathematical induction

(i) $\sum_{i=1}^{n} i\,(2^i) = 2 + (n-1)2^{n+1}$ (ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(5) Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$.

(6) Prove by mathematical induction that, for every positive integer n, 5 divides $(n^5 - n)$. etc.,

(II) Problems on BINOMIAL THEOREM:

Examples: (1) Find the coefficient of x^9y^3 in the expansion of $(2x-3y)^{12}$

(2) Find the coefficient of x^{12} in the expansion of $x^3(1-2x)^{10}$ etc.,

(III) Problems on MULTINOMIAL THEOREM:

Example: (1) Find the coefficient of $a^2b^3c^2d^5$ and $a^3bc^4d^2$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$

(2) Find the number of distinct terms in the expansion of $(x_1 + x_2 + x_3 + x_4 + x_5)^{16}$

(3) Find the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$ etc.,

(IV) Problems on PERMUTATIONS, COMBINATIONS AND PRODUCT RULE:

Examples:

1) In the word ENGINEERING, (i) Find the number of arrangements of all the letters of the word ENGINEERING? (ii) How many different strings of length 4 can be formed?
2) In the word SOCIOLOGICAL, (i) How many arrangements are there for all the letters? (ii) In how many of these arrangements all vowels are adjacent?
3) Find The number of nonnegative integer solutions of the equation $X_1+X_2+X_3+X_4+X_5=8$
4) A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting at least 2 questions from each part?
5) A certain question paper contains three parts A,B,C with four questions in part A, five questions in part B and six questions in part C. it is required to answer seven questions selecting at least two questions from each part. In how many ways different ways can a student select his seven questions for answering?
6) How many positive integers n can we form using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000? etc.,

******* END OF 2nd MODULE ******

NOTE:

For PASSING MARKS, prepare Q.No. (I), (II) and (III).
For good marks, prepare (I) to (IV) ,(all) .

MODULE-3

RELATIONS

(I) DEFINITIONS with example for each:

Relation, Types/properties of relations (reflexive, transitive, symmetric, anti-symmetric, irrreflexive, asymmetric), partial order relation, equivalence relation, lattice, Hasse diagram/poset diagram etc.,

(II) Problems on types of relations/properties of relations:

Examples:

(1) Let A= $\{1,2,3,4\}$, R= $\{(1,3),(1,1),(3,1),(1,2),(3,3),(4,4)\}$ be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive.

(2) Let A= $\{1,2,3,4\}$, and let R be the relation defined by $R = \{(x,y)|x,y \in A,\ x \le y\}$. Determine whether R is reflexive, symmetric, antisymmetric or transitive.

(III) Problems on EQUIVALENCE REALTION and PARTIAL ORDER RELATION :

Examples:

Type-1 problems:

(1) Define a relation R on AXA by $(x_1,y_1)R(x_2,y_2)$ iff $x_1+y_1=x_2+y_2$ where A={1,2,3,4,5}. Verify that R is an equivalence relation.

(2) Define a relation R on the set A={1,2,3} is as follows. R={(1,1)(2,3)(2,2)(3,3)(3,2}. Verify that R is an Equivalence relation and partial order relation.

Type-2 problems:

Prove the following:

(i) on the set of all integers, the relation R defined by aRb if and only if $a \leq b+1$ is reflexive but not irreflexiv.

(ii) On the set of all integers, the relation R defined by aRb if and only if |a-b|=2 is irreflexive and symmetric.

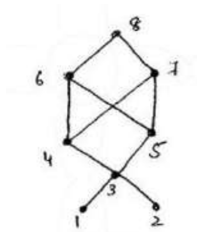
(IV) Problems on HASEE DIAGRAM/POSET DIAGRAM, LUB ,GLB etc.,:

Examples:

(1) Draw Hasse diagram representing the positive divisors of 36 (i.e., D_{36}).

(2) Let A={1,2,3,4,6,12}. On A define the relation R by aRb if and only if ‘a’ divides ‘b’. Draw Hasse diagram and write down the matrix relation,

(3) Consider the Hasse diagram of a Poset (A, R) as shown in figure. If B = {3,4,5}, find (i) all upper bounds of B (ii) all lower bounds of B (iii) the least upper bound of B (iv) the greatest lower bound of B (v) is this a Lattice (vi) maximal/greatest element of hasse diagram ?



(4) Let A={1,2,3,6,9,8} and R on A by xRy if x/y (i.e., x divides y). Draw the Hasse diagram for the post (A,R).

(V) MISCELLANEOUS PROBLEMS ON RELATIONS:

Examples:

(1) Matrix form/relation matrix of a given relation , Directed graph/digraph of a given relation, composition of relation RoS, R^2,R^3 etc., problems

(2) Number of relations on a given set problems, Equivalence classes etc., problems.

FUNCTIONS

(I) Problems on COMPOSITION OF FUNCTIONS:

Examples:

(1) Let f,g, and h be functions from Z to Z defined by,

f(x)=x-1, g(x)=3x and $h(x)=\begin{cases} 0, & if\ x\ is\ even \\ 1, & if\ x\ is\ odd \end{cases}$

- Determine (fo(goh))(x) and ((fog)oh)(x). (ii) Verify that fo(goh)=(fog)oh.

(2) Let f and g functions from R to R defined by

$f(x) = ax + b$ and $b(x) = 1 - x + x^2$ .If $(gof)(x) = 3 - 9x + 9x^2$. Determine a and b values.

(3) Let f,g,h be functions from R to R defined by f(x)=x+2,g(x)=x-2 and h(x)=3x. Find gof, fog, fof, hog and foh.

(II) Problems on No. Functions, one-to-one/ Injective and onto/ Surjective functions:

Examples:

(1) Let A= {1,2,3,4} and B = {1, 2, 3, 4, 5, 6} be two sets.

(i) Find how many functions are there from A to B? How many of these are one to one? How many are onto?

(ii) Find how many functions are there from B to A? How many of these are onto? How many are one to one?

(2) let A={1,2,3,4,5,6,7} and B={w,x,y,z}. Find the number of onto functions from A to B.

(3) The functions f:R->R and g:R->R are defined by, f(x)=3x+7 and $g(x)=x(x^3-1)$. Verify that f (x) is one-to-one but g(x) is not.

(4) Consider the function f:R->R defined by $f(x)=x^2$. Determine whether f is one-to-one or onto. If f is not onto, find its range. Is f invertible?

(III) Problems on INVERSE OF FUNCTION / INVERTIBLE FUNCTIONS:

Examples:

(1) Let f: R → R be defined by, $h(x)=\begin{cases} 3x - 5, & if\ x > 0 \\ 1 - 3x, & if\ x \le 0 \end{cases}$

Determine f(0), f(-1), $f^{-1}(1)$, $f^{-1}([-5, 5])$. $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(6)$, and $f^{-1}([-6, 5])$.

(2) If f: A → B, g: B → C are invertible functions, then prove that g ∘ f: A → C is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(3) Let A=B=R be the set of real numbers, the functions f and g from R to R defined by $f(x) = 2x^3 - 1$ and $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}$. Show that, each of f and g is the inverse of the other.

(4) Consider two functions f(x)=2x+5 and $g(x)=\frac{1}{2}(x-5)$. Prove that, g is an inverse of f.

---------------------------------------End of 3rd Module---------------------------------------

NOTE: **For PASSING MARKS, prepare either FUNCTIONS (or) RELATIONS.**

For good marks, prepare both Functions and Relations,(all) .

MODULE-4

(I) Problems on RECURRENCE RELATIONS:

Examples:

(1) Find the recurrence relation and the initial conditions for the sequence 0,2,6,12,20,30,42,……..and hence find the general term of the sequence.

(2) Slove the recurrence relation,

$$(i) a_n + a_{n-1} - 6a_{n-2} = 0 \text{ for } n \geq 2 \text{ with } a_0 = -1 \text{ and } a_1 = 8.$$

$$(ii) a_n = 2(a_{n-1} - a_{n-2}) \text{ for } n \geq 2 \text{ with } a_0 = 1 \text{ and } a_1 = 2.$$

(3) A sequence $\{C_n\}$ is defined recursively by, $C_n = 3C_{n-1} - 2C_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ $and C_2 = 3$ as the initial conditions, show that $C_n = -2^n + 7$.

(4) Solve the recurrence relation

$$D_n = bD_{n-1} - b^2 D_{n-2} \quad \text{for } n \geq 3, \text{ given that } D_1=b>0 \text{ and } D_2=0.$$

(5) Solve the following Recurrence Relation

$$a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0 \text{ for } n \geq 0, \text{ given } a_0 = 4 \text{ and } a_1 = 13.$$

(6) The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **etc.,**

(II) Problems on PRINCIPLE OF INCLUSION AND EXCLUSION:

Examples:

(1) How many integers between 1 and 300 (inclusive) which are divisible by (i) at least one of 5, 6, 8 (ii) divisible by none of 5, 6, 8? (iii) at least two of 5,6,8. (i) exactly two of 5,6,8 and (ii) divisible by at least two of 5,6,8.

(2) Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.

(3) In a survey of 260 students, the following data were obtained. 64 had taken mathematics, 94 had taken CS, 58 had taken EC, 28 had taken both mathematics and EC, 26 had taken both mathematics and CS, 22 had taken both CS and EC, and 14 had taken all three types of courses. Determine how many of these students had taken none of the three courses.

(4) In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG,P UN or BYTE occurs?

(5) In how many ways can one arrange the letters in the word CORRESPONDENTS so that there is no pair of consecutive identical letters? **etc.,**

(III) Problems on ROOK POLYNOMIAL:

Examples:

(1) Define Rook Polynomial. Find the ROOK polynomial for 3X3 board using the expansion formula.
(2) An apple ,a banana, a mango and an orange are to be distributed among 4 boys B1,B2,B3 and B4. The boys B1 and B2 do not wish to have an apple, the boy B3 does not want banana or mango and B4 refuses orange. In how many ways the distribution can made so that no boy is displeased.
(3) Five teachers T1,T2,T3,T4,T5 are to be made class teachers for five classes C1,C2 ,C3,C4,C5, one teacher for each class. T1 and T2 do not wish to become the class teachers for C1 or C2, T3 and T4 for C4 or C5 and T5 for C3 or C4 or C5. (i) Construct Board for the given constraints. (ii) Find Rook polynomial (iii) In how many ways can the teacher be assigned work without displeasing any teacher.
(4) Write expansion and product formula. Find the Rook polynomial for 2X2 board using the expansion formula.

(IV) Problems on DERANGEMENTS:

Examples: (1) Define derangements. Evaluate d_5 , d_6, d_7 , d_8 , d_9.
(2) Find the number of derangements of 1,2,3,4. Also list out all derangements.
(3) How many permutations of 1,2,3,4,5,6,7 are not derangements?

--- ***End of 4th Module*** ---

***NOTE*: For PASSING MARKS, prepare Q.No. (I) and (II).**
For good marks, prepare (I) to (IV) ,(all) .

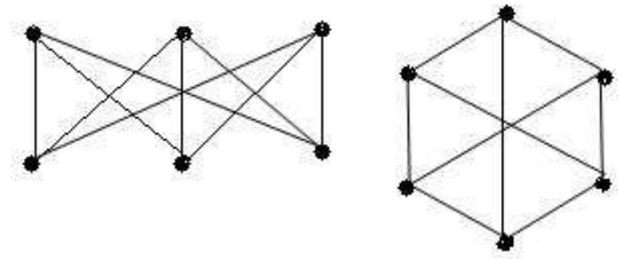
MODULE-5

***I) DEFINITIONS with example for each*:**

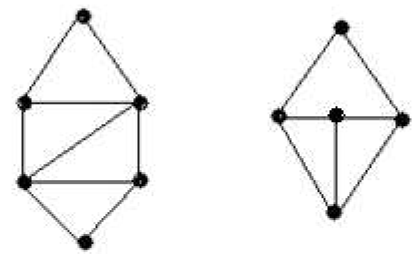
Graph, Simple Graph, Complement, Isomorphism, Multi-Graph, Complete Graph, Induced Sub Graph, Regular Graph, Degree, Sub Graph, Path, Cycle, Circuit, Walk, Trail, Spanning Sub Graph, Tree, Forest, Rooted Tree, Spanning Tree, Binary Tree, Complete Binary Tree, Full Binary Tree, Prefix Code, Balanced Binary Tree, Order of a Graph |V|, Connected Graph, Disconnected Graph, Pendant Vertex, Weight of a f A Tree, Optimal Tree Etc.,

II) Problems on TWO GRAPHS G AND H ARE ISOMORPHIC OR NOT:

Examples: (1) Define Isomorphism. Determine whether the following graphs are isomorphic.



(2) Verify the two graphs are isomorphic or not.



<u>III) Problems on OPTIMAL PREFIX CODE:</u> (Symbols with frequencies)

<u>Examples:</u>

(1) Construct an optimal prefix code for the symbols a,b,c,d,e,f,g,h,I,j that occur with respective frequencies 78,16,30,35,125,31,20,50,80,3.

(2) Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively.

<u>IV) Problems on OPTIMAL PREFIX CODE:</u> (For a given message)

<u>Examples:</u>

(1) Obtain the optimal prefix code for the message ROAD IS GOOD. Indicate the code.

(2)Obtain an optimal prefix code for the message LETTER RECEIVED. Indicate the code.

(3) Obtain the prefix code for the message MISSION SUCCESSFUL. Indicate the code.

<u>V) Problems on OPTIMAL TREE (OR) HUFFMAN TREE:</u>

<u>Example:</u> Define optimal tree and construct an optimal tree for a given set of weights {4,15,25,8,16}. Hence find the weight of the optimal tree.

<u>VI) PROBLEMS ON SORTING (MERGE SORT):</u>

<u>Example:</u>

(1) Apply the merge sort to the following given list of elements. {-1,0,2,-2,3,6,-3,5,1,4}

(2) Apply the merge sort the following list. Draw the splitting and merging trees for each application of the procedure. merge sort to the list {6,2,7,3,4,9,5,1,8}

<u>VII) Problems on DEGREE OF VERTEX and HANDSHAKING THEOREM:</u>

<u>Example:</u>

(1) Determine the order |V| of the graph G=(V,E) in the following cases.

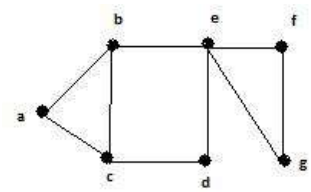
(i) G is regular graph with 15 edges . (ii) G has 10 edges with 2 vertices of degree 4 and all other of degree3. (iii) G has 20 edges with 4 vertices of degree 3, 2 vertices of degree 4 and remaining vertices of degree (iv) G is cubic graph with 9 edges.

(2) A tree has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, how many pendant vertices (degree one) does it have?
(3) If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, find the number of leaves (degree one)in T?

(4) Show that there exist no simple graphs corresponding to the following degree sequences:
i) 0, 2, 2, 3, 4 ii) 1, 1, 2, 3 iii) 2, 3, 3, 4, 5, 6 iv) 2, 2, 4, 6

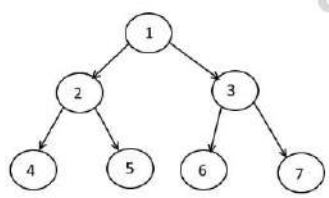
VIII) Problems on CYCLE, CIRCUIT, WALK, PATH, LENGTH OF A CYCLE:

Example: In the graph shown below, determine the following:
i) Walk from b to d that is not a trail. ii) b – d trail that is not a path iii) path from b to d
iv) closed walk from b to b that is not a circuit v) a circuit from b to b that is not a cycle
vi) a cycle from b to b. vii) Cycle from e to e with length 5.



IX) Problems on PREORDER, POSTORDER AND INORDER TRAVERSAL:

Example: For the tree shown below, list the vertices according to a ***preorder, inorder and postorder*** traversal.



(X) Important THEOREMS:

(1) Show that a tree with n vertices has n-1 edge. [or] Show that, in a tree |E|=|v|-1 [or] Prove that, in a tree, |V|=|E|+1

(2) Prove that in a graph, the number of vertices of odd degree is even.

(3) State and prove Handshaking theorem. [or] Prove that in a graph, the sum of the degrees of all the vertices is an even number.

---------------------------------------**End of 5th Module** ---------------------------------------

***NOTE*: For PASSING MARKS, prepare Q.No. (II), (III), (IV), (VII) and (X).**
For good marks, prepare (I) to (X) ,(all) .

For any further **queries/doubts, please** let me know.

****** **Good Luck for Your Exams and Do The Best** *******