Reg. No.										

END SEMESTER EXAMINATIONS APRIL / MAY 2014

FOURH SEMETER

B. Tech. Information Technology

MA8451 Discrete Mathematics

Time : 3 Hours **Answer ALL Questions** **Max. Mark : 100**

Part – A (10$\times$2 = 20 Marks)

1. Without using truth table show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

2. Symbolize the statement, "Given any positive integer, there is a greater positive integer", without using the set of positive integers as the universe of discourse.

3. Using mathematical induction show that $2^n < n!$ for every positive integer *n* with $n \geq 4$.

4. Show that if 30 dictionaries in a library contains a total of 61327 pages, then one of the dictionaries must have at least 2045 pages.

5. For a simple graph *G*, show that $\delta(G) \leq \frac{2|E(G)|}{|V(G)|} \leq \Delta(G)$, where $\delta(G)$ is the minimum degree of *G* and $\Delta(G)$ is the maximum degree of *G*.

6. Is there any disconnected graph *G* with $\delta(G) \geq \left|\frac{V(G)}{2}\right|$ and $|V(G)| \geq 3$? Justify your answer.

7. Show that if f is a homomorphism from a group $(G,*)$ into a group (H,Δ), then $f(e_G) = e_H$ and $f(a^{-1}) = (f(a))^{-1}$, for $a \in G$.

8. Let $(G,*)$ be a group and let $S = \{a \in G \mid a*x = x*a, \forall x \in G\}$. Then show that *S* is a subgroup of $(G,*)$.

9. Obtain the Hasse diagram of $(D_{30}, |)$, where D_{30} denotes the set of positive divisors of 30 and | is the relation "division".

10. Check whether the pentagon lattice is a distributive and complemented. Justify your answer.

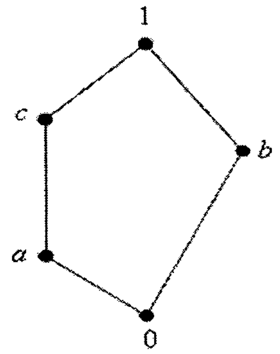

Figure Q10: Pentagon Lattice

Part – B (5× 16 = 80 Marks)

11(a)(i) Solve the recurrence relation $2a_{n+2} - 11a_{n+1} + 5a_n = 0$, where $n \geq 0, a_0 = 2$ and $a_1 = -8$. (8 Marks)

(ii) Determine the number of integers *n*, $1 \leq n \leq 1000$ that are not divisible by 2, 3 or 5. (8 Marks)

12(a)(i) Obtain the PDNF and PCNF of the formula $(P \wedge \neg(Q \wedge R)) \vee (P \rightarrow Q)$. How do you infer this formula? (10 Marks)

(ii) Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$. (6 Marks)

(OR)

12(b)(i) Show that the hypotheses, "If you send me an email message, then I will finish writing the programme", "If you do not send me an e-mail message, then I will go to sleep early", and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the programme, then I will wake up feeling refreshed". (10 Marks)

(ii) Explain the method of proof, "Proof by Contradiction". Using the Contradiction method, prove that if $n = ab$, where a and b are positive integers then $a \le \sqrt{n}$ or $b \le \sqrt{n}$. (6 Marks)

13(a)(i) When do we say two simple graphs are isomorphic? Check whether the following two graphs G and H given in ***Figure Q13a*** are isomorphic. Justify your answer. (10 Marks)

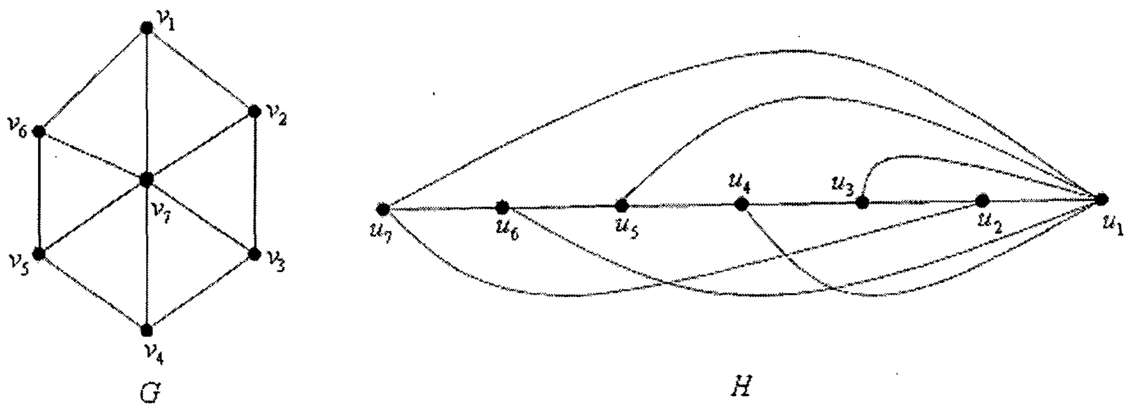

Figure Q13a

(ii) Show that if G is bipartite graph then G does not contain any odd cycle. Also show that every bipartite graph is 2-colorable. (6 Marks)

(OR)

13(b)(i) If G is a simple graph with $|V(G)| \ge 3$ and $\delta(G) \ge \frac{|V(G)|}{2}$ then prove that G is Hamiltonian. Check whether this sufficient condition is necessary for a Hamiltonian graph. Justify your answer. (12 Marks)

(ii) Give two non-isomorphic connected graphs having equal number of vertices, equal number of edges and the same degree sequence. (4 Marks)

14(a) Prove that in a finite group $(G, *)$, order of any subgroup divides the order of the group. Also show that if G is a finite group of order n then $a^n = e$, for any $a \in G$.

(16 Marks)

(OR)

14(b) Obtain all the elements of S_3 and construct its composition table with respect to the right composition of mapping $\Diamond$. Show that $(S_3, \Diamond)$ is a group but it is not an abelian group. Check whether the subgroup $H = \{p_1, p_5, p_6\}$ is a normal subgroup of $(S_3, \Diamond)$, where $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, $p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $p_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$. Justify your answer. (16 Marks)

15(a)(i) Show that the following are true in a lattice L. For $a, b, c \in L$,

1. if $b \le c$ then $a * b \le a * c$ and $a \oplus b \le a \oplus c$,
2. $a * (b \oplus c) \ge (a * b) \oplus (a * c)$ and $a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$. (10 Marks)

(ii) Show that every totally ordered set with at least three elements is a distributive lattice but not complemented lattice. (6 Marks)

(OR)

15(b)(i) Let $(L, *, \oplus)$ and $(M, \wedge, \vee)$ be two lattices. Then show that $(L \times M, \Delta, \nabla)$ is a lattice, where for $(a,b),(x,y) \in L \times M$, $(a,b)\Delta(x,y) = (a * x, b \wedge y)$ and $(a,b)\nabla(x,y) = (a \oplus x, b \vee y)$. (10 Marks)

(ii) Show that $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ hold in a complemented and distributive lattice. (6 Marks)

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