Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.Tech(ECE-NEW,PWE-NEW) / SEM-4 / PH-401 / 2012

2012

PHYSICS-II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The dimension of $\mu_0 t_0$ is

      a) $L^{-2} T^{-2}$ b) $L^{-2} T^{2}$

      c) $L T^{-1}$ d) $L^{-1} T^{-1}$.

   ii) The displacement current arises due to

      a) positive charge only

      b) negative charge only

      c) time varying electric field

      d) magnetic monopole.

iii) If the Fermi energy of metal (in $3d$) at thermal equilibrium is 15 eV, then the average energy of the electron is

a) 9 eV b) 10 eV

c) 15 eV d) 12 eV.

iv) Which of the following functions are eigen functions of the operator $\frac{d^2}{dx^2}$?

a) $\psi\ c \log x$ b) $\psi\ cx^2$

c) $\psi\ \frac{c}{x}$ d) $\psi\ ce^{-mx}$

(where c and m are arbitrary constants).

v) BE statistics is applicable for

a) ideal gas b) electron

c) proton d) photon.

vi) The degrees of freedom for a system of N particles with K constraint relation is given by

a) $N - K$ b) $N - 3K$

c) $3N - K$ d) $3(N - K)$.

vii) The number of ways in which 4 identical bosons can be distributed in 3 different energy states is

a) 15 b) 6

c) 144 d) 24.

viii) The equation of continuity essentially represents

a) conservation of mass

b) conservation of charge

c) conservation of potential

d) conservation of force.

ix) The ignorable co-ordinate corresponding to the motion of a particle under central force is given by

a) r b) θ

c) $\dot{r}$ d) $\dot{\theta}$.

x) An electric field in a certain region has the components $E_x = ax - bz$, $E_y = -ay + bz$ and $E_z = b(y - x)$. Then which of the following statements is correct ?

(a, b are positive constants)

a) $\vec{E}$ is an electrostatic field

b) There is free charge in space

c) $\vec{E}$ is irrotational

d) $\vec{E}$ is solenoidal.

xi) The vector potential $\vec{A}$ corresponding to a constant magnetic field $\vec{B}$ along z-axis can be represented by

a) $-Bz\,\hat{k}$

b) $\frac{B}{2}\left(\hat{i}x-\hat{j}y\right)$

c) $B\left(\hat{j}x-\hat{i}y\right)$

d) $\frac{B}{2}\left(\hat{j}x-\hat{i}y\right)$.

xii) Skin depth for a conductor in reference to electromagnetic wave varies

a) inversely as frequency

b) directly as frequency

c) inversely as square root of frequency

d) directly as square root of frequency.

xiii) The expectation value of the position of a particle in a one-dimensional potential box of length

$L\ (\ V(x)=0;\ 0<x<L,\ V(x)=\infty$ at $x=\neq L\ )$ is

a) L

b) $\frac{L}{2}$

c) $\frac{L}{3}$

d) $\frac{L}{4}$.

xiv) The force experienced by a charged particle in a magnetic field is independent of

a) velocity of the particle

b) strength of the field

c) charge of the particle

d) mass of the particle.

xv) The electronic polarizability (α_e) of an atom is related to its radius (R) as

a) $\alpha_e \propto R^3$ b) $\alpha_e \propto R^2$

c) $\alpha_e \propto R$ d) $\alpha_e \propto R^0$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Write down the Maxwell's equations of an electromagnetic field. Hence, obtain the wave equation for electric filed in free space. 3 + 2

3. State Stokes theorem in vector calculus. Find the unit vectors perpendicular to $x^2 + y^2 - z^2 = 100$ at the point (1, 2, 3). 2 + 3

4. State the Amperei's law of mangnetostatics. Obtain its differential form from the integral one. Apply Ampere's law of magnetostatics to deduce an expression of magnetic field B due to a straight conductor of infinite length carrying current I. 1 + 2 + 2

5. a) Four distinguishable particles each of which can be in one of the energy states ∈, 2∈, 4∈ and 6∈ having total energy 6∈. Find all possible number of distributions of all the particles in the energy states. Write the number of microstates possible and the number of microstates corresponding to each macrostate.

   b) Sketch the nature of Fermi-Dirac distribution function at $T = 0$ and $T > 0$ K in the same graph. 3 + 2

6. Show that if the Lagrangian does not depend on time, then the Hamiltonian is a constant of motion. Write down the Hamiltonian and obtain the equation of motion for a simple harmonic oscillator. 3 + 1 + 1

7. a) Find the value of $[\hat{L}_x, \hat{z}]$.

   b) Show that the eigenvalues of a Hermitian operation are real. Give an example of a Hermitian operator in quantum mechanics. 2 + 2 + 1

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Distinguish between holonomic and non-holonomic constraints. 2

b) Write down the equation of constant, specify the nature of constant and calculate the degrees of freedom in each case :

i) A particle constrained to move on the surface of a sphere

ii) A simple pendulum with a fixed suppert. 3 + 3

c) Show that if a generalized coordinate is cyclic in Lagrangian, then the corresponding generalized momentum will be conserved. 3

d) Find the equation of motion using Hamilton's canonical equation for a system comprising masses m_1 and m_2 connected by a massless string of length L through a frictionless pulley such that $m_1 > m_2$. 4

9. a) What do you mean by μ and Γ-phase space ? Find the area in the phase space of a one-dimensional harmonic oscillator of mass m whose total energy is E. 2 + 2

b) Derive Planck's radiation law from *BE* statistics. State clearly the assumptions made in the theory. 3 + 2

c) What is Fermi energy ? Calculate the degeneracy function $g(E)$ as a function of energy E for an ideal Fermi gas. 1 + 3

d) Evaluate the temperature at which there is one per cent probability that a state with energy of 0·6 eV above the Fermi energy will be occupied by an electron. 2

10. a) Give the physical interpretation of the wave of function $\psi(x)$. 2

b) Show that for a stationary state given by the wave function $\psi(x, t) = \psi(x)\, e^{-\frac{i}{\hbar}Ent}$, the expectation value of energy is equal to the energy eigenvalue. 3

c) A particle is in a cubic box with infinitely hard walls whose edges are L units long. The wave function is given by

$$\psi(x, y, z) = A \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) \sin\left(\frac{n\pi z}{L}\right).$$

Find the value of A. Find the ground state and first excited energy eigenvalues. Are they non-degenerate ? Explain. 2 + 2 + 2

d) Show that the function $\psi(x) = Cx\, e^{-\frac{x^2}{2}}$ is an eigen-function of the operator $\left(x^2 - \frac{d^2}{dx^2}\right)$. Find the corresponding eigenvalue. 3 + 1

11. a) If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, show that $2 \sin\frac{\theta}{2} = \left| \hat{a} - \hat{b} \right|$. 2

b) Show that the electric field is always perpendicular to the equipotential surface. 3

c) Show the

$$\vec{A} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$$ is

irrotational. Find ϕ such that $\vec{A} = \vec{\Delta}\phi$. 2 + 3

d) Calculate the work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 4z\hat{j} + 8y\hat{k}$ along the curve $x = t^2 + 1$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. 3

e) Show that $\int_S (ax\hat{i} + by\hat{j} + cz\hat{k}) \cdot d\vec{S} = \frac{4\pi}{3}(a + b + c)$

where S is the surface of the sphere

$x^2 + y^2 + z^2 = 1$. 2

12. a) A spherically symmetric charge distribution is given by $\rho(r) = \rho_0 \left(1 - \frac{r^2}{a^2}\right)$ for $0 \le r \le a$, ρ_0 is a constant.

$= 0$ for $r > a$

Calculate the

i) total charge

ii) the electric field intensity $\vec{E}$ and potential V both inside ($r < a$) and outside ($r > a$) regimes.

1 + 2 + 2

b) If ϕ is a scalar potential associated with the electric field $\vec{E}$ and $\vec{A}$ is the vector potential associated with the magnetic induction $\vec{B}$, show that they must satisfy the equation $\Delta^2\phi + \frac{\partial}{\partial t}\left(\vec{\Delta}.\vec{A}\right) = -\frac{\rho}{\in_0}$. 5

c) The intensity of sunlight reaching the earth's surface is about 1300 W/m^2. Calculate the strength of the electric and magnetic fields of the incoming sunlight. 3

d) N charged spherical water drops, each having a radius r and charge q, coalesce into a single big drop. What is the potential of the big spherical drop ? 2