NORMAL FORM OF A MATRIX (OR) CANONICAL FORM

If a Matrix $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ or $[I_r \; 0]$ (or) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ or $[I_r]$ are called normal form (or) Canonical form where 'I_r' is a Identity matrix of order 1 & 0 is a null matrix.

In the normal form of a matrix the rank of the matrix is 'r'

Ex:- $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2$.

→ Every $m \times n$ matrix of rank r can be reduced to the form I_r, $[I_r, 0]$ or $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ by a finite chain of elementary row or column operations, where I_r is the r-rowed unit matrix. The above form is called "normal form" or "first canonical form" of a matrix.

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Method to reduce a matrix to the canonical form

Step 1:- Make $a_{11}=1$ by applying suitable row or column operations or both.

Step 2:- Make all the elements below a_{11} in 1st column and on the right of a_{11} in the 1st row equal to zero with the help of suitable row and column operations.

Step 3:- If the resulting matrix is in the normal form, the process ends. otherwise repeat the above steps regarding a_{22} such that no row or column operation should involve 1st row or 1st column. If the matrix is still not in the normal form proceed to a_{33} and apply the same process. The process is continued till the normal form is obtained.