Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH(NEW)APM / TT / AUE / CHE / ME / PE / CE / SEM-4 / M-402 / 2012

2012

MATHEMATICS-III

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Short Answer Type Questions)

Answer any *ten* questions : 10 × 2 = 20

1. a) If $f(x) = x + x^2$, $-\pi \le x \le \pi$ be represented in a Fourier series as $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then find the value of a_0.

   b) State Fourier Integral theorem.

   c) If $F(s)$ is the Fourier transform of $f(x)$, then find the Fourier transform of $f(ax)$.

   d) Find the residue of $f(z) = \dfrac{2 + 3 \sin \pi z}{z(z-1)^2}$ at $z = 1$.

   e) Find the value of $\int_c \dfrac{z}{z-1} dz$ where C is the curve defined by $| z | = \dfrac{1}{2}$.

   f) Find the poles of the function $f(z) = z^2 / \left\{ (z-1)^2 \ (z+2) \right\}$.

g) If a Poisson variate X is such that $P(X=1)=P(X=2)$, Find $P(X=4)$.

h) For any two events A and B (may not be mutually exclusive), prove that
$P(A+B)=P(A)+P(B)-P(AB)$.

i) Find the value of p so that the function $(2x-x^2+py^2)$ is harmonic.

j) Find the value of $J_{\frac{1}{2}}(x)$.

k) Prove that $(2n+1)x P_n=(n+1)P_{n+1}+nP_{n-1}$

l) Find the ordinary and singular points of the differential equation $x^2(1+x)^2 \dfrac{d^2y}{dx^2}+(x^2-1)\dfrac{dy}{dx}+2xy=0$.

m) Find the bilinear transformation which *maps* $z=0,\ 1,\ \infty$ onto $\omega=-1,\ -i,\ 1$ respectively.

n) Find the value of $\int_{-1}^{1} P_0(x)\,dx$ where $P_n(x)$ is a Legendre's polynomial of degree n.

o) Write down the Bessel's equation of order 2.

GROUP – B

Answer any *five* questions taking at least *one* question from each Modules. : $5 \times 10 = 50$

Module I : Fourier Series and Fourier Transform

2. a) Expand $f(x)=x,\ -\pi \le x \le \pi$ in Fourier series. Hence deduce that $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\ \ldots$ 4 + 1

b) Find the Fourier sine transform of $f(x)$, where

$$f(x)=\begin{cases}1, & 0<x\le\pi\\ 0, & x>\pi\end{cases}$$

and hence evaluate the integral $\displaystyle\int_0^{\infty}\left(\frac{1-\cos p\pi}{p}\right)\sin px\,dp$.

3 + 2

3. a) Find the Fourier series of $f(x) = x^2, \ -\pi \le x \le \pi$.

Hence prove that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$. 4 + 1

b) Find the function whose Fourier cosine transform is $\frac{\sin as}{s}$. 5

Module II : Calculus of Complex variable

4. a) Using Cauchy's Residue theorem, prove that

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} = \frac{2\pi}{\sqrt{3}}.$$ 5

b) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4}\, dz$

where C is the circle $|z| = 3$. 5

5. a) If $u = x^2 - y^2$ and $v = -\frac{y}{x^2 + y^2}$, then prove that both u and v are harmonic. 5

b) Find the zeros and their orders of the function $f(z) = \frac{z^5 - 1}{z^2 + 5}$. 5

6. a) Expand $f(z) = \frac{z}{(z-1)(z-2)}$ in Laurent's series for $1 < |z| < 2$. 5

b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy-Riemann conditions are satisfied at that point. 5

Module III : Probability

7. a) Two urns contain respectively 5 white, 7 black balls and 4 while, 2 black balls. One of the urns is selected by the toss of a fair coin and then 2 balls are drawn without replacement from the selected urn. If both balls are white, what is the probability that the first urn is selected ? 5

b) X is a continuous random variable having probability dimity function $f(x) = \begin{cases} \frac{4x}{5}, & 0 < x \le 1 \\ \frac{2(3-x)}{5}, & 1 < x \le 2 \\ 0, & \text{elsewhere} \end{cases}$

Find the mean value of x and the distribution function. 3 + 2

8. a) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or by 8 ? 5

b) A random variable X has the following probability function :

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Obtain the value of K and estimate $P(X < 6)$ and $P(0 < X < 5)$. 5

Module IV : PDE and Series solution of ODE.

9. Use Laplace transform to solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ $(x > 0,\ t > 0)$

where $u(x,0) = 0,\ \frac{\partial u}{\partial t}(x,0) = 0,\ x > 0$

and $u(0,\ t) = F(t),\ u(\infty,\ t) = 0,\ t \ge 0$. 10

10. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. 10

11. a) Find the series solution of *ODE*

$$\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + x^2 y = 0 \quad \text{about } x = 0. \qquad 5$$

b) Show that $\int_{-1}^{1} x^2 p_{n-1}(x)\, p_{n+1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$

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