(b) Evaluate the following by changing the order of integeration :

$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx.$$

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BAM/A-15

MATHEMATICS
(Advanced Calculus)
Paper : BM-201

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) If $<I_n = [a_n\ b_n]>$ is a sequence of closed interval s.t.

(i) $I_{n+1} \subseteq I_n$ for all $n \in N$,

(ii) $\lim_{n\to\infty}$ (length of I_n) = 0,

then show that $\bigcap_{n\in N}^{\infty} I_n$ is a singelton set.

(b) Show that $\lim_{n\to\infty}\left[\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdots\frac{n}{n-1}\right]^{1/n} = 1.$

2. (a) Show that every absolutely convergent series is convergent. Is the converse true ? If not, show an example.

(b) Test the following series for convergence :

$$1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\cdots,\ x>0.$$

3. (a) Show that the series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ is convergent if and only if $|x| < 1$.

(b) Prove that

$$\lim_{n\to\infty}\left[\frac{(n+1)(n+2)\cdots(n+n)}{n^n}\right]^{1/n} = \frac{4}{e}.$$

SECTION–II

4. (a) Show that there is no real number 'k' for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0 1].

(b) Expand $x^4 + x^2y^2 - y^2$ about the point (1, 1) upto the term of second degree.

5. (a) Prove that between any two real roots of $e^x \sin x = x$, there is at least one real root of $\cos x + \sin x = e^{-x}$.

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cdot \cos 2u}{4\cos^3 u}.$$

6. (a) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, find the Jacobian of transformation.

(b) State and prove Langrage Mean Value (LMV) theorem.

SECTION–III

7. (a) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.

(b) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$.

8. (a) Find the envelope of the family of lines

$$x\cos\theta + y\sin\theta = a,$$

θ being the parameter.

(b) Find the value of a, b, c so that

$$\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2.$$

SECTION–IV

9. (a) Show that $B(m, n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$ if m, n are positive integers.

(b) Prove that $\left|\overline{\left(\frac{1}{2}\right)}\right. = \sqrt{\pi}$.

10. (a) Evaluate $\iint_D e^{-(x^2+y^2)}\,dx\,dy$

where D is the region bounded by $x^2 + y^2 = a^2$.

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BAM/A-15

DIFFERENTIAL EQUATIONS

Paper : BM-202

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) Obtain the power series solution of

$$\frac{d^2y}{dx^2} + (x-3)\frac{dy}{dx} + y = 0$$

in powers of $(x-2)$. (2½)

(b) Find the solution of the equation in terms of Bessel's function $\frac{d^2y}{dx^2} + \frac{1}{x}\cdot\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0$. (2½)

2. (a) Prove that

$(n+1)\,P_{n+1}(x) + nP_{n-1}(x) = (2n+1)\,xP_n(x)$. (2½)

(b) Solve the equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2dx\frac{dy}{dx} + n(n+1)y = 0$$

by changing it to a hypergeometric equation. (2½)

SECTION–II

3. (a) Find the Laplce transform of

$$f(t) = |t-1| + |t+1|, \; t \geq 0. \qquad (2½)$$

(b) State and prove Convolution theorem. (2½)

4. (a) Solve the integral equation

$$f(t) = 1 + \int_0^t f(u) \sin(t-u)\, du$$

and verify your solution. (2½)

(b) Solve the initial value problem

$$\frac{d^2y}{dt} + y = 6\cos 2t, \; y'(0) = 1, \; y(0) = 3. \qquad (2½)$$

SECTION–III

5. (a) Form a partial differential equation by eliminating the function f from $f(x+y+z, \;\; x^2+y^2-z^2) = 0$. (2½)

(b) Solve the partial differential equation

$$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx. \qquad (2½)$$

6. (a) Solve the differential equation

$$z = px + qy + p^2 - q^2. \qquad (2½)$$

(b) Find the complete integral of the equation using Charpit's method

$$(p^2 + q^2)y = qz. \qquad (2½)$$

7. (a) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x+2y). \qquad (2½)$$

(b) Solve the equation

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} - x\frac{\partial z}{\partial x} = \frac{x^3}{y^2}. \qquad (2½)$$

SECTION–IV

8. (a) Find the external of the functional

$$\int_0^1 [(y')^2 + 12xy]\; dx. \qquad (2½)$$

(b) Find the extremals of the functional

$$\int_0^{\pi/2} (y''^2 - y^2 + x^2)\, dx$$

that satisfies the conditions

$$y(0) = 1, \; y'(0) = 0, \;\; y(\pi/2) = 0, \; y'\pi/2 = -1. \qquad (2½)$$

9. Find the shortest distance between the circle $x^2 + y^2 = 1$ and the straight line $x + y = 4$. (5)

10. Find the shortest distance of the point (0, 2, 4) to the straight line

$$\frac{x-1}{1} = \frac{y}{3} = \frac{z}{4}. \qquad (5)$$

10. (a) A heavy particle moves in a smooth sphere. Show that if the velocity be that due to the level of the centre, the reaction of the surface will vary at the depth below the centre.

(b) A particle moves with a central acceleration $\frac{\lambda}{(\text{distance})^3}$. Find the path and disinguish the cases.

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BAM/A-15

MECHANICS

Paper : BM-203

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) Show that if three forces acting on a rigid body keep it in equilibrium, they must be co-planar.

(b) Prove that a system of co-planar forces shall be in equilibrium if the algebraic sum of the moments of all the forces about any three non-collinear points in their plane vanish separately.

2. (a) The virtual works done by the tension in a virtual extension of a string from length l to $l + \delta l$ is $-T.\delta l$, where T is the tension in the string.

(b) A heavy uniform rod of length $2a$ rests with its ends in contact with two smooth inclined planes of inclination α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principle of virtual work that

$$\tan\theta = \frac{1}{2}[\cot\alpha - \cot\beta].$$

3. (a) A hemisphere of radius 'a' and weight W is placed with its curved surface on a smooth table and a string of length $l(< a)$ is attached to a point on its rim and to a point on the table. Find the position of equilibrium and prove that the tension of the string is $\frac{3W}{8} \cdot \frac{a-l}{\sqrt{2al - l^2}}$.

(b) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Prove that its tension is 3W.

SECTION–II

4. (a) Show that any system of forces acting on a rigid body can be reduced in general to a force acting at an arbitrary chosen point of the body and a couple.

(b) Find the equation of the conjugate line of the given line $\frac{x-f}{l} = \frac{y-g}{m} = \frac{z-h}{n}$.

5. (a) A uniform beam of thickness $2b$, rests symmetrically on a perfect rough horizontal cylinder of radius a. Show that equilibrium of the beam will be stable or unstable according as b is less or greater than a.

(b) Show that the maximum distance between two forces which are equivalent to a given system (R, K) and which are inclined at a given angle 2α is $\frac{2K}{R} \cot \alpha$ and the forces are then each equal to $\left(\frac{R}{2}\right) \sec \alpha$.

SECTION–III

6. (a) A particle moves along a circle $r = 2a \cos \theta$ in such a way that its acceleration towards the origin is always zero. Show that the transverse acceleration varies as the fifth power of cosec θ.

(b) A particle moving with S.H.M. of period 12 seconds travels 10 cm from the position of rest in 2 seconds. Find the amplitude, the maximum velocity and the velocity at the end of 2 seconds.

7. (a) Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions.

(b) The rate of change of direction of the velocity of a particle moving in a cycloid is constant. Prove that acceleration must be constant in magnitude.

8. (a) A heavy particle of mass 'm' is made to move on a smooth curve in a vertical plane. Discuss its motion.

(b) A bead moves along a rough curved wire which is such that it changes its direction of motion with constant angular velocity. Show that a possible form of wire is an equiangular spiral.

SECTION–IV

9. (a) Find the differential equation of central orbit in polar form.

(b) The greatest and least velocities of a certain planet in its orbit round the sum are 30 km/sec. and 29.2 km/sec. respectively. Find the eccentricity of the orbit.