Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

Seat No.	

[4162]-101

S.E. (Civil) (First Semester) EXAMINATION, 2012

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator and steam tables is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^x \cos 2x$$

(*ii*) $\frac{d^3y}{dx^3} + y = \sin(2x + 3)$

(*iii*) $\frac{d^2y}{dx^2} + 4y = \sec^2(2x)$, (by the method of variation of parameters)

(*iv*) $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$

(*b*) Solve the following simultaneous linear differential equations :

$$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} = 2x + 5y.$$ [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - y = e^x \cosh 3x$

(*ii*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

(*iii*) $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = x^2$

(*iv*) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ (by the method of variation of parameters)

(*b*) Solve :

$$\frac{dx}{x(y - z)} = \frac{dy}{y(z - x)} = \frac{dz}{z(x - y)}.$$ [5]

3. (*a*) The deflection of a strut of length L with one end at $x = 0$ fixed and the other end supported and subjected to end thrust P, statisfies the equation :

$$\frac{d^2y}{dx^2} + a^2y = \frac{a^2\text{R}}{\text{P}}(\text{L} - x),$$

where a, R, P are constants. Prove that the deflection of the curve is given by :

$$y = \frac{\text{R}}{\text{P}}\left(\frac{\sin ax}{a} - \text{L}\cos ax + \text{L} - x\right),$$

where $a\text{L} = \tan(a\text{L})$. [8]

(*b*) The temperature at any point of an insulated metal rod is governed by the differential equation, provided that the length of rod is L,

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}.$$

Find $u(x, t)$, subject to the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u_t(\text{L}, t) = 0$

(*iii*) $u(x, 0) = \dfrac{u_0 x}{\text{L}}$

where $u_t = \dfrac{\partial u}{\partial t}$. [8]

Or

4. (*a*) Solve the Laplace's equation :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0,$$

subject to the conditions :

(*i*) $z(0, y) = 0$

(*ii*) $z(1, y) = 0$

(*iii*) $z(x, \infty) = 0$

(*iv*) $z(x, 0) = \sin^3(\pi x), \quad 0 < x < 1.$ [8]

(*b*) A body weighing 20 kg is hung from a spring. A pull of 40 kg will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time t seconds, the maximum velocity and period of oscillation. [8]

5. (*a*) Using Gauss-Seidel method, solve the following system of equations starting with initial values as $x = y = z = 0$ where :

$$5x - y = 9,$$

$$x - 5y + z + 4 = 0,$$

$$y - 5z = 6.$$ [9]

(*b*) Use Runge-Kutta method of 4th order to solve :

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

to find y at 0.2 with $h = 0.1$. [8]

Or

6. (*a*) Solve :

$$\frac{dy}{dx} = y - \frac{2x}{y}, \quad y(0) = 1$$

in the range $0 \leq x \leq 0.2$ using modified Euler's method with $h = 0.1$. [9]

(*b*) Solve the system of the following algebraic equations by Gauss elimination method :

$$x + 9y - 6z = 1$$

$$2x - 7y + 4z = 9$$

$$3x - 8y - 5z + 6 = 0.$$

[8]

SECTION II

7. (*a*) Lives of two models of refrigerators turned for new models in a recent years are :

Life (No. of Years)	**No. of Refrigerators**	
	Model A	**Model B**
0—2	5	2
2—4	16	7
4—6	13	12
6—8	7	9
8—10	5	9
10—12	4	1

Find which model has more uniformity ? [6]

(*b*) Obtain the correlation between population density (per square mile) and death rate (per thousand persons) from the data to 5 cities : [6]

Population Density	**Death Rate**
200	12
500	18
400	16
700	21
300	10

(*c*) 10 coins thrown simultaneously. Find the probability of getting :

(*i*) 8 heads

(*ii*) At least 8 heads. [5]

Or

8. (*a*) Find the four moments about the mean of the following :

x	f
61	5
64	18
67	42
70	27
73	8

Also find β_1 and β_2. [6]

(*b*) In a certain factory turning out razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10,000 packets. [6]

(*c*) A random sample of 200 screws is drawn from a population which represents the size of screws. If a sample is distributed normally with a mean 3.15 cm and standard deviation 0.025 cm, find expected number of screws whose size falls between 3.12 cm and 3.2 cm.

Given :

Area corresponding to 1.2 is 0.3849.

Area corresponding to 2.0 is 0.4772. [5]

9. (*a*) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = t$, where t is the time. Find the velocity and acceleration at time $t = 1$. [4]

(*b*) Find the directional derivative of $\phi = 2xz^4 - x^2y$ at the point (2, –2, 1) in the direction of the tangent to the curve $x = e^t$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$. [5]

(*c*) Prove the following (any *two*) : [8]

(*i*) $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})}{r^{n+2}}\,\bar{r}$

(*ii*) $\nabla^4\left(r^2 \log r\right) = \frac{6}{r^2}$

(*iii*) $\bar{a}\cdot\nabla\left[\bar{b}\cdot\nabla\left(\frac{1}{r}\right)\right] = \frac{3(\bar{a}\cdot\bar{r})(\bar{b}\cdot\bar{r})}{r^5} - \frac{\bar{a}\cdot\bar{b}}{r^3}.$

Or

10. (*a*) Verify whether the vector field given by :

$$\bar{F} = \left(y^2\cos x + z^3\right)\bar{i} + (2y\sin x - 4)\,\bar{j} + \left(3xz^2 + 2\right)\bar{k}$$

is irrotational. If so find corresponding scalar potential ϕ such that $\bar{F} = \nabla\phi$. [6]

(*b*) Find the directional derivative of $xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x - 2) = (y + 1) = (z - 1)$. [5]

(*c*) If

$$\bar{u} = 2xy^3\,\bar{i} + 3xyz^2\,\bar{j} - x^2yz\,\bar{k}$$

and $\phi = 3x^2 - yz$, then find :

(*i*) $\bar{u}\cdot\nabla\phi$

(*ii*)

at $(1, 2, -1)$. [6]

11. (*a*) If

$$\overline{F} = \left(2xy + 3z^2\right)\overline{i} + \left(x^2 + 4yz\right)\overline{j} + \left(2y^2 + 6xz\right)\overline{k},$$

evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where C is the curve $x = t, \; y = t^2, \; z = t^3$ joining the points (0, 0, 0) and (1, 1, 1). [6]

(*b*) Evaluate by Stokes' theorem

$$\iint_S \nabla \times \overline{F} \cdot \hat{n} \, dS$$

where S is the surface of paraboloid $z = 4 - x^2 - y^2$ $(z \geq 0)$ and $\overline{F} = y^2\overline{i} + z\overline{j} + xy\overline{k}$. [5]

(*c*) Evaluate :

$$\iint_S \left(x\overline{i} + y\overline{j} + z^2\overline{k}\right) \cdot d\overline{S}$$

where S is the curved surface of cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 2$. [5]

Or

12. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r} \quad \text{for} \quad \overline{F} = \sin y\,\overline{i} + x(1 + \cos y)\,\overline{j}$$

where C is the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad z = 0.$$ [5]

(*b*) Evaluate :

$$\iint_S \overline{r} \cdot \hat{n}\, dS$$

over the surface of a sphere of radius 1 with centre at origin. [5]

(*c*) Show that the velocity potential

$$\phi = \frac{1}{2}a\left(x^2 + y^2 - 2z^2\right)$$

satisfies the Laplace's equation. Also determine the stream lines. [6]