	Utech
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### **MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

### **GROUP - A**

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following:

 $10 \times 1 = 10$ 

- i) The rank of the matrix  $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 2 \\ 1 & 4 & 3 \end{bmatrix}$  is
  - a) 3

b) 5/2

c) 2

- d) 1.
- ii) In the Newton's forward interpolation formula the value of  $u = (x x_0)/h$  lies between
  - a) 1 and 2
- b) 1 and 1
- c) 0 and ∞
- d) 0 and 1.

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- The eigenvalues of a matrix A are 2 and 4. Then the eigenvalues of  $A^{-1}$  are
  - 2, 4 a)

- 4, 4 b)
- c) 0.25, 0.5
- d) 0.5, 0.25.
- The value of a for which the vectors (1, 2, 1), (a, 1, 1)and (1, 1, 2) are linearly dependent is
  - a) 2

- 1 c)

- d)  $\frac{3}{2}$ .
- The value of the determinant  $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix}$  is v)
  - 0 a)

b) abc

– abc c)

- 2 abc. d)
- If the system of equations 4x + 2y 5z = 0,  $x + \lambda y + 2z = 0$ , 2x + y - z = 0 has a non-zero solution, then  $\lambda$  is

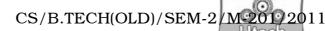
c)

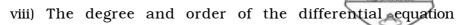
- d) none of these.
- The integrating factor of  $\frac{dy}{dx} + y = 1$  is
  - $e^{\chi}$ a)

b)  $\boldsymbol{x}$ 

 $e^2$ c)

d) 2.





$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = y^5$$
 are



ix) The general solution of 
$$y = px + f(p)$$
, where  $p = \frac{dy}{dx}$ 

a) 
$$y = c^2 x + f(c)$$
 b)  $y = cx + f(c^2)$ 

b) 
$$y = cx + f(c^2)$$

c) 
$$y = cx + f(c)$$

c) 
$$y = cx + f(c)$$
 d)  $y = cx^2 + f'(c)$ .

x) The value of the integral 
$$\int_{0}^{\infty} e^{5t} t^3 dt$$
 is

a) 
$$\frac{1}{625}$$

b) 
$$\frac{6}{625}$$

c) 
$$\frac{6}{25}$$

a) 
$$\frac{s}{s^2 - a^2}$$

b) 
$$\frac{s}{s^2 + a^2}$$

c) 
$$\frac{a}{s^2 + a^2}$$

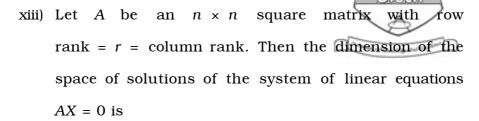
d) 
$$\frac{1}{s^2 - a^2}$$
.

#### The relation between E and $\Delta$ is xii)

a) 
$$E = 1 + \Delta$$

b) 
$$E = 1 - \Delta$$

c) 
$$E = \Delta - 1$$



a) r

b) n-i

c) m-r

d) min (m, r) - r.

#### GROUP - B

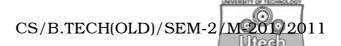
## (Short Answer Type Questions)

Answer any *three* of the following.  $3 \times 5 = 15$ 

- 2. Solve the differential equation by Laplace transform  $\frac{d^2y(t)}{dt^2} + 4y(t) = \sin t, y(0) = \frac{dy(0)}{dx} = 0.$
- 3. a) If  $S_1$  and  $S_2$  are two subspaces of a vector space V, then prove that  $S_1 \, \cap \, S_2$  is a subspace of V.
  - b) Show that  $S = \{ (x, y, z) \in \mathbb{R}^3 : x 3y + 4z = 0 \}$  is a subspace of  $\mathbb{R}^3$ .
- 4. Show that

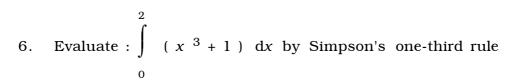
$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1+d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

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5. Solve:

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} + y = e^{t} \\ \frac{\mathrm{d}y}{\mathrm{d}t} - x = e^{-t} \end{cases}$$



taking 4 intervals.

#### **GROUP - C**

## (Long Answer Type Questions)

Answer any *three* of the following.  $3 \times 15 = 45$ 

7. a) Investigate for what values of  $\lambda$  and  $\mu$  the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

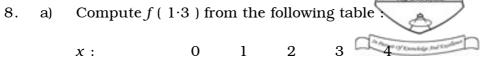
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

c) Show that every square matrix can be expressed as a sum of a symmetric and skew-symmetric matrix.



$$f(x):$$
 1 1.5 2.2 3.1 4.3.

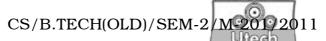
b) Solve by the method of variation of parameters

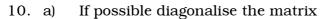
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3e^{-x} + x$$

- c) If  $\alpha$ ,  $\beta$  and  $\gamma$  form a basis of a vector space V, then prove that  $\alpha + \beta + \gamma$ ,  $\beta + \gamma$  and  $\gamma$  also form a basis of V.
- 9. a) Show that the matrix

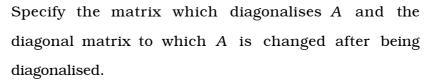
$$A = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
 is orthogonal and hence obtain  $A^{-1}$ .

- b) Using the method of separation of symbols, prove that  $u_0 + u_1 + u_2 + \dots + u_n = {^{n+1}} C_1 u_0 + {^{n+1}} C_2 \Delta u_0 + \dots + {^{n+1}} C_3 \Delta^2 u_0 + \dots + {^{n+1}} C_{n+1} \Delta^n u_0.$
- c) Show that  $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle/ \begin{matrix} a, b, c, d \in R \\ a = d \end{matrix} \right\} \text{ is a subspace of the }$  vector space of  $2 \times 2$  real matrices. Obtain a basis and dimension of M.





$$A = \left(\begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{array}\right),$$



- b) Find  $L\left(\frac{1-e^t}{t}\right)$ .
- c) Assuming orthogonal property of Legendre function, prove that

$$\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}.$$

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