Name : ....

Roll No. : ....

Invigilator's Signature : ....

2011

MATHEMATICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The rank of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 2 \\ 1 & 4 & 3 \end{bmatrix}$ is

a) 3 b) 5/2

c) 2 d) 1.

ii) In the Newton's forward interpolation formula the value of $u = (x - x_0) / h$ lies between

a) 1 and 2 b) – 1 and 1

c) 0 and ∞ d) 0 and 1.

iii) The eigenvalues of a matrix A are 2 and 4. Then the eigenvalues of A^{-1} are

a) 2, 4 b) 4, 4

c) 0·25, 0·5 d) 0·5, 0·25.

iv) The value of a for which the vectors (1, 2, 1), (a, 1, 1) and (1, 1, 2) are linearly dependent is

a) 2 b) $\frac{2}{3}$

c) -1 d) $\frac{3}{2}$.

v) The value of the determinant $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix}$ is

a) 0 b) abc

c) $-abc$ d) $2abc$.

vi) If the system of equations $4x + 2y - 5z = 0$, $x + \lambda y + 2z = 0$, $2x + y - z = 0$ has a non-zero solution, then λ is

a) $\frac{1}{3}$ b) $\frac{1}{2}$

c) 1 d) none of these.

vii) The integrating factor of $\frac{dy}{dx} + y = 1$ is

a) e^x b) x

c) e^2 d) 2.

viii) The degree and order of the differential equation

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = y^5$ are

a) 1, 2 b) 3, 2

c) 5, 2 d) 2, 1.

ix) The general solution of $y = px + f(p)$, where $p = \frac{dy}{dx}$ is

a) $y = c^2 x + f(c)$ b) $y = cx + f(c^2)$

c) $y = cx + f(c)$ d) $y = cx^2 + f'(c)$.

x) The value of the integral $\int_0^{\infty} e^{5t} t^3 \, dt$ is

a) $\frac{1}{625}$ b) $\frac{6}{625}$

c) $\frac{6}{25}$ d) none of these.

xi) Laplace transform of the function $\cos at$ is

a) $\frac{s}{s^2 - a^2}$ b) $\frac{s}{s^2 + a^2}$

c) $\frac{a}{s^2 + a^2}$ d) $\frac{1}{s^2 - a^2}$.

xii) The relation between E and Δ is

a) $E \equiv 1 + \Delta$ b) $E \equiv 1 - \Delta$

c) $E \equiv \Delta - 1$ d) none of these.

xiii) Let A be an $n \times n$ square matrix with row rank = r = column rank. Then the dimension of the space of solutions of the system of linear equations $AX = 0$ is

a) r b) $n - r$

c) $m - r$ d) $\min(m, r) - r$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the differential equation by Laplace transform $\frac{d^2y(t)}{dt^2} + 4y(t) = \sin t,\ y(0) = \frac{dy(0)}{dx} = 0.$

3. a) If S_1 and S_2 are two subspaces of a vector space V, then prove that $S_1 \cap S_2$ is a subspace of V.

   b) Show that $S = \{(x, y, z) \in R^3 : x - 3y + 4z = 0\}$ is a subspace of R^3.

4. Show that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

5. Solve :

$$\begin{cases} \dfrac{dx}{dt} + y = e^{t} \\ \dfrac{dy}{dt} - x = e^{-t} \end{cases}$$

6. Evaluate : $\int_0^2 (x^3 + 1)\,dx$ by Simpson's one-third rule taking 4 intervals.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Investigate for what values of λ and μ the following equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

c) Show that every square matrix can be expressed as a sum of a symmetric and skew-symmetric matrix.

8. a) Compute $f(1\cdot3)$ from the following table :

x :	0	1	2	3	4
$f(x)$:	1	1·5	2·2	3·1	4·3.

b) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3e^{-x} + x$$

c) If α, β and γ form a basis of a vector space V, then prove that $\alpha + \beta + \gamma$, $\beta + \gamma$ and γ also form a basis of V.

9. a) Show that the matrix

$$A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$ is orthogonal and hence obtain A^{-1}.

b) Using the method of separation of symbols, prove that

$$u_0 + u_1 + u_2 + \dots\dots + u_n = {}^{n+1}C_1\, u_0 + {}^{n+1}C_2\, \Delta u_0 + {}^{n+1}C_3\, \Delta^2 u_0 + \dots\dots\dots + {}^{n+1}C_{n+1}\, \Delta^n u_0 .$$

c) Show that

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Big/ \begin{matrix} a, b, c, d \in R \\ a = d \end{matrix} \right\}$$ is a subspace of the vector space of 2 × 2 real matrices. Obtain a basis and dimension of M.

10. a) If possible diagonalise the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix},$$

Specify the matrix which diagonalises A and the diagonal matrix to which A is changed after being diagonalised.

b) Find $L\left(\frac{1-e^t}{t}\right)$.

c) Assuming orthogonal property of Legendre function, prove that

$$\int_{-1}^{1} [P_n(x)]^2 \, dx = \frac{2}{2n+1} .$$